Part III: Uncertainty and Strategy

7. Uncertainty
8. Game Theory

Chapter 8
Game Theory
Part II

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Repeated Games

- The simple constituent game (e.g. the Prisoners' Dilemma) that is played repeatedly is called the *stage game*.
- Repeated play of the stage game opens up the possibility of cooperation in equilibrium.
- Players can adopt *trigger strategies*, whereby they continue to cooperate as long as everyone else does, but revert to playing the Nash equilibrium if anyone deviates from cooperation.
- We will investigate the conditions under which trigger strategies work to increase players' payoffs.

Finitely repeated games

- Repeating a stage game for a known, finite number of times does not increase the possibility for cooperation. This can be shown by backward induction.
- Reinhard Selten, winner of the Nobel Prize for his contributions to game theory, showed that for any stage game with a unique Nash equlibrium, the unique subgame-perfect equilibrium of the finitely repeated game involves playing the Nash equilibrium every period.
- If the stage game has multiple Nash equilibria, it may be possible to achieve cooperation in a finitely repeated game.
- Players can use trigger strategies to maintain cooperation by threatening to play the Nash equilibrium that yields a worse outcome for the player who deviates from cooperation.

Infinitely repeated games

- With infinitely repeated games, there is no definite ending period T from which to start backward induction.
- The crucial issue with an infinitely repeated game is not that it goes on forever but that its end is indeterminate.
- Players can sustain cooperation in infinitely repeated games using trigger strategies. The trigger strategy must be severe enough to deter the deviation.
- Suppose both players use the following specific trigger strategy in the Prisoners' Dilemma:

  Continue being silent if no one has deviated; fink forever afterward if anyone has deviated to fink in the past.
- To show that this trigger strategy forms a subgame-perfect equilibrium, we need to check that a player can not gain from deviating.

Suspect 1 \ Suspect 2	Fink	Silent
Fink	$U_1 = 1, U_2 = 1$	$U_1 = 3, U_2 = 0$
Silent	$U_1 = 0, U_2 = 3$	$U_1 = 2, U_2 = 2$

- If both players are silent every period, the present discount value of payoffs, with discount factor δ, over time would be

$$\begin{aligned} V^{eq} &= 2 + 2\delta + 2\delta^2 + 2\delta^3 + \cdots \\ &= 2(1 + \delta + \delta^2 + \delta^3 + \cdots) = \frac{2}{1-\delta} \end{aligned}$$

- If a player deviates and then the other finks every period, that player's payoff is

$$\begin{aligned} V^{dev} &= 3 + 1 \cdot \delta + 1 \cdot \delta^2 + \cdots \\ &= 3 + \delta(1 + \delta + \delta^2 + \cdots) = 3 + \frac{\delta}{1-\delta} \end{aligned}$$

- The trigger strategies form a subgame-perfect equilibrium if $V^{eq} \geq V^{dev}$, implying that

$$\frac{2}{1-\delta} \geq 3 + \frac{\delta}{1-\delta}, \; or \; \delta \geq \frac{1}{2}$$

Continued cooperative play is desirable provided players do not discount future gains from cooperation too highly.

- If $\delta < 1/2$, no cooperation is possible in the infinitely repeated Prisoner's Dilemma; the only subgame-perfect equilibrium involves finking every period.
- The trigger strategy in which players revert to stage-game Nash equilibrium forever is the harshest punishment possible is called the *grim strategy*.
- Less harsh punishment include the so-called *tit-for-tat strategy*, which involves only one round of punishment for cheating.

- The grim strategy elicits cooperation for the largest range of cases (the lowest value of δ) of any strategy.
- As δ approaches 1, grim-strategy punishments become infinitely harsh because they involve an unending stream of undiscounted losses.
- Infinite punishments can be used to sustain a wide range of possible outcomes. This is the logic behind the *folk theorem for infinitely repeated games.*
- Take any stage-game payoff for a player between Nash equilibrium one and the highest one, with payoff V, that appears anywhere in the payoff matrix, the folk theorem says that the player can earn V in some subgame-perfect equilibrium for δ close enough to 1.

Incomplete Information

- In the games studies thus far, players knew everything there was to know about the setup of the game, including each others' strategy sets and payoffs.
- Matters become more complicated, and potentially more interesting, if some players have information about the game that others do not.
- Games of incomplete information can quickly become complicated. Players that lack full information will use what they do know to make inferences about what they do not.
- Probability theory provides a formula, called *Bayes' rule*, for making inferences about hidden information.
- The relevance of Bayes' rule in games of incomplete information has led them to be called *Bayesian games*.

- To limit the complexity of the analysis, we will focus on two-player games in which one of the players (player 1) has private information and the other (player 2) does not.
- The next section begins with the simple case in which the players move simultaneously.
- The subsequent section then analyzes games in which the informed player 1 move first.
- Such games, called *signaling games*, are more complicated than simultaneous games because player 1's action may signal something about his or her private information to uninformed player 2.
- We will introduce Bayes' rule at that point to help analyze player 2's inference about player 1's hidden information based on observations of player 1's ation.

Simultaneous Bayesian Games

- A two-player, simultaneous-move game in which player 1 has private information, but player 2 does not.
- We use “he" for player 1 and “she" for player 2 to facilitate exposition.

Player types and beliefs

- Player 1 can be of a number of possible *types*, denoted t. Players knows his own type.
- Player 2 is uncertain about t and must decide on her strategy based on beliefs about t.
- The game begins at a chance node, at which a value of t_k is drawn for player 1 from a set of possible types, $T = \{t_1, \cdots, t_k, \cdots, t_K\}$.

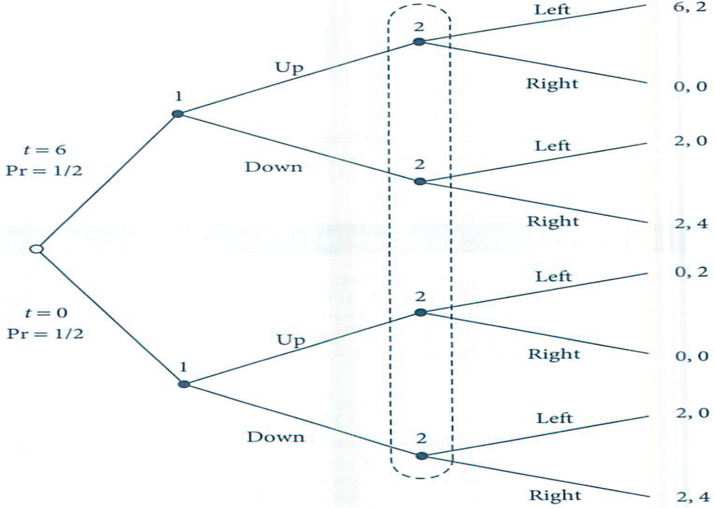
- Let $Pr(t_k)$ be the probability of drawing type t_k.
- Player 1 sees which type is drawn. Player 2 does not see and only knows the probabilities, $Pr(t_k)$.
- Since player 1 observes t before moving, his strategy can be conditioned on t. Let $s_1(t)$ be player 1's strategy contingent on his type.
- Player 2's strategy is an unconditional one, s_2.
- We write player 1's payoff as $U_1(s_1(t), s_2, t)$ and player 2's as $U_2(s_2, s_1(t), t)$.
- Note that Player 1's type may affect player 2's payoff in two ways: directly and indirectly through player 1's strategy.

Figure 8.13 Simple Game of Incomplete Information

		Player 2	
		Left	Right
Player 1	Up	t, 2	0, 0
	Down	2, 0	2, 4

- All payoffs are known to both players except for t in the upper left.
- Player 2 only knows the distribution: an equal chance (1/2. 1/2) that t = 0 or t = 6.
- Player 1 knows the realized value of t, equivalent to knowing his type.

Figure 8.14 Extensive Form for Simple Game of Incomplete Information



Bayesian-Nash equilibrium

- Equilibrium requires that player 1's strategy be the best response for each and every one of his types.
- Equilibrium also requires player 2's strategy maximize an expected payoff, where the expectation is taken with respect to her beliefs about player 1's type.
- The calculations involved in computing the best response to the pure strategies of different types of rivals in a game of incomplete information are similar to the calculations involved in computing the best response to a rival's mixed strategy in a game of complete information.

- **Bayesian-Nash equilibrium**. In a two-player simultaneous move game in which player 1 has private information, a Bayesian-Nash equilibrium is a strategy profile $(s_1^*(t), s_2^*)$ such that $s_1^*(t)$ is a best response to s_2^* for each type $t \in T$ of player 1,

$$U_1(s_1^*(t), s_2^*, t) \geq U_1(s_1', s_2^*, t) \text{ for all } s_1' \in S_1,$$

and such that s_2^* is a best response to $s_1^*(t)$ given player 2's beliefs $Pr(t_k)$ about player 1's types:

$$\begin{aligned} & \sum_{t_k \in T} Pr(t_k) U_2(s_2^*, s_1^*(t_k), t_k) \\ \geq \quad & \sum_{t_k \in T} Pr(t_k) U_2(s_2', s_1^*(t_k), t_k) \text{ for all } s_2' \in S_2 \end{aligned}$$

Example 8.5 Bayesian-Nash Equilibrium of Game in Figure 8.14

First solve for the informed player's (player 1's) best response for each of his types. There are two possible candidates for an equilibrium in pure strategies.

- 1 plays (Up|t=6, Down|t=0) and 2 plays Left

  This is not an equilibrium because player 2 would gain by deviating to right, earning an expected payoff of 2, instead of earning an expected payoff of 1.

- 1 plays (Down|t=6, Down|t=0) and 2 plays Right

  This is a Bayesian-Nash equilibrium because both would not gain by deviating to the other strategy.

Example 8.6 Tragedy of the Commons as a Bayesian Game

- For an example with continuous actions, suppose that Herder 1 has private information regarding his value of grazing per sheep,

$$v_1(q_1, q_2, t) = t - (q_1 + q_2)$$

where herder 1's type is $t = 130$ ("high" type) with probability 2/3 and $t = 100$ ("low" type) with probability 1/3.
- Herder 2's value remains the same as

$$v_2(q_1, q_2) = 120 - q_1 - q_2$$

- Herder 1's value-maximization problem is

$$\max_{q_1}\{q_1 v_1(q_1, q_2, t)\} = \max_{q_1}\{q_1(t - q_1 - q_2)\}.$$

The first-order condition for a maximum is

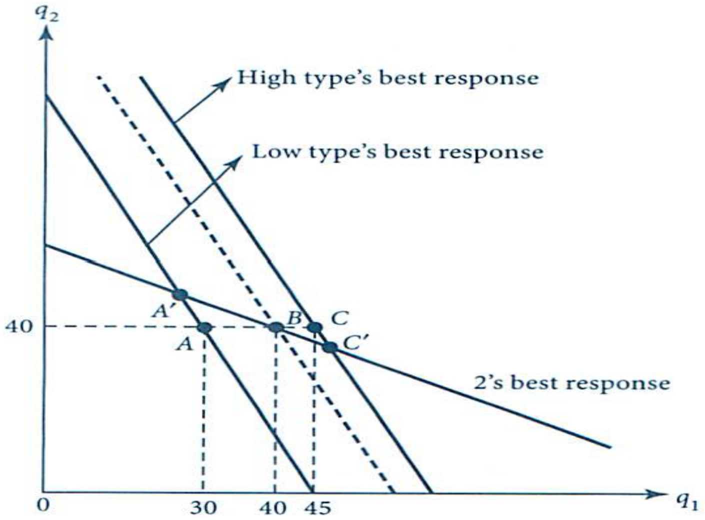
$$t - 2q_1 - q_2 = 0$$

- Therefore,
$$q_{1H} = 65 - \frac{q_2}{2}, q_{1L} = 50 - \frac{q_2}{2}$$
where q_{1H} and q_{1L} are the “high type" and “low type" of herder 1 respectively.

- For herder 2's best response. Herder 2's expected payoff is
$$\frac{2}{3}q_2(120 - q_{1H} - q_2) + \frac{1}{3}q_2(120 - q_{1L} - q_2) = q_2(120 - \bar{q}_1 - q_2),$$
where $\bar{q}_1 = \frac{2}{3}q_{1H} + \frac{1}{3}q_{1L}$.

- The first-order condition for maximizing herder 2's expected payoff is
$$q_2 = 60 - \frac{\bar{q}_1}{2} = 60 - \frac{60 - \frac{q_2}{2}}{2} = 30 + \frac{q_2}{4},$$
implying that $q_2^* = 40$ and $q_{1H} = 45, q_{1L} = 30$.

Figure 8.15 Equilibrium of the Bayesian Tragedy of the Commons



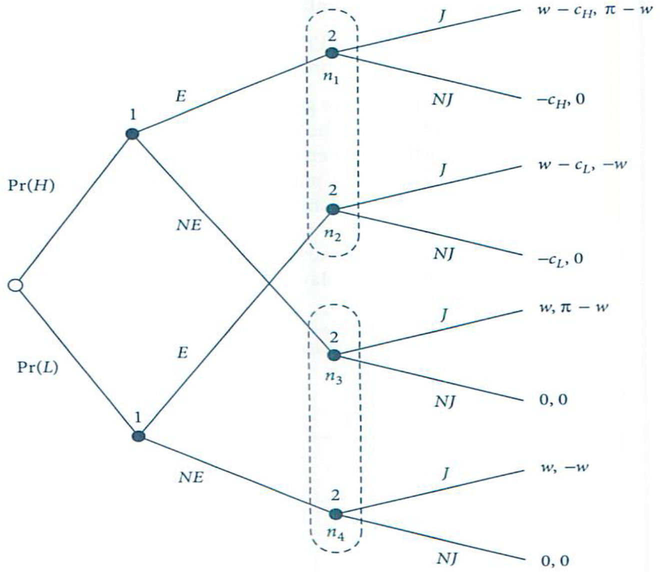
Signaling Games

- In this section we move from simultaneous-move games of private information to sequential game in which the informed player, player 1, takes an action that is observable to player 2 before player 2 moves.
- Player 1's action can serve as a signal to player 2 that can be used to update her beliefs about player 1's type.
- For example, a prestigious college degree may signal that a job applicant is highly skilled.
- Signaling games are more complicated because we need to model how player 2 processes the information in player 1's signal and then updates her beliefs about player 1's type.

Job-market signaling

- A version of Michael Spence's model of job-market signalling, for which he won the Nobel Price in economics.
- Player 1 is a worker who can be of two types, high skilled ($t = H$) and low skilled ($t = L$).
- Player 2 is a firm considering hiring player 1.
- Low-skilled worker generates no revenue for the firm while high-skilled worker generates revenue of π.
- If a worker is hired, the firm must pay the worker w, with $\pi > w > 0$.
- The firm cannot observe the worker's skill, only his education level.
- Let c_L be the cost of obtaining an education for the low type. Let c_H be the cost of obtaining an education for the high type, and $c_H < c_L$.

Figure 8.16 Job-Market Signaling



- Player 1 observes his type at the start; player 2 observes only player 1's action (educational signal) before moving.
- Let $Pr(H)$ and $Pr(L)$ be player 2's beliefs before observing player 1's educational signal that player 1 is high- or low-skilled. These are called player 2's *prior beliefs*.
- Observing player 1's action will lead player 2 to revise his or her beliefs to form what are called *posterior beliefs*.
- Suppose player 2 sees player 1 choose E, then player 2's expected pay-off from playing J is

$$
\begin{aligned}
& Pr(H|E)(\pi - w) + Pr(L|E)(-w) \\
= \quad & Pr(H|E)\pi - (Pr(H|E) + Pr(L|E)) \cdot w = Pr(H|E)\pi - w
\end{aligned}
$$

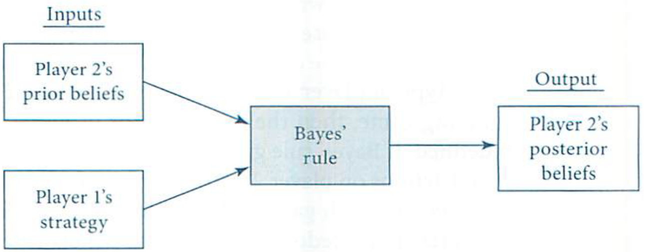
Player 2's payoff from playing NJ is 0. Therefore, player 2's best response is J if and only if $Pr(H|E) > \frac{w}{\pi}$.

Bayes' rule

- Bayes' rule gives the following formula for computing player 2's posterior belief $Pr(H|E)$ and $Pr(H|NE)$;

$$
\begin{aligned}
Pr(H|E) &= \frac{Pr(E|H)Pr(H)}{Pr(E|H)Pr(H) + Pr(E|L)Pr(L)} \\
Pr(H|NE) &= \frac{Pr(NE|H)Pr(H)}{Pr(NE|H)Pr(H) + Pr(NE|L)Pr(L)}
\end{aligned}
$$

Figure 8.17 Bayes' Rule as a Black Box



- When player 1 plays a pure strategy, Bayes' rule often gives a simple result. Suppose that $Pr(E|H) = 1$ and $Pr(E|L) = 0$, that is, player 1 obtains an education if and only if he or she is high-skilled.

$$Pr(H|E) = \frac{1 \cdot Pr(H)}{1 \cdot Pr(H) + 0 \cdot Pr(L)} = 1$$

- Suppose that $Pr(E|H) = Pr(E|L) = 1$, that is, player 1 obtains an education regardless of his or her type.

$$Pr(H|E) = \frac{1 \cdot Pr(H)}{1 \cdot Pr(H) + 1 \cdot Pr(L)} = Pr(H)$$

- More generally, when player 1 plays the mixed strategy $Pr(E|H) = p$ and $Pr(E|L) = q$, then

$$Pr(H|E) = \frac{p \cdot Pr(H)}{p \cdot Pr(H) + q \cdot Pr(L)}$$

Perfect Bayesian equilibrium

A perfect Bayesian equilibrium consists of a strategy profile and a set of beliefs such that

- at each information set, the strategy of a player moving there maximizes his or her expected payoff, where the expectation is taken with respect to his or her beliefs, and
- at each information set, the beliefs of the player moving there are formed using Bayes' rule (based on prior beliefs and other players' strategies).

- Signaling games may have multiple equilibria, which can be divided into three classes— separating, pooling and hybrid.
- In a *separating equilibrium*, each type of player 1 chooses a different action. Player 2 learns player 1's type with certainty after observing player 1's action.
- In a *pooling equilibrium*, different types of player 1 choose the same action. Observing player 1's action provides player 2 with no additional information about player 1's type.
- In a *hybrid equilibrium*, one type of player 1 plays a strictly mixed strategy. Player 2 learns a little about player 1's type but doesn't learn it with certainty. Player 2 may respond to the uncertainty by also playing a mixed strategy.

Example 8.7 Separating Equilibrium in Job-Market Signaling Game

- A good guess for a separating equilibrium is that the high-skilled worker signals his or her type by getting an education and the low-skilled worker does not.
- Given these strategies, player 2's beliefs must be $Pr(H|E) = Pr(L|NE) = 1$ and $Pr(H|NE) = Pr(L|E) = 0$.
- Player 2's best response if he observes that player 1 obtains an education is to offer a job (J), given the payoff of $\pi - w > 0$, and not offer a job (NJ) if player 1 does not obtain an education because $0 > -w$.
- Lastly, we need to check that player 1 would not want to deviate from the separating strategy $(E|H, NE|L)$ given that player 2 plays $(J|E, NJ|NE)$.

- Type H player 1 would not deviate if $w - c_H > 0$, where $w - c_H$ and 0 are payoffs of obtaining and not obtaining an education.
- Type L player 1 would not deviate if $0 > w - c_L$, where 0 and $w - c_L$ are payoffs of obtaining and not obtaining an education.
- There is separating equilibrium in which the worker obtains an education if and only if he or she is high-skilled and in which the firm offers a job only to applicants with an education if and only if $c_H < w < c_L$.

- Another possible separating equilibrium if for player 1 to obtain an education if and only if he or she is low-skilled.
- Player 2's best response would be to offer a job if and only if player 1 did not obtain an education.
- Under this circumstance, Type L player 1 would earn $0 - c_L$ from playing E because he is not offered an job, and $w - 0$ from playing NE, so he would deviate to NE since $-c_L < w$.
- Therefore, this will be ruled out as a perfect Bayesian equilibrium.

Example 8.8 Pooling Equilibrium in Job-Market Signalling Game

- A possible pooling equilibrium in which both types of player 1 choose E.
- For player 1 not to deviate from choosing E, player 2's strategy must be to offer a job if and only if the worker is educated— that is, $(J|E, NJ|NE)$. (Why?)
- Player 2's expected payoff from choosing J when player 1 obtains an education is

$$\begin{aligned} & Pr(H|E)(\pi - w) + Pr(L|E)(-w) \\ = \quad & Pr(H)(\pi - w) + Pr(L)(-w) = Pr(H)\pi - w \end{aligned}$$

For J to be a best response for player 2, we need $Pr(H) > \frac{w}{\pi}$.

- For NJ to be a best response to NE for player 2, we need

  $$0 > Pr(H|NE)(\pi - w) + Pr(L|NE)(-w) = Pr(H|NE)\pi - w$$

  or

  $$Pr(H|NE) \leq \frac{w}{\pi}.$$

- In sum, we need $Pr(H|NE) \leq w/\pi \leq Pr(H)$ to have a pooling equilibrium is which both types of player 1 obtain an education. That is, $Pr(H)$ must be sufficiently high and $Pr(H|NE)$ must be sufficiently low.
- In this equilibrium, type L pools with type H to prevent player 2 from learning anything about the worker's skill from the education signal.
- The other possibility for a pooling equilibrium is for both types of player 1 to choose NE. (See Problem 8.10)

Example 8.9 Hybrid Equilibrium in Job-Market Signalling Game

- One possible hybrid equilibrium is for type H always to obtain an education and type L to randomize between playing E and NE with probabilities e and $1-e$.
- Player 2's strategy is to offer a job to an educated applicant with probability j and not to offer a job to an uneducated applicant.
- We need to solve for the equilibrium values of e^* and j^* and the posterior beliefs $Pr(H|E)$ and $Pr(H|NE)$ that are consistent with perfect Bayesian equilibrium.
- The posterior beliefs are

$$
\begin{aligned}
Pr(H|E) &= \frac{Pr(H)}{Pr(H)+ePr(L)} = \frac{Pr(H)}{Pr(H)+e[1-Pr(H)]} \\
Pr(H|NE) &= 0
\end{aligned}
$$

- For type L of player 1 to be willing to play a strictly mixed strategy, he or she must get the same expected payoff from playing E and NE. That is

$$jw - c_L = 0, \text{ or } j^* = \frac{c_L}{w}$$

- Player 2 will play a strictly mixed strategy (conditional on observing E) only if he or she gets the same expected payoff from playing J and playing NJ. That is

$$Pr(H|E)(\pi - w) + Pr(L|E)(-w) = Pr(H|E)\pi - w = 0,$$

or

$$Pr(H|E) = \frac{w}{\pi} = \frac{Pr(H)}{Pr(H) + e[1 - Pr(H)]}$$

$$e^* = \frac{(\pi - w)Pr(H)}{w[1 - Pr(H)]}$$

Experimental Games

- Experimental economics is a recent branch of research that explores how well economic theory matches the behavior or experimental subjects in a laboratory setting.
- The importance of experimental economics was highlighted in 2002 when Vernon Smith received the Nobel Prize in economics for his pioneering work in the field.
- An important area in this field is the use of experimental methods to test game theory.

Experiments with the Prisoners' Dilemma

- In one experiment, subjects played Prisoners' Dilemma game 20 times with each other being matched with a different, anonymous opponent to avoid repeated-game effects.
- Players converged to the Nash equilibrium as subjects gained experience with the game.
- Players played the cooperative action 43% of the time in the first 5 rounds, falling to only 20% of the time in the last five rounds.
- Although 80% of the decisions were consistent with Nash equilibrium play by the end of the experiment, 20% of them still were anomalous.

Experiments with the Ultimatum Game

- In the subgame-perfect equilibrium of the Ultimatum Game, the Proposer offers a minimal share of the pot, and this is accepted by the Responder.
- In experiments, the division tends to be much more than in the subgame-perfect equilibrium.
- The most common offer is a 50-50 split. Responders tend to receive offers giving them less than 30% of the pot.
- Players may care about other factors such as fairness and thus obtain a benefit from a more equal division of the pot.

Experiments with the Dictator Game

- In the Dictator Game, the Proposer chooses a split of the pot, and this split is implemented without input from the Responder.
- Proposer tend to offer a less-even split than in the Ultimatum Game but still offer the Responder some of the pot, suggesting that Proposers have some residual concern for fairness.
- In one experiment designed so that the experimenter would never learn which Proposer made the offers, Proposers almost never gave an equal split to Responders and indeed took the whole pot for themselves 2/3 of the time.

Evolutionary Games and Learning

Evolutionary model

- In the evolutionary model, players do not make rational choices. They play the way they are genetically programmed.
- The more successful a player's strategy in the population, the more fit is the player and the more likely the player survive to pass his or her genes on the future generations and thus the more likely the strategy spreads in the population.

Learning model

- In a learning model, players are matched at random with others from a large population. Players usually are assumed to have a degree of rationality.
- Players are not fully rational in that they do not distort their strategies to affect others' learning and thus future play.

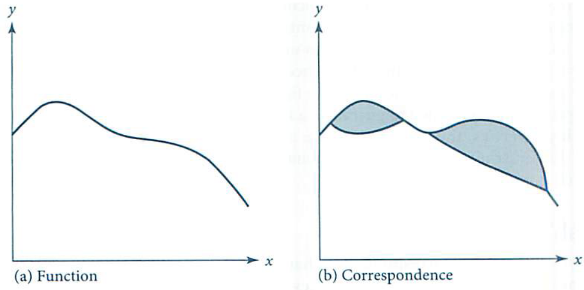
Extensions: Existence of Nash Equilibrium

- This section will sketch John Nash's original proof (1950) that all finite games have at least one Nash Equilibrium (in mixed if not in pure strategies).
- Nash's proof is similar to the proof of the existence of a general competitive equilibrium in Chapter 13. Both reply on a fixed point theorem.
- The proof of the existence of Nash equilibrium requires a slightly more powerful theorem. Instead of Brouwer's fixed point theorem, which applies to functions. Nash's proof replies on Kakutani's fixed point theorem, which applies to correspondence.

E8.1 Correspondences versus functions

- A function maps each point in a first set to a single point in a second set.
- A correspondence maps a single point in the first set to possibly many points in the second set. The best response, $BR_i(s_{-i})$, is an example.

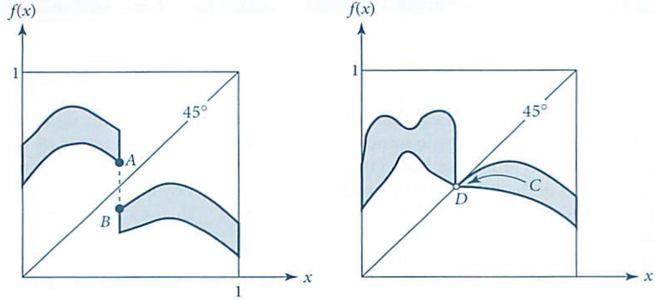
Figure E8.1 Comparison of Functions and Correspondences



E8.2 Kakutani's fixed point theorem

- *Any* *convex*, *upper-semicontinuous* *correspondence* $[f(x)]$ *from a closed, bounded, convex set into itself has at least one fixed point* (x^*) *such that* $x^* \in f(x^*)$.

Figure E8.2 Kakutani's Conditions on Correspondence



(a) Correspondence that is not convex

(b) Correspondence that is not upper semicontinuous

E8.3 Nash's proof

- We use $R(s)$ to denote the correspondence that underlies Nash's existence proof.
- This correspondence takes any profile of players' strategies $s = (s_1, s_2, \cdots, s_n)$ and maps it into another mixed strategy profile, the profile of best responses:

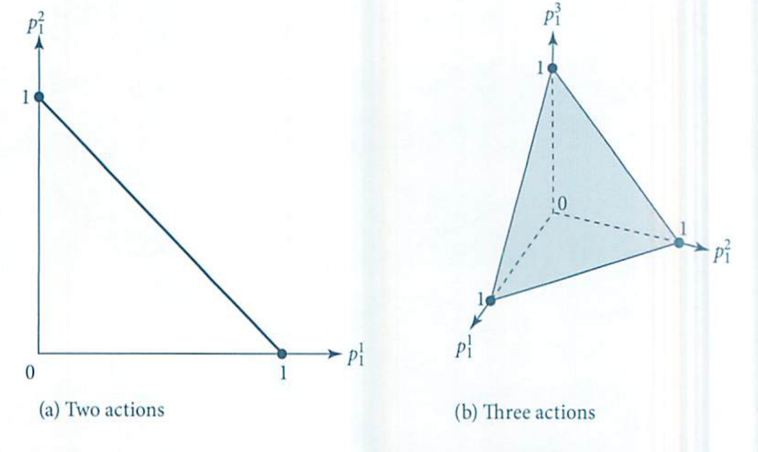
$$R(s) = (BR_1(s_{-1}), BR_2(s_{-2}), \cdots, BR_n(s_{-n}))$$

- A fixed point of the correspondence is a strategy for which $s^* \in R(s^*)$; this is a Nash equilibrium because each player's strategy is a best response to others' strategies.
- The proof checks that all the conditions involved in Kakutani's fixed point theorem are satisfied by the best-response correspondence $R(s)$.

1. *Show that the set of mixed-strategy profiles is closed, bounded, and convex.*

   - A strategy profile is just a list of individual strategies, the set of strategy profiles will be closed, bounded, and convex if each player's strategy set S_i has these properties individually.
   - Figure E8.3 shows for the case of two and three actions, the set of mixed strategies over actions has a simple shape.
   - The set is closed (contains its boundary), bounded (does not go off to infinity in any direction), and convex (the segment between any two point in the set is also in the set).

Figure E8.3 Set of Mixed Strategies for an Individual



2. *Check that the best-response correspondence $R(s)$ is convex.*

   - Individual best responses cannot look like Figure E8.2a because if any two mixed strategies such as A and B are best responses to others' strategies, then mixed strategies between them must also be best responses.
   - For example, in the Battle of Sexes, if $(1/3, 2/3)$ and $(2/3, 1/3)$ are best responses for husband against his wife playing $(2/3, 1/3)$, then mixed strategies between the two such as $(1/2, 1/2)$ must also be best responses for him. In fact, all possible mixed strategies for the husband are best response to the wife's playing $(2/3, 1/3)$.

3. *Check that R(s) is upper semicontinuous*

- Individual best responses cannot look like in Figure E8.2b, they can not have holes like point D punched out of them because payoff functions $U_i(s_i, s_{-i})$ are continuous.
- Recall that payoffs, when written as functions of mixed strategies, are actually expected values with probabilities given by the strategies s_i and s_{-i}.
- Expected values are linear functions of the underlying probabilities. Linear functions are, of course, continuous.

E8.4 Games with continuous actions

- Nash's existence theorem applies to finite games. Nash's theorem does not apply to game that feature continuous actions, such as the Tragedy of the Commons in Example 8.5.
- Is a Nash equilibrium guaranteed to exist for these games? Glicksberg(1952) proved that the answer is "yes" as long as payoff functions are continuous.

References

- Glickberg, I. L. "A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points." *Proceedings of the National Academy of Sciences* 38 (1952): 170-84.
- Nash, John. "Equilibrium Points in n-Person Games." *Proceedings of the National Academy of Sciences* 36 (1950): 48-49.