

Preferences and Utility

Ming-Ching Luoh

2020.10.16.

Axioms of Rational Choice

Utility

Trades and Substitution

The Mathematics of Indifference Curves

Utility Functions for Specific Preferences

The Many-Good Case

Extensions

- This chapter shows the way in which economists characterize individuals' **preferences**.
- It begins with an abstract discussion of the “preference relation.”
- Then turns to the primary tool for studying individual choices— the **utility function**.

Axioms of Rational Choice

- One way to begin an analysis of individuals' choices is to state a basic set of postulates, or axioms, that characterize "rational" behavior.
- The preference relation is **assumed** to have **three** basic properties as follows.
 - I. **Completeness**. If A and B are any two situations, an individual can **always** specify exactly one of these possibilities:
 - A is preferred to B
 - B is preferred to A
 - A and B are **equally** attractive

- II. *Transitivity*. If A is preferred to B, and B is preferred to C, then A is preferred to C.

This assumes that the individual's choices are internally consistent.

- III. *Continuity*. If A is preferred to B, then situations suitably "close to" A must also be preferred to B.

This is used to analyze individuals' responses to relatively small changes in income and prices.

Utility

- Given the Assumptions of completeness, transitivity, and continuity, it is possible to show that people are able to **rank** all possible situations **from** the least desirable **to** the most desirable ones.
- Following the terminology introduced by the 19th-century political theorist Jeremy Bentham, economists call this ranking *utility*.
- If A is preferred to B, then the utility **assigned** to A exceeds the utility assigned to B: $U(A) > U(B)$

Non-uniqueness and utility measures

- Individuals' preferences are assumed to be represented by a **utility function** of the form $U(x_1, x_2, \dots, x_n)$, where x_1, x_2, \dots, x_n are the quantities of each of n goods that might be consumed in a period.
- This function is unique only up to an **order-preserving** transformation.
- Utility rankings are **ordinal** in nature, it records the **relative desirability** of commodity bundles.
- It makes no sense to consider **how much** more utility is gained from A than from B.
- The lack of uniqueness in the assignment of utility numbers implies that it is not possible to compare utilities **between** different people.

The *ceteris paribus* assumption

- Because *utility* refers to overall **satisfaction**, such a measure is clearly affected by a variety of factors such as
 - the **consumption** of physical commodities,
 - psychological attitudes,
 - peer group pressures,
 - personal experiences,
 - the general cultural environment.
- A common practice is to devote attention exclusively to choices among **quantifiable options**, while holding constant the other things that affect behavior. This is the *ceteris paribus*, "other things being equal", assumption.

Utility from consumption of goods

- Consider an individual's problem of choosing among n consumption goods x_1, x_2, \dots, x_n .
- Assume that the individual's rankings of these goods can be represented by a utility function of the form

$$\text{utility} = U(x_1, x_2, \dots, x_n, \text{other things}),$$

- "other things" are held constant, so

$$\text{utility} = U(x_1, x_2, \dots, x_n)$$

- For two goods, x and y :

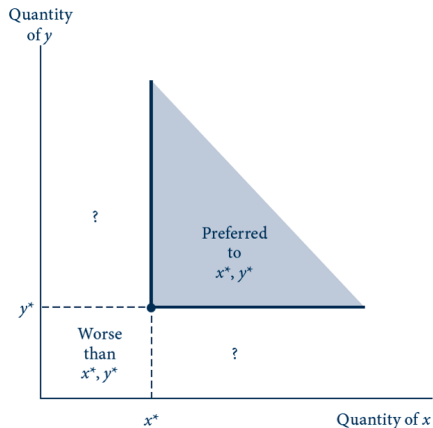
$$\text{utility} = U(x, y)$$

Arguments of utility functions

- $U(W)$ = utility an individual receives from real wealth (W)
- $U(c, h)$ = utility from consumption (c) and leisure (h), individual's labor-leisure choice in Chapter 16.
- $U(c_1, c_2)$ = utility from consumption in two different **periods** (Chapter 17), where c_1 is consumption in this period and c_2 is consumption in the next period.

Economic goods

Figure 3.1 More of a Good Is Preferred to Less



Two-good utility function $U(x, y)$

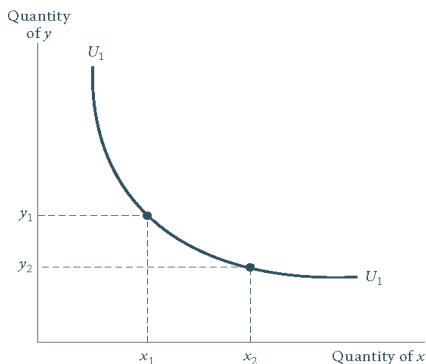
Trades and Substitution

- Most economic activity involves **voluntary** trading between individuals.
- When someone buys, say, a load of bread, he or she is voluntarily giving up one thing (money) for something else (bread) that is of **greater value** to that individual.
- We will develop a formal apparatus for illustrating trades in the utility function context to examine this kind of voluntary transaction.
- We first motivate our discussion with a **graphical presentation** and then turn to some more **formal mathematics**.

Indifference curves and the marginal rate of substitution

- **Indifference curve** shows a set of consumption bundles about which the individual is indifferent.
- That is, all consumption bundles that the individual ranks equally. The bundles all provide the same level of utility.

Figure 3.2 A Single Indifference Curve



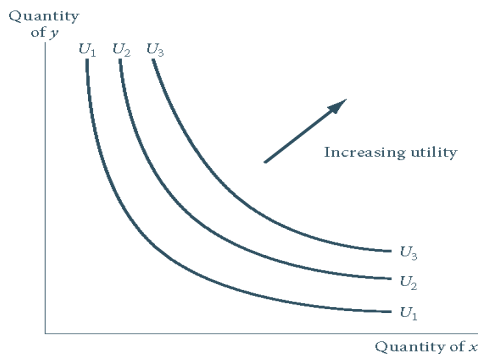
Marginal rate of substitution, MRS

- The **negative** of the slope of an indifference curve (U_1) at some point is the **marginal rate of substitution (MRS)** at that point.
- People become progressively **less willing** to trade away y to get more x . The absolute value of the slope **diminishes** as x increases.
- MRS changes as x and y change. This reflects the individual's **willingness to trade** y for x

$$MRS = - \left. \frac{dy}{dx} \right|_{U=U_1}$$

Indifference curve map

Figure 3.3 There Are **Infinitely Many** Indifference Curves in the $x - y$ Plane

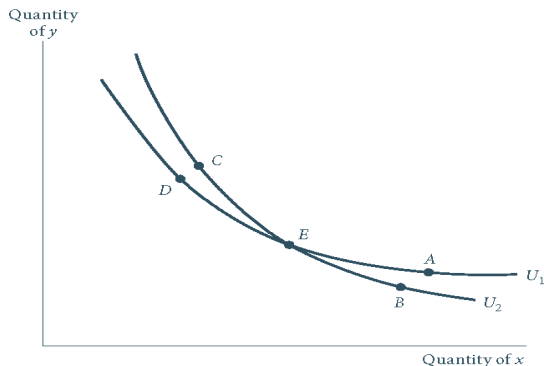


- Level of utility represented by these curves increases as we move in a northeast direction, because more of a good is preferred to less.

Indifference curves and transitivity

- Indifference curves **cannot** intersect.

Figure 3.4 Intersecting Indifference Curves Imply Inconsistent Preferences

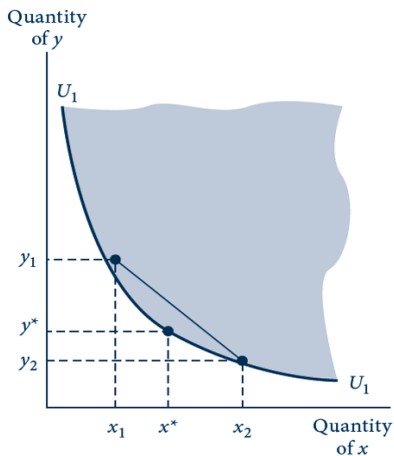


- $A > B$, $C > D$ and $B \approx C$, then $A > D$. Contradicts with $A \approx D$.

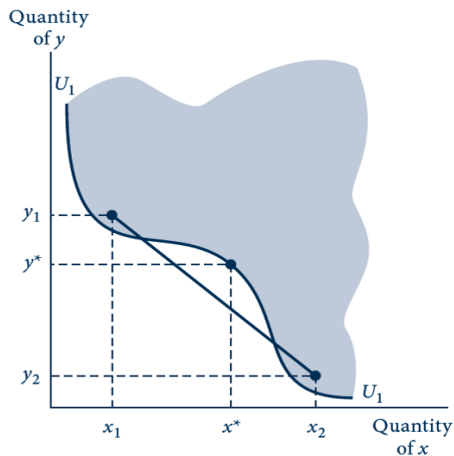
Convexity of indifference curves

- A set of points is said to be *convex* if any two points within the set can be joined by a straight line that is contained completely within the set.
- The assumption of diminishing *MRS* is *equivalent* to the assumption that all combinations of x and y that are *preferred or indifferent* to a particular x^* , y^* form a convex set.
- In Figure 3.5a, any two of the combinations— say, x_1, y_1 and x_2, y_2 — can be joined by a straight line also contained in the shaded area.
- In Figure 3.5b, the set of points preferred or indifferent to x^*, y^* is *not convex*.

Figure 3.5 The Notion of Convexity as an Alternative Definition of a Diminishing MRS



(a)



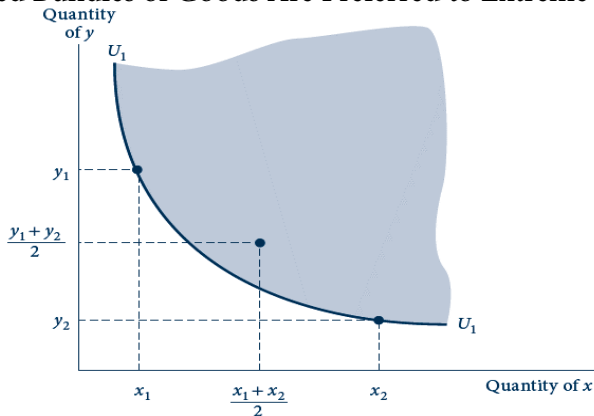
(b)

Convexity and balance in consumption

- “Well-balanced” bundles of commodities are preferred to bundles that are heavily weighted toward one commodity.

Figure 3.6

Balanced Bundles of Goods Are Preferred to Extreme Bundles



Example 3.1: Utility and the *MRS*

- A person's ranking of hamburgers (y) and soft drinks (x),

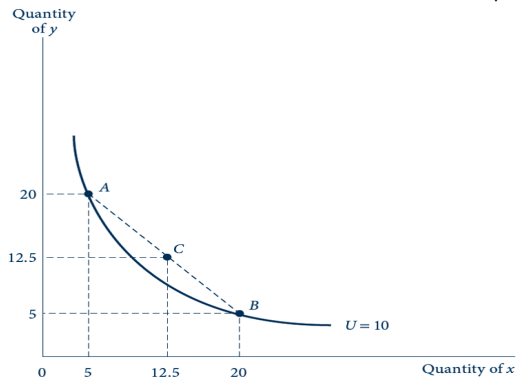
$$\text{utility} = \sqrt{X \cdot Y}.$$

- An indifference curve for this function is found by identifying that set of combinations of x and y for which utility has the same value.

$$\text{Utility} = 10 = \sqrt{X \cdot Y}, \text{ so } 100 = x \cdot y,$$

therefore

$$y = \frac{100}{x}.$$

Figure 3.7 Indifference Curve for Utility = $\sqrt{x \cdot y}$ 

- Along $U = 10$, $y = \frac{100}{x}$, $MRS = -\frac{dy}{dx} = \frac{100}{x^2}$
- As x rises, MRS falls. When $x = 5$, $MRS = 4$, when $x = 20$, $MRS = 0.25$.

The Mathematics of Indifference Curves

The marginal rate of substitution

- Suppose an individual receives utility $U(x, y)$ from consuming two goods whose quantities are given by x and y .
- A specific level of utility, k , such that $U(x, y) = k$. Or,

$$U_x dx + U_y dy = dk = 0.$$

- The trade-offs between x and y , the rate at which x can be traded for y is given by the negative of the ratio of the “marginal utility” of good x to that of good y .

$$MRS = - \frac{dy}{dx} \Big|_{U(x,y)=k} = \frac{U_x}{U_y}$$

Convexity of Indifference Curves

- A function will have convex indifference curves, providing it is **quasi-concave** (from Chapter 2).
- Diminishing MRS requires that the utility function be quasi-concave.
- The assumption of diminishing marginal utility will **not** ensure that the utility is quasi-concave.
- Diminishing *MRS* and diminishing marginal utility are two different concepts.

Example 3.2: Showing Convexity of Indifference Curves

1. $U(x, y) = \sqrt{x \cdot y}$

Since taking logs is **order preserving**, let

$$U^*(x, y) = \ln[U(x, y)] = 0.5 \ln x + 0.5 \ln y,$$

then

$$MRS = \frac{\partial U^* / \partial x}{\partial U^* / \partial y} = \frac{0.5/x}{0.5/y} = \frac{y}{x}$$

- Clearly, MRS is diminishing as x increases and y decreases. Therefore, the indifference curves are **convex**.

$$2. U(x, y) = x + xy + y$$

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{1 + y}{1 + x}$$

- MRS is diminishing as x increases and y decreases. Therefore, the indifference curves are **convex**.

$$3. U(x, y) = \sqrt{x^2 + y^2}$$

$$\text{Let } U^*(x, y) = [U(x, y)]^2 = x^2 + y^2$$

$$MRS = \frac{\partial U^* / \partial x}{\partial U^* / \partial y} = \frac{x}{y}$$

- As x increases and y decreases, the MRS increases!
- The indifference curves are **concave**, **not** convex. This is clearly **not** a quasi-concave function.

Utility Functions for Specific Preferences

- Individuals' rankings of commodity bundles and the utility functions implied by these rankings are **unobservable**.
- All we can learn about people's **preferences** must come from the **behavior** we **observe** to changes in income, prices, and other factors.
- Nevertheless, it is usual to examine a few of the forms particular utility functions might take.
- This may offer insights into observed behavior, and understanding the properties of such functions can be of some help in **solving problems**.

Cobb-Douglas Utility

- Figure 3.8a shows the familiar shape of an indifference curve. One commonly used utility function that generates such curves has the form

$$U(x, y) = x^\alpha y^\beta,$$

where α and β are positive constants, each less than 1.

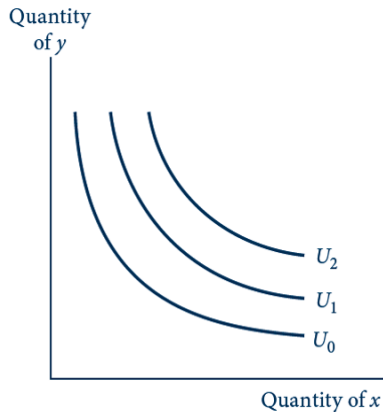
- The relative sizes of α and β indicate the relative importance of the goods.
- It is convenient to **normalize** these parameters so that $\alpha + \beta = 1$. In this case, utility would be given by

$$U(x, y) = x^\delta y^{1-\delta}$$

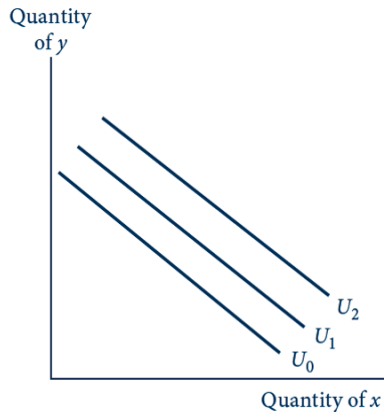
where $\delta = \alpha/(\alpha + \beta)$ and $1 - \delta = \beta/(\alpha + \beta)$, and

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{\delta}{1-\delta} \frac{y}{x}$$

Figure 3.8 Examples of Utility Functions (a, b)



(a) Cobb-Douglas



(b) Perfect substitutes

Perfect substitutes

- The linear indifference curves in Figure 3.8b are generated by a utility function of the form

$$U(x, y) = \alpha x + \beta y$$

where α and β are positive constants.

- The MRS is **constant** along the entire indifference curves.

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{\alpha}{\beta}$$

- For example, many people do not care where they buy gasoline. A gallon of gas is a gallon of gas despite the advertising efforts of **Exxon** and **Shell**.

Perfect complements

- A case directly opposite to perfect substitutes is the L-shaped indifference curves in Figure 3.8c. The utility function is of the form

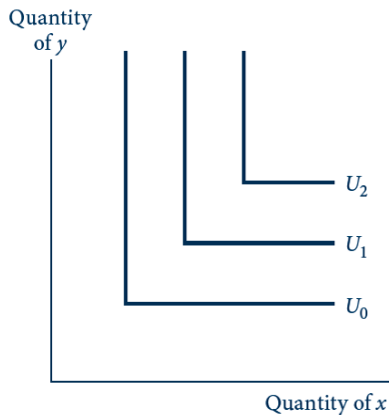
$$U(x, y) = \min(\alpha x, \beta y)$$

where α and β are positive parameters.

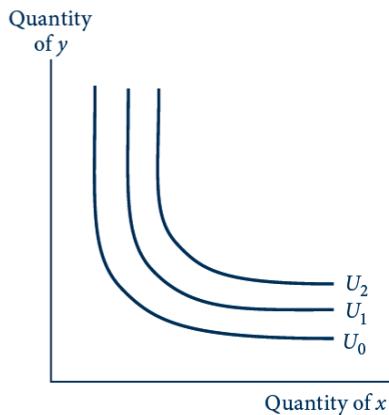
- The consumption bundle will be $\alpha x = \beta y$, or

$$\frac{y}{x} = \frac{\alpha}{\beta}.$$

Figure 3.8 Examples of Utility Functions (c, d)



(c) Perfect complements



(d) CES

CES Utility (constant elasticity of substitution)

- A function that permits a variety of shapes to be known is the *Constant Elasticity of Substitution* (CES) function.

$$U(x, y) = [x^\delta + y^\delta]^{1/\delta},$$

where $\delta \leq 1$, $\delta \neq 0$.

- Monotonic transformation** $U^* = U^\delta / \delta$, so that

$$U^*(x, y) = \frac{x^\delta}{\delta} + \frac{y^\delta}{\delta}$$

Thus,

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \left(\frac{y}{x}\right)^{1-\delta}.$$

- Since $\ln\left(\frac{y}{x}\right) = \frac{1}{1-\delta} \ln MRS$, the elasticity of substitution is

$$\frac{d \ln\left(\frac{y}{x}\right)}{d \ln MRS} = \frac{1}{1-\delta} \equiv \sigma$$

The elasticity of substitution, σ

- The elasticity of substitution, σ , is corresponding to δ in the CES utility functions with $\sigma = 1/(1 - \delta)$.
- For $\delta = 1$, $\sigma = \infty$, $U(x, y)$ corresponds to **perfect substitutes** case.
- For δ approaching $-\infty$, $\sigma = 0$, $U(x, y)$ approaches the case of **perfect complements**
- For $\delta = 0$, $\sigma = 1$, $U(x, y)$ approaches the case of **Cobb-Douglas**.

- One specific shape of the CES function whose indifference curves fall between the Cobb-Douglas and fixed proportion cases is for the case $\delta = -1$ and thus $\sigma = \frac{1}{1-\delta} = \frac{1}{2}$, that is

$$U(x, y) = -\frac{1}{x} - \frac{1}{y}$$

with

$$MU_x = \frac{1}{x^2} > 0$$

$$MU_y = \frac{1}{y^2} > 0$$

$$MRS = \frac{MU_x}{MU_y} = \frac{y^2}{x^2}$$

Example 3.3 Homothetic Preferences

- Utility function is **homothetic** if the MRS depends only on the **ratio** of the amounts of the two goods, not on the total quantities of the goods.
- For the case of **perfect substitutes**, MRS is the same at every point.
- For the case of **perfect complements**,
 - MRS is ∞ if $y/x > \alpha/\beta$
 - MRS is undefined if $y/x = \alpha/\beta$
 - MRS is 0 if $y/x < \alpha/\beta$

- For the general **Cobb-Douglas** function,

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x},$$

the MRS depends only on the ratio $\frac{y}{x}$.

- For homothetic functions, the slopes of the indifference curves depend only on the **ratio y/x** , not on how far the curve is **from the origin**.
- Indifference curves for higher utility are simple copies of those for lower utility.

Example 3.4 Non-homothetic Preferences

- Not all proper utility functions exhibit homothetic preferences.
- Consider the quasi-linear utility function

$$U(x, y) = x + \ln y$$

- Good y exhibits diminishing marginal utility, but good x does not.

$$MRS = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{1}{1/y} = y$$

- The MRS diminishes as the chosen quantity of y decreases, but it is independent of the quantity of x consumed.

The Many-Good Case

- Suppose utility is a function of n goods given by $U(x_1, x_2, \dots, x_n)$, then the equation

$$U(x_1, x_2, \dots, x_n) = k$$

defines an indifference **surface** in n dimensions.

- This surface shows all those combinations of the n goods that yield the same level of utility.

The MRS with many goods

$$MRS = - \frac{dx_2}{dx_1} \bigg|_{U(x_1, x_2, \dots, x_n) = k} = \frac{U_{x_1}(x_1, x_2, \dots, x_n)}{U_{x_2}(x_1, x_2, \dots, x_n)}$$

Extensions: Special Preferences

E3.1 Threshold effects

- People may be “set in their ways” and may require a rather large change in circumstances to change what they do.
- For example, people may stick with an old favorite TV show even though it has declined in quality.
- One way to capture such behavior is to assume that individuals make decisions as though they faced thresholds of preference.
- Bundle A might be chosen over B **only** when

$$U(A) > U(B) + \epsilon$$

where ϵ is the threshold that must be overcome.

E3.2 Quality

- Many consumption items differ in quality.
- One approach is simply to regard items of different quality as totally separate goods that are relative close substitutes.
- An **alternative** approach focuses on quality as a direct item of choice. Utility in this case is reflected by

$$\text{Utility} = U(q, Q)$$

where q is the quantity consumed, and Q is the quality of that consumption.

- A more general approach where good q provides a well-defined set of attributes of goods (a), so that

$$\text{Utility} = U[q, a_1(q), a_2(q)]$$

Assumes that those attributes provide utility.

E3.3 Habits and addiction

- **Habits** are formed when individuals discover they enjoy using a commodity in one period and this increases their consumption in subsequent periods.
- An extreme case of habits is **addiction** where past consumption significantly increases the utility of present consumption.
- One way to portray these ideas is to assume that utility in period t depends on consumption in period t and the total of all previous consumption of the habit-forming good x .

$$\text{Utility} = U(x_t, y_t, s_t)$$

where

$$s_t = \sum_{i=1}^{\infty} x_{t-i}$$

- However, data on all past levels of consumption usually do not exist. A common way to proceed is to assume that utility is given by

$$\text{Utility} = U(x_t^*, y_t)$$

where x_t^* is a function of current consumption (x_t) and consumption in the previous period (x_{t-1}), such as $x^* = x_t - x_{t-1}$ or $x^* = x_t/x_{t-1}$.

- Such functions imply that the higher the value of x_{t-1} , the more x_t will be chosen in the current period.
- Becker, Grossman and Murphy (1994) adapt the models to studying cigarette smoking and other addictive behavior. They show that reductions in smoking **early in life** can have large effects on eventual cigarette consumption because of the dynamics in individuals' utility functions.

E3.4 Second-party preferences

- Individuals clearly care about the well-being of other individuals. e.g. charitable contributions or **bequests to children**.
- Second-party preferences can be incorporated into the utility function of person i by

$$\text{Utility} = U_i(x_i, y_i, U_j)$$

where U_j is the utility of someone else.

- If $\partial U_i / \partial U_j > 0$, this person will engage in **altruistic** behavior.
- If $\partial U_i / \partial U_j < 0$, this person will demonstrate the malevolent behavior associated with **envy**.
- Gary Becker explored a variety of topics, including the general theory of **social interactions** (1976) and the importance of **altruism** in the theory of family (1981).

Evolution biology and genetics

- Biologists have suggested a particular form for the second-party preferences, drawn from the theory of genetics.

$$\text{Utility} = U_i(x_i, y_i) + \sum_j r_j U_j$$

where r_j measures **closeness** of the genetic relationship between person i and person j .

- For parents and children, for example, $r_j = 0.5$, whereas for cousins $r_j = 0.125$.