Part II: Choice and Demand

3. Preferences and Utility
4. Utility Maximization and Choice
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Preferences and Utility

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- This chapter shows the way in which economists characterize individuals' preferences.
- It begins with an abstract discussion of the "preference relation."
- Then turns to the primary tool for studying individual choices— the utility function.

Axioms of Rational Choice

- One way to begin an analysis of individuals' choices is to state a basic set of postulates, or axioms, that characterize "rational" behavior.
- The preference relation is assumed to have three basic properties as follows.

I. *Completeness*. If A and B are any two situations, an individual can always specify exactly one of these possibilities:

  - A is preferred to B
  - B is preferred to A
  - A and B are equally attractive

II. *Transitivity*. If A is preferred to B, and B is preferred to C, then A is preferred to C.

This assumes that the individual's choices are internally consistent.

III. *Continuity*. If A is preferred to B, then situations suitably "close to" A must also be preferred to B.

This is used to analyze individuals' responses to relatively small changes in income and prices.

Utility

- Given the Assumptions of completeness, transitivity, and continuity, it is possible to show that people are able to rank all possible situations from the least desirable to the most desirable ones.
- Following the terminology introduced by the 19th-century political theorist Jeremy Bentham, economists call this ranking *utility*.
- If A is preferred to B, then the utility assigned to A exceeds the utility assigned to B: $U(A) > U(B)$

Non-uniqueness and utility measures

- Individuals' preferences are assumed to be represented by a utility function of the form $U(x_1, x_2, \cdots, x_n)$, where $x_1, x_2, \cdots, x_n$ are the quantities of each of n goods that might be consumed in a period.
- This function is unique only up to an order-preserving transformation.
- Utility rankings are ordinal in nature, it records the relative desirability of commodity bundles.
- It makes no sense to consider how much more utility is gained from A than from B.
- The lack of uniqueness in the assignment of utility numbers implies that it is not possible to compare utilities between different people.

The ceteris paribus assumption

- Because *utility* refers to overall satisfication, such a measure is clearly affected by a variety of factors such as
  - the consumption of physical commodities,
  - psychological attitudes,
  - peer group pressures,
  - personal experiences,
  - the general cultural environment.
- A common practice is to devote attention exclusively to choices among quantifiable options, while holding constant the other things that affect behavior. This is the *ceteris paribus*, “other things being equal", assumption.

Utility from consumption of goods

- Consider an individual's problem of choosing among n consumption goods $x_1, x_2, \cdots, x_n$.
- Assume that the individual's rankings of these goods can be represented by a utility function of the form

$$\text{utility} = U(x_1, x_2, \cdots, x_n, \text{other things}),$$

- "other things" are held constant, so

$$\text{utility} = U(x_1, x_2, \cdots, x_n)$$

- For two goods, x and y:

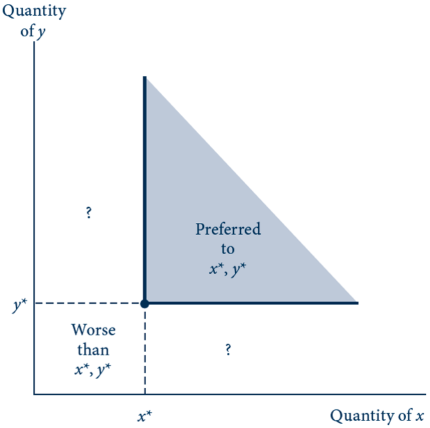
$$\text{utility} = U(x, y)$$

Arguments of utility functions

- $U(W)$ = utility an individual receives from real wealth (W)
- $U(c, h)$ = utility from consumption (c) and leisure (h), individual's labor-leisure choice in Chapter 16.
- $U(c_1, c_2)$ = utility from consumption in two different periods (Chapter 17), where c_1 is consumption in this period and c_2 is consumption in the next period.

Economic goods

Figure 3.1 More of a Good Is Preferred to Less



Two-good utility function $U(x, y)$

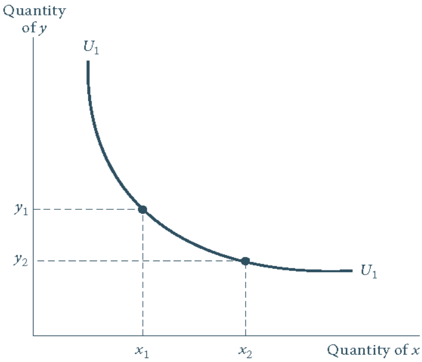
Trades and Substitution

- Most economic activity involves voluntary trading between individuals.
- When someone buys, say, a load of bread, he or she is voluntarily giving up one thing (money) for something else (bread) that is of greater value to that individual.
- We will develop a formal apparatus for illustrating trades in the utility function context to examine this kind of voluntary transaction.
- We first motivate our discussion with a graphical presentation and the turn to some more formal mathematics.

Indifference curves and the marginal rate of substitution

- Indifference curve shows a set of consumption bundles about which the individual is indifferent.
- That is, all consumption bundles that the individual ranks equally. The bundles all provide the same level of utility.

Figure 3.2 A Single Indifference Curve



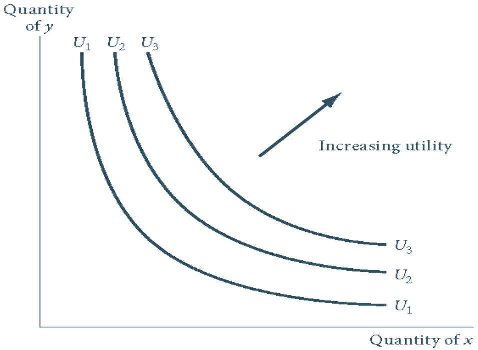
Marginal rate of substitution, MRS

- The negative of the slope of an indifference curve (U_1) at some point is the marginal rate of substitution (MRS) at that point.
- People become progressively less willing to trade away y to get more x. The absolute value of the slope diminishes as x increases.
- MRS changes as x and y change. This reflects the individual's willingness to trade y for x

$$MRS = -\frac{dy}{dx}\bigg|_{U=U_1}$$

Indifference curve map

Figure 3.3 There Are Infinitely Many Indifference Curves in the $x - y$ Plane

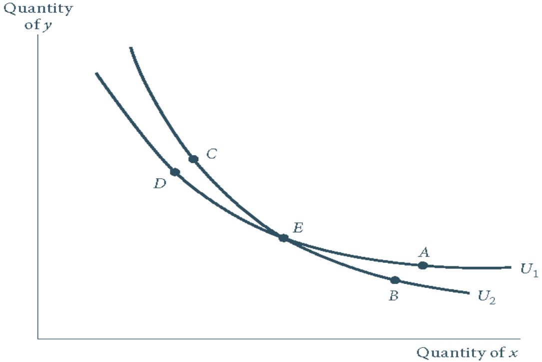


- Level of utility represented by these curves increases as we move in a northeast direction, because more of a good is preferred to less.

Indifference curves and transitivity

- Indifference curves cannot intersect.

Figure 3.4 Intersecting Indifference Curves Imply Inconsistent Preferences

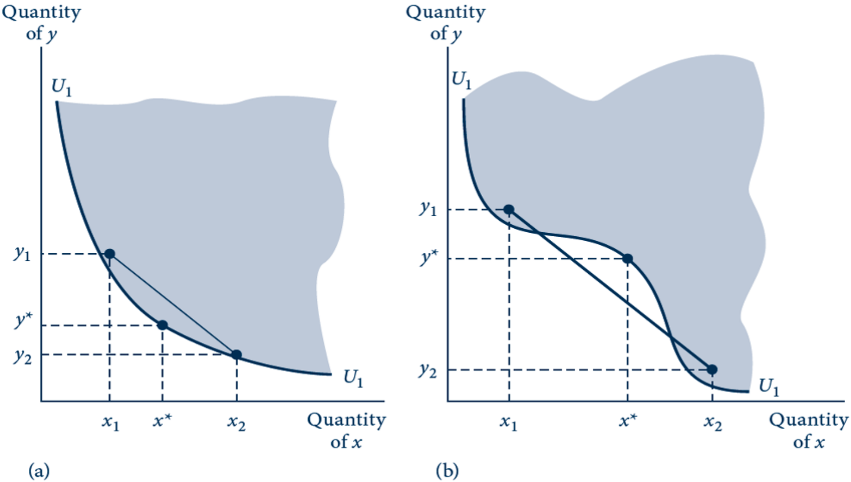


- $A \succ B$, $C \succ D$ and $B \approx C$, then $A \succ D$. Contradicts with $A \approx D$.

Convexity of indifference curves

- A set of points is said to be *convex* if any two points within the set can be joined by a straight line that is contained completely within the set.
- The assumption of diminishing *MRS* is equivalent to the assumption that all combinations of x and y that are preferred or indifferent to a particular x^*, y^* form a convex set.
- In Figure 3.5a, any two of the combinations— say, x_1, y_1 and x_2, y_2— can be joined by a straight line also contained in the shaded area.
- In Figure 3.5b, the set of points preferred or indifferent to x^*, y^* is not convex.

Figure 3.5 The Notion of Convexity as an Alternative Definition of a Diminishing MRS

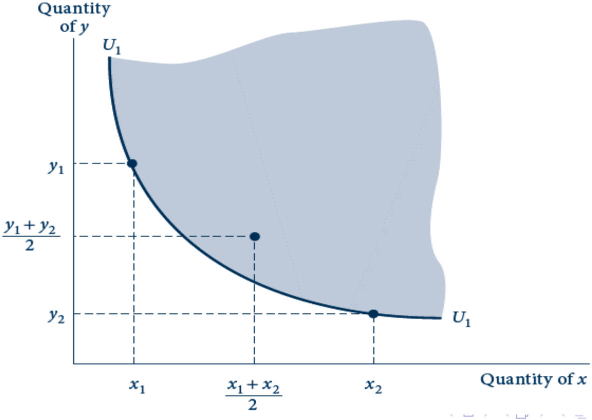


Convexity and balance in consumption

- “Well-balanced" bundles of commodities are preferred to bundles that are heavily weighted toward one commodity.

Figure 3.6

Balanced Bundles of Goods Are Preferred to Extreme Bundles



Example 3.1: Utility and the *MRS*

- A person's ranking of hamburgers (y) and soft drinks (x),

$$\text{utility} = \sqrt{X \cdot Y}.$$

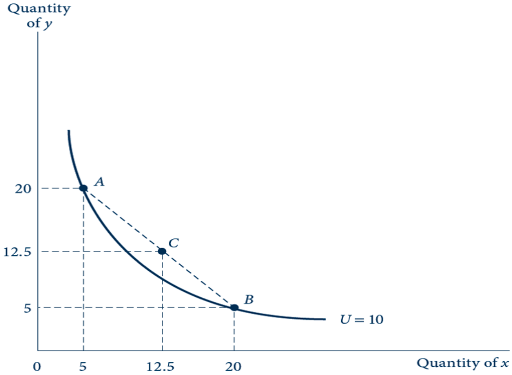
- An indifference curve for this function is found by identifying that set of combinations of x and y for which utility has the same value.

$$\text{Utility} = 10 = \sqrt{X \cdot Y}, \text{ so } 100 = x \cdot y,$$

therefore

$$y = \frac{100}{x}.$$

Figure 3.7 Indifference Curve for Utility=$\sqrt{x \cdot y}$



- Along $U = 10$, $y = \frac{100}{x}$, $MRS = -\frac{dy}{dx} = \frac{100}{x^2}$
- As x rises, MRS falls. When $x = 5$, $MRS = 4$, when $x = 20$, $MRS = 0.25$.

The Mathematics of Indifference Curves

The marginal rate of substitution

- Suppose an individual receives utility $U(x, y)$ from consuming two goods whose quantities are given by x and y.
- A specific level of utility, k, such that $U(x, y) = k$. Or,

$$U_x dx + U_y dy = dk = 0.$$

- The trade-offs between x and y, the rate at which x can be traded for y is given by the negative of the ratio of the "marginal utility" of good x to that of good y.

$$MRS = -\left.\frac{dy}{dx}\right|_{U(x,y)=k} = \frac{U_x}{U_y}$$

Convexity of Indifference Curves

- A function will have convex indifference curves, providing it is quasi-concave (from Chapter 2).
- Diminishing MRS requires that the utility function be quasi-concave.
- The assumption of diminishing marginal utility will not ensure that the utility is quasi-concave.
- Diminishing MRS and diminishing marginal utility are two different concepts.

Example 3.2: Showing Convexity of Indifference Curves

1. $U(x, y) = \sqrt{x \cdot y}$

Since taking logs is order preserving, let

$$U^*(x, y) = \ln[U(x, y)] = 0.5 \ln x + 0.5 \ln y,$$

then

$$MRS = \frac{\partial U^* / \partial x}{\partial U^* / \partial y} = \frac{0.5/x}{0.5/y} = \frac{y}{x}$$

- Clearly, MRS is diminishing as x increases and y decreases. Therefore, the indifference curves are convex.

2. $U(x, y) = x + xy + y$

$$MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{1 + y}{1 + x}$$

- MRS is diminishing as x increases and y decreases. Therefore, the indifference curves are convex.

3. $U(x, y) = \sqrt{x^2 + y^2}$
Let $U^*(x, y) = [U(x, y)]^2 = x^2 + y^2$

$$MRS = \frac{\partial U^* / \partial x}{\partial U^* / \partial y} = \frac{x}{y}$$

- As x increases and y decreases, the MRS increases!
- The indifference curves are concave, not convex. This is clearly not a quasi-concave function.

Utility Functions for Specific Preferences

- Individuals' rankings of commodity bundles and the utility functions implied by these rankings are unobservable.
- All we can learn about people's preferences must come from the behavior we observe to changes in income, prices, and other factors.
- Nevertheless, it is usual to examine a few of the forms particular utility functions might take.
- This may offer insights into observed behavior, and understanding the properties of such functions can be of some help in solving problems.

Cobb-Douglas Utility

- Figure 3.8a shows the familiar shape of an indifference curve. One commonly used utility function that generates such curves has the form

$$U(x, y) = x^{\alpha} y^{\beta},$$

where α and β are positive constants, each less than 1.

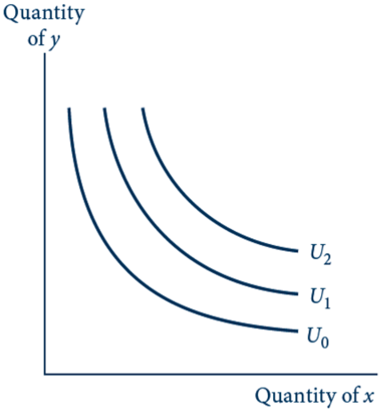
- The relative sizes of α and β indicate the relative importance of the goods.
- It is convenient to normalize these parameters so that $\alpha + \beta = 1$. In this case, utility would be given by

$$U(x, y) = x^{\delta} y^{1-\delta}$$

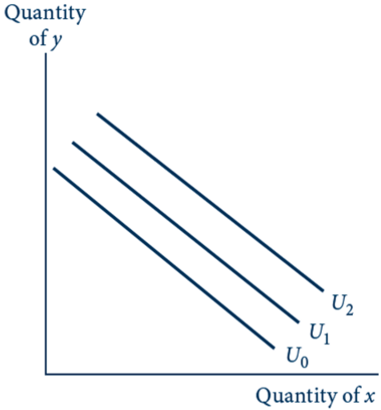
where $\delta = \alpha/(\alpha + \beta)$ and $1 - \delta = \beta/(\alpha + \beta)$, and

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{\delta}{1-\delta}\frac{y}{x}$$

Figure 3.8 Examples of Utility Functions (a, b)



(a) Cobb-Douglas



(b) Perfect substitutes

Perfect substitutes

- The linear indifference curves in Figure 3.8b are generated by a utility function of the form

$$U(x, y) = \alpha x + \beta y$$

where α and β are positive constants.

- The MRS is constant along the entire indifference curves.

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{\alpha}{\beta}$$

- For example, many people do not care where they buy gasoline. A gallon of gas is a gallon of gas despite the advertising efforts of Exxon and Shell.

Perfect complements

- A case directly opposite to perfect substitutes is the L-shaped indifference curves in Figure 3.8c. The utility function is of the form

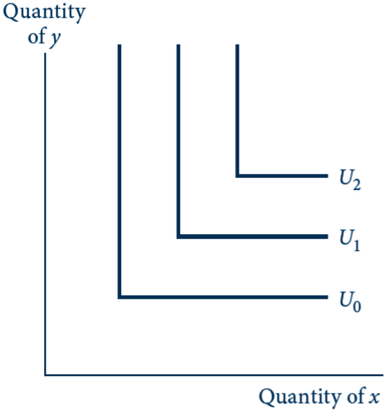
$$U(x, y) = \min(\alpha x, \beta y)$$

where α and β are positive parameters.

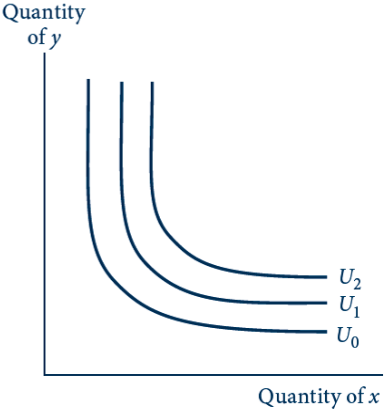
- The consumption bundle will be $\alpha x = \beta y$, or

$$\frac{y}{x} = \frac{\alpha}{\beta}.$$

Figure 3.8 Examples of Utility Functions (c, d)



(c) Perfect complements



(d) CES

CES Utility (constant elasticity of substitution)

- A function that permits a variety of shapes to be known is the *Constant Elasticity of Substitution* (CES) function.

$$U(x, y) = [x^\delta + y^\delta]^{1/\delta},$$

where $\delta \leq 1, \delta \neq 0$.

- Monotonic transformation $U^* = U^\delta/\delta$, so that

$$U^*(x, y) = \frac{x^\delta}{\delta} + \frac{y^\delta}{\delta}$$

Thus,

$$MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y} = \left(\frac{y}{x}\right)^{1-\delta}.$$

- Since $\ln(\frac{y}{x}) = \frac{1}{1-\delta} \ln MRS$, the elasticity of substitution is

$$\frac{d\ln(\frac{y}{x})}{d\ln MRS} = \frac{1}{1-\delta} \equiv \sigma$$

The elasticity of substitution, σ

- The elasticity of substitution, σ, is corresponding to δ in the CES utility functions with $\sigma = 1/(1-\delta)$.
- For $\delta = 1, \sigma = \infty, U(x,y)$ corresponds to perfect substitutes case.
- For δ approaching $-\infty, \sigma = 0, U(x,y)$ approaches the case of perfect complements
- For $\delta = 0, \sigma = 1, U(x,y)$ approaches the case of Cobb-Douglas.

- One specific shape of the CES function whose indifferent curves fall between the Cobb-Douglas and fixed propoetion cases is for the case $\delta = -1$ and thus $\sigma = \frac{1}{1-\delta} = \frac{1}{2}$, that is

$$U(x, y) = -\frac{1}{x} - \frac{1}{y}$$

with

$$\begin{aligned} MU_x &= \frac{1}{x^2} > 0 \\ MU_y &= \frac{1}{y^2} > 0 \\ MRS &= \frac{MU_x}{MU_y} = \frac{y^2}{x^2} \end{aligned}$$

Example 3.3 Homothetic Preferences

- Utility function is homothetic if the MRS depends only on the ratio of the amounts of the two goods, not on the total quantities of the goods.
- For the case of perfect substitutes, MRS is the same at every point.
- For the case of perfect complements,
  - MRS is ∞ if $y/x > \alpha/\beta$
  - MRS is undefined if $y/x = \alpha/\beta$
  - MRS is 0 if $y/x < \alpha/\beta$

- For the general Cobb-Douglas function,

$$MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{\alpha x^{\alpha-1}y^{\beta}}{\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha}{\beta}\cdot\frac{y}{x},$$

  the MRS depends only on the ratio $\frac{y}{x}$.

- For homothetic functions, the slopes of the indifference curves depend only on the ratio y/x, not on how far the curve is from the origin.

- Indifference curves for higher utility are simple copies of those for lower utility.

Example 3.4 Non-homothetic Preferences

- Not all proper utility functions exhibit homothetic preferences.
- Consider the quasi-linear utility function

$$U(x, y) = x + \ln y$$

- Good y exhibits diminishing marginal utility, but good x does not.

$$MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{1}{1/y} = y$$

- The MRS diminishes as the chosen quantity of y decreases, but it is independent of the quantity of x consumed.

The Many-Good Case

- Suppose utility is a function of n goods given by $U(x_1, x_2, \cdots, x_n)$, then the equation

$$U(x_1, x_2, \cdots, x_n) = k$$

defines an indifference surface in n dimensions.

- This surface shows all those combinations of the n goods that yield the same level of utility.

The MRS with many goods

$$MRS = -\left.\frac{dx_2}{dx_1}\right|_{U(x_1,x_2,\cdots,x_n)=k} = \frac{U_{x_1}(x_1, x_2, \cdots, x_n)}{U_{x_2}(x_1, x_2, \cdots, x_n)}$$

Extensions: Special Preferences

E3.1 Threshold effects

- People may be “set in their ways" and may require a rather large change in circumstances to change what they do.
- For example, people may stick with an old favorite TV show even though it has declined in quality.
- One way to capture such behavior is to assume that individuals make decisions as though they faced thresholds of preference.
- Bundle A might be chosen over B only when

$$U(A) > U(B) + \epsilon$$

where ϵ is the threshold that must be overcome.

E3.2 Quality

- Many consumption items differ in quality.
- One approach is simply to regard items of different quality as totally separate goods that are relative close substitutes.
- An alternative approach focuses on quality as a direct item of choice. Utility is this case is reflected by

$$\text{Utility} = U(q, Q)$$

  where q is the quantity consumed, and Q is the quality of that consumption.
- A more general approach where good q provides a well-defined set of attributes of goods (a), so that

$$\text{Utility} = U[q, a_1(q), a_2(q)]$$

  Assumes that those attributes provide utility.

E3.3 Habits and addiction

- Habits are formed when individuals discover they enjoy using a commodity in one period and this increases their consumption in subsequent periods.
- An extreme case of habits is addiction where past consumption significantly increases the utility of present consumption.
- One way to portray these ideas is to assume that utility in period t depends on consumption in period t and the total of all previous consumption of the habit-forming good x.

$$\text{Utility} = U(x_t, y_t, s_t)$$

where

$$s_t = \sum_{i=1}^{\infty} x_{t-i}$$

- However, data on all past levels of consumption usually do not exist. A common way to proceed is to assume that utility is given by

$$\text{Utility} = U(x_t^*, y_t)$$

where x_t^* is a function of current consumption (x_t) and consumption in the previous period (x_{t-1}), such as $x^* = x_t - x_{t-1}$ or $x^* = x_t / x_{t-1}$.

- Such functions imply that the higher the value of x_{t-1}, the more x_t will be chosen in the current period.
- Becker, Grossman and Murphy (1994) adapt the models to studying cigarette smoking and other addictive behavior. They show that reductions in smoking early in life can have large effects on eventual cigarette consumption because of the dynamics in individuals' utility functions.

E3.4 Second-party preferences

- Individuals clearly care about the well-being of other individuals. e.g. charitable contributions or bequests to children.
- Second-party preferences can be incorporated into the utility function of person i by

$$\text{Utility} = U_i(x_i, y_i, U_j)$$

where U_j is the utility of someone else.

- If $\partial U_i / \partial U_j > 0$, this person will engage in altruistic behavior.
- If $\partial U_i / \partial U_j < 0$, this person will demonstrate the malevolent behavior associated with envy.
- Gary Becker explored a variety of topics, including the general theory of social interactions (1976) and the importance of altruism in the theory of family (1981).

Evolution biology and genetics

- Biologists have suggested a particular form for the second-party preferences, drawn from the theory of genetics.

$$\text{Utility} = U_i(x_i, y_i) + \sum_j r_j U_j$$

where r_j measures closeness of the genetic relationship between person i and person j.

- For parents and children, for example, $r_j = 0.5$, whereas for cousins $r_j = 0.125$.