Part I: Introduction

1. Economic Models
2. Mathematics for Microeconomics

Chapter 1
Economic Models

Ming-Ching Luoh

2020.9.18.

Theoretical Models

A modern economy is a complicated place. Take peanuts as an example. Peanuts must be

- harvested at the right time, and
- shipped to processors who turn them into
- peanut butter, peanut oil and numerous peanut delicacies.
- These processors, in turn, must make certain that their products arrive at thousands of retail outlets in the proper quantities to meet demand.

- It would be impossible to describe the features of even these peanut markets in complete details,
- Economists must abstract from the complexities of the real world and develop rather simple economics models that capture the “essentials."
- Economic models provide aid in understanding economic behavior.

Verification of Economic Models

Not all models prove to be “good." Two general methods have been used to verify economic models.

- Direct approach: Establishes the validity of the model’s assumptions.
- Indirect approach: Shows that the model correctly predicts real-world events.

The profit maximization model

- The model assumes that a firm seeks to maximize profit.
- It ignores the personal motivations for the firm's managers and does not consider conflicts among them.
- It assumes that other possible goals such as obtaining power or prestige are unimportant.
- It assumes that the firm has sufficient information about its costs and the nature of the market.
- No model can exactly describe reality. The real question is whether this simple model has an claim to being a good one.

Testing assumptions

Do firms really seek maximum profits?

- Examine this question by sending questionnaire to executives to ask the goals they pursue.
- The results of such studies have been varied.

Testing predictions

- Some economists, most notably Milton Friedman conclude that the only way to determine the validity of a model is to see whether it is capable of predicting and explaining real-world events.
- The ultimate test of an economic model comes when it is confronted with data from the economy itself.
- A test of the profit-maximization model would be provided by predicting the behavior of real-world firms by assuming that these firms behave *as if* they were maximizing profit.
- The ultimate test of any theory is its ability to predict *real-world events*.

Importance of empirical analysis

- The goal of theoretical models is to learn something about the real world.

General Features of Economic Models

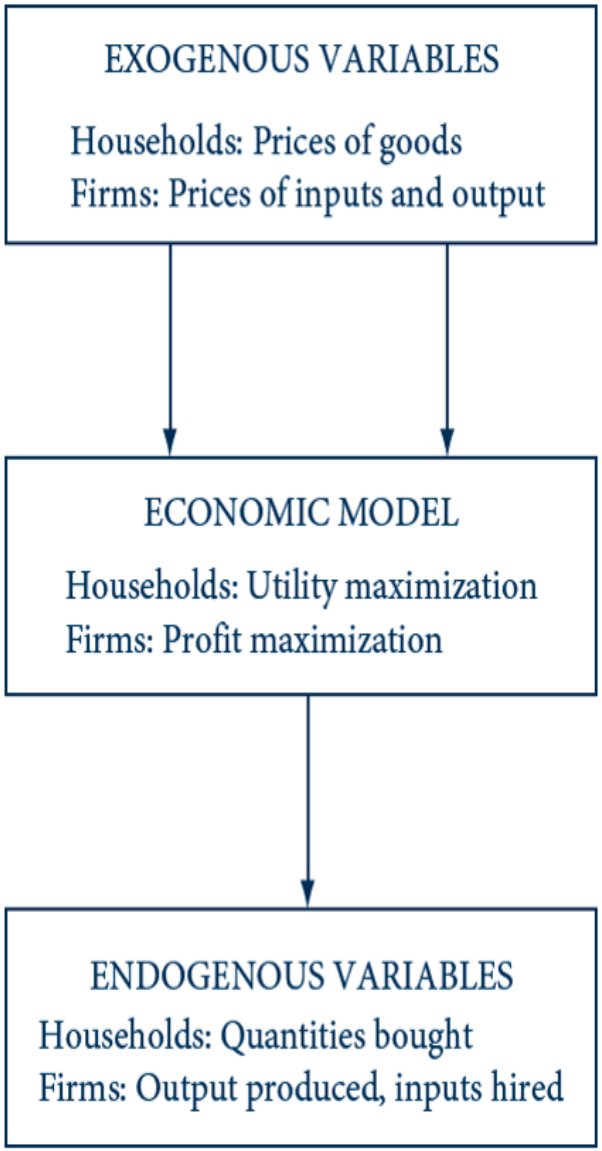
All economic models incorporate three common elements:

1. the *Ceteris Paribus* (other things equal) assumption;
    - However, economists have not been able to conduct controlled experiments to test their models.
    - They have been forced to rely on various statistical methods to control for other forces when testing their theories.
2. the supposition that economic decision-makers seek to optimize something;
3. a careful distinction between “positive" and “normative" questions.

Structure of Economic Models

- Economic models highlight the relationships between factors that affect the decisions of households and firms and the results of those decisions.
- Exogenous variables are variables outside of a decision-maker's control.
- Endogenous variables are variables determined within the models.

Figure 1.1 Structure of a Typical Microeconomic Model



Optimization assumptions

- Economic actors are rationally pursuing a goal.
  - Consumers: maximize utility
  - Firms: maximize profits (or minimize costs)
  - Government regulators: maximize public welfare
- Optimization assumptions are useful for generating precise, solvable models.
- Optimization models appear to perform fairly well in explaining reality.

Example 1.1 Profit Maximization

- A firm can sell all the output that it wishes at an exogenously determined price of p per unit. Total costs of production, C, depend on the amount produced, q

$$\text{Profits} = \pi = pq - C(q)$$

The first-order condition for a maximum:

$$\frac{d\pi}{dq} = p - C'(q) = 0, \text{ or } p = C'(q)$$

The second-order condition:

$$\frac{d^2\pi}{dq^2} = -C''(q) < 0, \text{ or } C''(q^*) > 0$$

Marginal cost must be increasing at q^*

- The model can now be used to "predict" how a firm will react to a change in price (comparative statics).
- Differentiate the first order condition with respect to p yields

$$\frac{d[p - C'(q^*) = 0]}{dp} = 1 - C''(q^*)\frac{dq^*}{dp} = 0$$

$$\Rightarrow \frac{dq^*}{dp} = \frac{1}{C''(q^*)} > 0$$

- This is a testable proposition that a price-taking firm should respond to an increase in price by increasing output.

Positive-normative distinction

- The third feature of economic models is the attempt to differentiate between “positive" and “normative" questions.
- Positive economic theories seek to explain the economic phenomena that are observed.
- Normative economic theories focus on what “should” be done.
- Some economists believe that the only proper economic analysis is positive analysis.
- For other economists, the positive-normative distinction seems artificial, the stud of economics necessarily involves the researchers’ own views.

Development of Economic Theory of Value

Early economic thoughts on “value"

- The theory of value concerns the determinants of the “value" of a commodity.
- Today we regard value as being synonymous with the price of a commodity.
- Most of the early writings about economics, however, sought to establish the idea of a “just price" for some items and examine how actual market prices conformed to this ideal.
- This distinction was most clearly illustrated by arguments about whether interest payment on loans were “just," throughout the 14th and 15th centuries.

The founding of modern economics

- During the latter part of the 18th century, philosophers began to take a scientific approach to economic questions by focusing more explicitly on the mechanism by which prices are determined.
- The 1776 publication of *The wealth of nations* by Adam Smith is considered the beginning of modern economics.
- Smith and his successors, such as David Ricardo continued to struggle in finding a way to describe the relationship between value and price.
- To Smith, the value of a commodity meant its “value in use," whereas the price represented its “value in exchange."
- The distinction was illustrated by the famous water-diamond paradox. Water, has great value in use, has little value in exchange, while diamonds are of little use but have a great value in exchange.

Labor theory of exchange value

- The concept of value in use was left for philosophers to debate, while economists turned their attention to explaining the determinants of value in exchange (i.e. to explain relative prices).
- One possible explanation is that exchange values of goods are determined by the costs of producing them. And cost of production are primarily affected by labor costs. Therefore, it was a short step to embrace a labor theory of value.
- For the diamond-water paradox, diamonds are relatively costly because their production requires more labor input, whereas water is freely available.
- Long-run exchange values were assumed to be determined solely by labor costs of production.

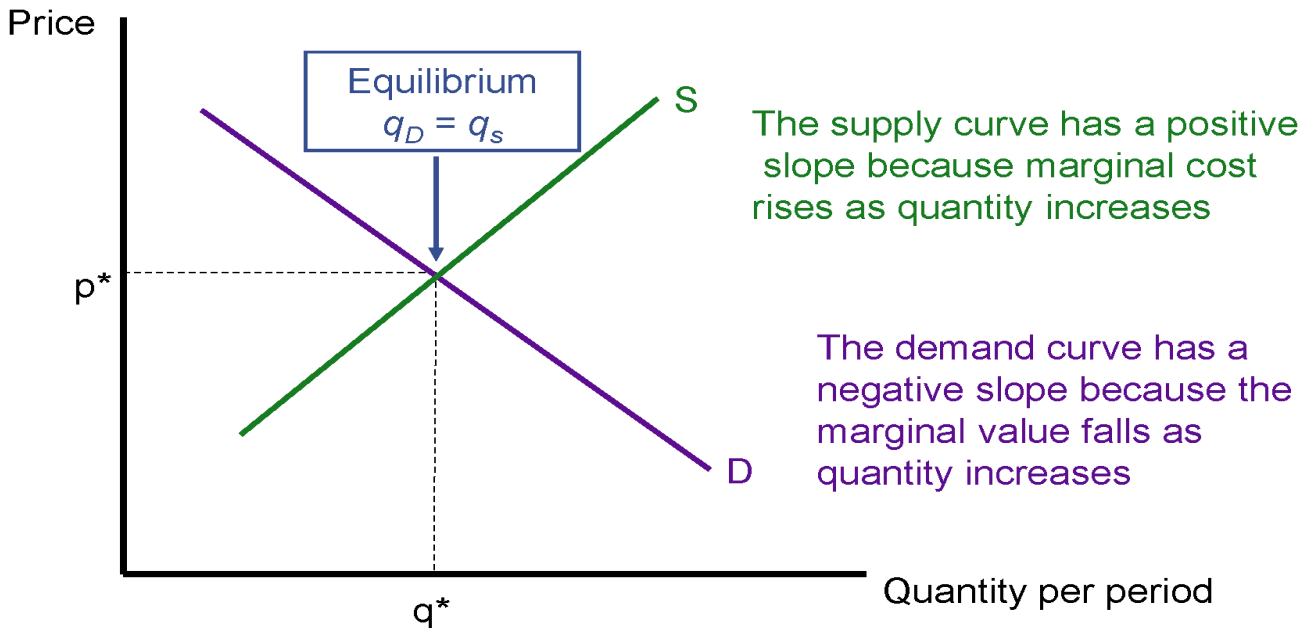
The marginalist revolution

- Between 1850 and 1880, economists became increasingly aware that to construct an adequate alternative to the labor theory of value, they had to devise a theory of value in use.
- During the 1870s, several economists discovered that the exchange value of an item is determined by the usefulness of the *last unit consumed*.
- Since water is plentiful, consuming an additional unit has a relatively low value.

Marshallian supply-demand synthesis

- The clearest statement of these marginal principles was presented by the English economist Alfred Marshall (1842-1924) in his *Principles of Economics*, published in 1890.
- Marshall showed that demand and supply *simultaneously* operate to determine price.
- That analysis is illustrated by the famous Marshallian cross shown in Figure 1.2.
- Prices reflect both the marginal valuation that consumers place on goods and the marginal costs of producing the goods.

Figure 1.2 The Marshallian Supply-Demand Cross



- Marshall theorized that demand and supply interact to determine the equilibrium price (p^*) and the quantity (q^*) that will be traded in the market.
- He concluded that it is not possible to say that either demand or supply alone determines price or therefore that either costs or usefulness to buyers alone determines exchange value.

Example 1.2 Supply-Demand Equilibrium

Peanut market:

- Quantity demanded = $q_D = 1000 - 100p$
- Quantity supplied = $q_S = -125 + 125p$
- Equilibrium $\Rightarrow q_D = q_S$

$$\begin{aligned} 1000 - 100p &= -125 + 125p \\ 225p &= 1125 \\ p^* &= 5 \\ q^* &= 500 \end{aligned}$$

- A more general model is

$$\begin{aligned} q_D &= a + bp \\ q_S &= c + dp \end{aligned}$$

Equilibrium $\Rightarrow q_D = q_S$

$$\begin{aligned} a + bp &= c + dp \\ p^* &= \frac{a - c}{d - b} \end{aligned}$$

- What happens to the equilibrium price if either demand or supply shift?
- An increase in demand (an increase in a) increases equilibrium price
- An increase in supply (an increase in c) reduces price

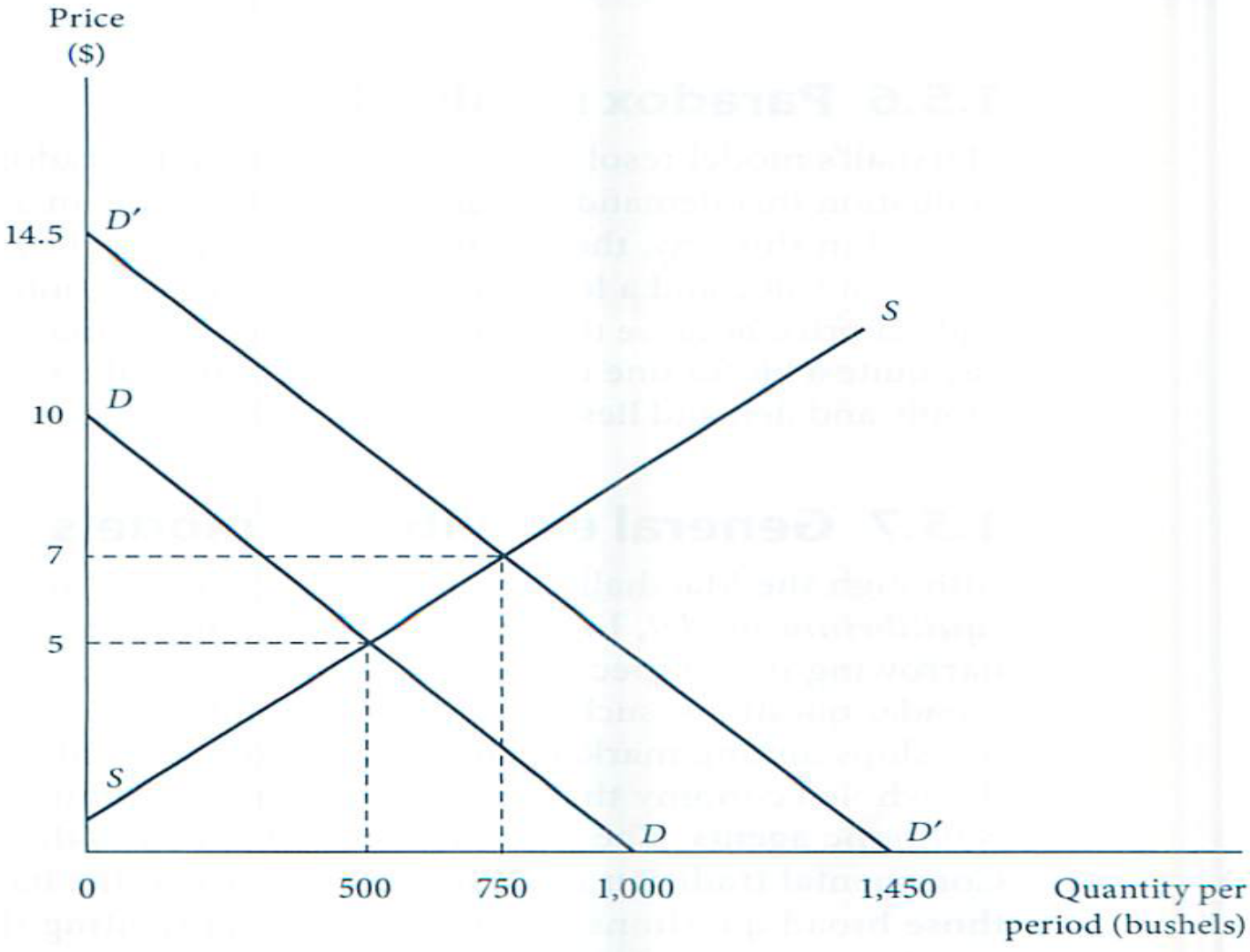
$$\frac{dp^*}{da} = \frac{1}{d-b} > 0$$
$$\frac{dp^*}{dc} = \frac{-1}{d-b} < 0$$

- A shift in demand will lead to a new equilibrium:

$$q'_D = 1450 - 100p$$

$$\begin{aligned} q'_D &= 1450 - 100p = q_S = -125 + 125p \\ 225p &= 1575 \\ p^* &= 7 \\ q^* &= 750 \end{aligned}$$

Figure 1.3 Changing Supply–Demand Equilibria



Paradox resolved

- Marshall's model resolves the water-diamond paradox.
- Water

  Low marginal value

  Low marginal cost of production

  Low price
- Diamonds

  High marginal value

  High marginal cost of production

  High price

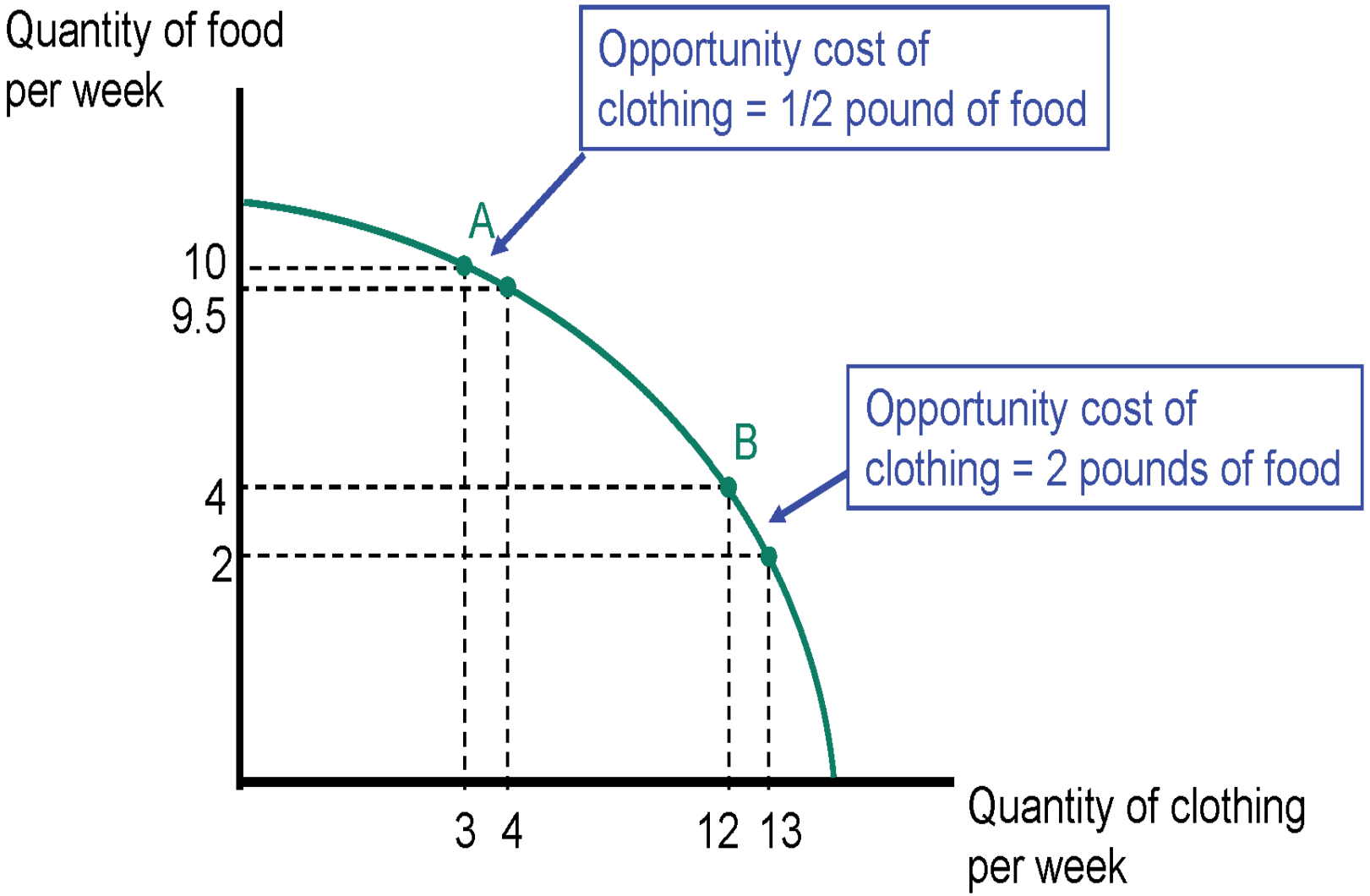
General equilibrium models

- The Marshallian model is a *partial equilibrium model* It focuses only on one market at a time.
- For more general questions, we need a model of the entire economy that mirrors the connections among various markets and economic agents.
- French economist Leon Walras (1831-1910) created the basis for modern investigations into those broad questions.
- His method of representing the economy by a large number of simultaneous equations forms the basis for understanding the interrelationships implicit in *general equilibrium analysis*.
- Several models of this type are described in Chapter 13.

Production possibility frontier

- Because the production possibility frontier shows two goods, rather than the single good in Marshall's model, it is used as a basic building block for general equilibrium models.
- Production possibility frontier shows the combinations of two outputs that can be produced with an economy's resources.
- Producing more of one good means producing less of another good because resources are scarce.
- The production possibility frontier shows that opportunity costs depend on how much of each good is produced.

Figure 1.4 Production Possibility Frontier



Example 1.3 A Production Possibility Frontier and Economic Inefficiency

- An economy produces two goods, x and y, using labor as the only input.
- Production function for good x is $x = l_x^{0.5}$, where l_x is the quantity of labor used in x production.
- The production function for good y is $y = 2l_y^{0.5}$, where l_y is the quantity of labor used in y production.

  Total labor available: $l_x + l_y \leq 200$
- Production possibilities frontier is

$$l_x + l_y = x^2 + 0.25y^2 \leq 200$$

- Assuming this economy is on the frontier, the opportunity cost of good y in terms of good x can be derived by solving for y as

$$\begin{aligned} y^2 &= 800 - 4x^2, \\ \text{or } y &= \sqrt{800 - 4x^2} = \left[800 - 4x^2\right]^{0.5} \end{aligned}$$

Differentiating this expression, we have

$$\frac{dy}{dx} = 0.5(800 - 4x^2)^{-0.5}(-8x) = \frac{-4x}{y}$$

When $x = 10$, $y = 20$, $\frac{dy}{dx} = \frac{-4 \cdot 10}{20} = -2$.

Concavity

- The slope of the frontier becomes steeper (more negative) as x output increases and y output decreases.
- When $x = 12$, $y \approx 15$, $dy/dx = -4(12)/15 = -3.2$

Inefficiency

- Economy operating inside its production possibility frontier is inefficient.

  For example, 20 workers are permanently unemployed.

  $x^2 + 0.25y^2 = 200 - 20 = 180$

  When $x = 10$, then $y \approx 17.9$

Welfare economics

- The tools used in general equilibrium analysis have been applied to the study of normative questions about the welfare properties of various economic arrangement.
- The most significant advances in welfare properties were made by the British economist Francis Y. Edgeworth (1848-1926) and the Italian economist Vilfredo Pareto (1848-1923) in the early years of the 20th cnetury.
- These economists helped to provide a precise definition for the concept of “economic efficiency" and to demonstrate the conditions under which markets will be able to achieve it.
- By clarifying the relationship between the allocation and pricing of resources they provided support for the idea that properly functioning markets provide an “invisible hand" that helps allocate resources efficiently.

Modern Developments

The mathematical foundations of economic models

- A major postwar development in microeconomics theory was the clarification and formalization of the basic assumptions that are made about individuals and firms.
- The first landmark in this development was the 1947 publication of Paul Samuelson's *Foundation of Economic Analysis,* in which the author laid out a number of models of otpimizing behavior.
- Samuelson demonstrated the importance of basing behavior models on well-specified mathematical postulates so that various optimization techniques from mathematics can be applied.

New tools for studying markets

- A second feature of the new development is a number of new tools for explaining market equilibria.
- These include techniques for describing pricing in single markets, such as increasing sophisticated models of monopolistic pricing or models of the strategic relationships among firms that use game theory.
- They also include general equilibrium tools for simulataneously exploring relationships among many markets.

The economics of uncertainty and information

- The third major theoretical advance was the incorporation of uncertainty and imperfect information into economic models.
- Some of the basic assumptions used to study behavior in uncertain situations were originally developed in 1940s in connection with the theory of games.
- Later developments showed how these ideas could be used to explain why individuals tend to be averse to risks and how they might gather information to reduce the uncertainties they face.

Behavioral economics

- A final theoretical advance is reflected in attempts to make economic models more realistic in terms of how they describe the decision economic actors make,
- By drawing on insights from psychology and other social sciences, these models seek to illustrate how imperfect information or various systematic biases can be used to explain why decisions may not always conform to the "rational" assumptions.

Computers and empirical analysis

- One other aspect of the postwar development of microeconomics is the increasing use of computers to analyze economic data and build economic models.
- Economists' ability to test their theories has dramatically improved.
- Today's economists have available a wide variety of sophisticated techniques together with extensive microeconomic data with which to test their models.