Suggested Answers for Problem Set 6

Jan. 10, 2003

1a All the regression results can be found in the log file, ps6q1.log.
The OLS regression, the result is

$$\begin{aligned} Y &= 1906.16 + 0.241X, \quad R^2 = 0.463 \\ s.e. &= (920.66) \;\; (0.098) \\ t &= \;\; 2.07 \qquad 2.46 \end{aligned}$$

1b The coefficient of $\ln X$ is -2.477 with standard error 4.14 and t-value 0.60. The result indicate that there is no heteroscedasticity according to the Park test.

1c The results are

$$\begin{aligned} |\hat{u}_i| &= 383.36 - 0.018X_i \\ s.e. &= (629.85) \;\; (0.067) \\ t &= \;\; 0.61 \qquad -0.27 \\ |\hat{u}_i| &= 530.67 - 3.31\sqrt{X_i} \\ s.e. &= (1277.4) \;\; (13.24) \\ t &= \;\; 0.42 \qquad -0.25 \end{aligned}$$

Both are insignificant, there is no hoteroscedasticity according to the specifications of Glejser test.

2a All the regression results can be found in the log file, ps6q2.log.
The OLS regression is

$$\begin{aligned} Y &= 193.06 + 0.032X \\ s.e. &= (990.97) \;\; (0.008) \\ t &= \;\; 0.19 \qquad 3.83 \end{aligned}$$

2b The explained sum of squares of regressing p_i on X_i is 17.82. Therefore, the χ^2 statistic is $\frac{1}{2}17.82 = 8.91$ with degree of freedom 1. It is significant at a 95 % level of confidence. The null hypothesis that the error term is homoscedastic can be rejected.

2c The White $n \cdot R^2 = 18 \cdot 0.2896 = 5.213$ with d.f.=2. Therefore, at the 95% level, do not reject the null that the model is homoscedastic. However, at the 90% level, one would reject the null that the model is homoscedastic.

2d The White's heteroscedasticity-consistent standard error is 0.010, which is greater than the standard error in (a).

3a

$$\begin{aligned}
\text{Var}(u_t) &= \rho^2\text{Var}(u_{t-1}) + \text{Var}(\epsilon_t) = \rho^2\text{Var}(u_t) + \text{Var}(\epsilon_t) \\
(1-\rho^2)\text{Var}(u_t) &= \sigma^2 \\
\text{Var}(u_t) &= \frac{\sigma^2}{1-\rho^2}
\end{aligned}$$

3b

$$\begin{aligned}
\text{Cov}(u_t, u_{t-1}) &= \text{E}[(\rho u_{t-1} + \epsilon_t)u_{t-1}] = \rho\text{Var}(u_{t-1}) = \rho\frac{\sigma^2}{1-\rho^2} \\
\text{Cov}(u_t, u_{t-2}) &= \text{E}[(\rho u_{t-1} + \epsilon_t)u_{t-2}] = \text{E}[(\rho^2 u_{t-2} + \rho\epsilon_{t-1} + \epsilon_t)u_{t-2}] \\
&= \rho^2\text{Var}(u_{t-2}) = \rho^2\frac{\sigma^2}{1-\rho^2}
\end{aligned}$$

In general, the covariance between u_t and u_{t-s} is $\rho^s\text{Var}(u_{t-s}) = \rho^s\frac{\sigma^2}{1-\rho^2}$.

3c

$$\begin{aligned}
\text{Var}(u) &= \text{E}(uu') = \begin{bmatrix} \text{Var}(u_1) & \text{Cov}(u_1, u_2) & \cdots & \text{Cov}(u_1, u_T) \\ \text{Cov}(u_2, u_1) & \text{Var}(u_2) & \cdots & \text{Cov}(u_2, u_T) \\ \vdots & \vdots & \cdots & \vdots \\ \text{Cov}(u_1, u_T) & \text{Cov}(u_2, u_T) & \cdots & \text{Var}(u_T) \end{bmatrix} \\
&= \begin{bmatrix} \frac{\sigma^2}{1-\rho^2} & \rho\frac{\sigma^2}{1-\rho^2} & \cdots & \rho^{T-1}\frac{\sigma^2}{1-\rho^2} \\ \rho\frac{\sigma^2}{1-\rho^2} & \frac{\sigma^2}{1-\rho^2} & \cdots & \rho^{T-2}\frac{\sigma^2}{1-\rho^2} \\ \vdots & \vdots & \cdots & \vdots \\ \rho^{T-1}\frac{\sigma^2}{1-\rho^2} & \rho^{T-2}\frac{\sigma^2}{1-\rho^2} & \cdots & \frac{\sigma^2}{1-\rho^2} \end{bmatrix} \\
&= \frac{\sigma^2}{1-\rho^2}\begin{bmatrix} 1 & \rho & \cdots & \rho^{T-1} \\ \rho & 1 & \cdots & \rho^{T-2} \\ \vdots & \vdots & \cdots & \vdots \\ \rho^{T-1} & \rho^{t-2} & \cdots & 1 \end{bmatrix}
\end{aligned}$$

4 Since there is no intercept, the OLS estimate of β_2 is

$$\begin{aligned}
\hat{\beta}_2 &= \frac{X_i Y_i}{\sum X_i^2} = \frac{\sum X_i(\beta_2 X_i + u_i)}{\sum X_i^2} = \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2} \\
\text{Var}(\hat{\beta}_2) &= \text{E}[(\hat{\beta}_2 - \beta_2)^2] = \text{E}\left[\left(\frac{\sum X_i u_i}{\sum X_i^2}\right)^2\right] = \frac{\sum X_i^2\text{E}(u_i^2)}{(\sum X_i^2)^2} \\
&= \frac{\sum X_i^2\sigma^2 X_i^2}{(\sum X_i^2)^2} = \frac{\sigma^2\sum X_i^4}{(\sum X_i^2)^2}
\end{aligned}$$