Suggested Answers for Problem Set 4

Dec. 6, 2002

1a

$$\begin{aligned} ABC &= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 \\ 7 & 4 \end{bmatrix} \\ CAB &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 1 \\ 5 & 1 \end{bmatrix} \\ C'B'A' &= \begin{bmatrix} 4 & 7 \\ 2 & 4 \end{bmatrix} \end{aligned}$$

1b From (a), $(ABC)' = \begin{bmatrix} 4 & 7 \\ 2 & 4 \end{bmatrix} = C'B'A'$

1c

$$\begin{aligned} (ABC)^{-1} &= \frac{1}{|ABC|} \begin{bmatrix} 4 & -2 \\ -7 & 4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3.5 & 2 \end{bmatrix} \\ C^{-1} &= \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \\ (AB)^{-1} &= \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ -1.5 & 1 \end{bmatrix} \end{aligned}$$

1d From (c), $C^{-1}(AB)^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ -1.5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3.5 & 2 \end{bmatrix} = (ABC)^{-1}$

2 Let $A : n \times K$, $B : K \times T$, $C : K \times T$ and $D = A(B+C)$, $E = AB$, $F = AC$.

$$\begin{aligned} d_{ij} &= \sum_{k=1}^{K} a_{ik}(b_{kj} + c_{kj}) = \sum_{k=1}^{K} a_{ik}b_{kj} + a_{ik}c_{kj} \\ &= e_{ij} + f_{ij} \\ \text{therefore } A(B+C) &= AB + AC \end{aligned}$$

3a The test statistics are

$$t_2 = \frac{\hat{\beta}_2 - 1}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{0.9576 - 1}{0.3022} = -0.1403$$
$$t_3 = \frac{\hat{\beta}_3 - 1}{\hat{\sigma}_{\hat{\beta}_3}} = \frac{0.8242 - 1}{0.3571} = -0.4923$$

The critical value is 2.179. Therefore, we do not reject the null that both elasticities are statistically equal to one.

3b The test statistic, assuming zero covariancem, is

$$t = \frac{0.9576 - 0.8242}{\sqrt{0.0913 + 0.1275}} = 0.2852$$

The test statistic, assuming a -0.0972 covariance, is

$$t = \frac{0.9576 - 0.8242}{\sqrt{0.0913 + 0.1275 - 2(-0.0972)}} = 0.2075$$

The critical value is 2.179. Therefore, in either case, we do not reject the null that the elasticities are statistically equal.

4 One can see model B as

$$\begin{aligned} Y_t &= \beta_1 + X_{2t} + \beta_2 X_{2t} + \beta_3 X_{3t} + u_{2t} \\ &= \beta_1 + (1 + \beta_2) X_{2t} + \beta_3 X_{3t} + u_{2t} \\ &= \beta_1 + \beta_2^* X_{2t} + \beta_3 X_{3t} + u_{2t} \end{aligned}$$

where $\beta_2^* = 1 + \beta_2$. Thus, Models A and B are very similar, they both explain Y_t with the same explanatory variables.

4a Yes, the estimates for α_1 and β_1 will be the same.

4b Yes, the estimates for α_3 and β_3 are the same.

4c $\beta_2^* = 1 + \beta_2 = \alpha_2$.

4d No, since the dependent variables are not the same.

5a Let the coefficient for $\ln K$ be $\beta^* = \beta_2 + \beta_3 - 1$. Setting up the null hypothesis that $\beta^* = 0$ can test whether $\beta_2 + \beta_3 = 1$.

5b No, the analysis is symmetric. We can test whether the coefficient of $\ln L$ is zero.

6a

就讀大學比例 (%)		
1980	1990	2000
8.53	10.59	24.33

6b

就讀大學比例 (%)			
	1980	1990	2000
男性	9.25	10.16	22.77
女性	7.68	11.12	26.22
男女差異	1.57	-0.96	3.45

6c β_2 表示男性就讀大學比例與女性就讀大學比例的差距, 其估計係數分別爲 1.58%,-0.96% 和-3.46%, 男女差距縮小, 2000 年時女性已超越男性。

6d 詳細迴歸係數請見附檔。除了各年的「台北縣」和 1990 年的「性別」不顯著之外, 其他係數在各年都是正向且顯著的, 尤其「父親教育年數」與「母親教育年數」在 1990-2000 年間的重要性是增加的。