Econometrics I
Problem Set 4
Nov. 29, 2002
Due: Dec. 6, 2002

1. Consider $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

   (a) Compute ABC, CAB and $C'B'A'$.

   (b) Verifty that $(ABC)' = C'B'A'$.

   (c) Find the inverse of ABC and C and AB.

   (d) Verify that $(ABC)^{-1} = C^{-1}(AB)^{-1}$

2. Show that $A(B + C) = AB + AC$, where A is a $n \times K$ matrix, B and C are both $K \times T$ matrices.

3. In an application of the Cobb-Douglas production function the following results were obtained:

$$\begin{aligned} \ln \hat{Y_i} &= 2.3542 + 0.9576 \ln X_{2i} + 0.8242 \ln X_{3i} \\ & \qquad\qquad (0.3022) \qquad\qquad (0.3571) \\ & R^2 = 0.8432, \; df = 12 \end{aligned}$$

   where Y = output, X_2 = labor input, and X_3 = capital input, and where the figures in parentheses are the estimated standard errors.

   (a) The coefficients of the labor and capital inputs give the elasticities of output with respect to labor and capita. Test the hypotheis that these elasticities are *individually* equal to unity.

   (b) Test the hypotheis that the labor and capital elasticities are equal, assuming (i) the covariance between the estimated labor and capital coefficients is zero, and (ii) it is -0.0972.

4. Consider the following models.

$$\text{Model A} : Y_t = \alpha_1 + \alpha_2 X_{2t} + \alpha_3 X_{3t} + u_{1t}$$
$$\text{Model B} : Y_t - X_{2t} = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_{2t}$$

   (a) Will OLS estimates of α_1 and β_1 be the same? Why?

   (b) Will OLS estimates of α_3 and β_3 be the same? Why?

(c) What is the relationship between α_2 and β_2?

(d) Can we compare the R^2 of the two models? Why or why not?

5. Consider the Cobb-Douglas production function

$$Y = \beta_1 L^{\beta_2} K^{\beta_3} \tag{1}$$

where Y = output, L = labor input, and K = capital input. Dividing (1) through by K, we get

$$\frac{Y}{K} = \beta_1 \left(\frac{L}{K}\right)^{\beta_2} K^{\beta_2+\beta_3-1}$$

Takeing the natural log, we obtain

$$\ln\left(\frac{Y}{K}\right) = \beta_0 + \beta_2 \ln\left(\frac{L}{K}\right) + (\beta_2 + \beta_3 - 1) \ln K \tag{2}$$

where $\beta_0 = \ln \beta_1$.

(a) Suppose you had data to run regression (2). How would you test the hyothesis that there are constant returns to scale, i.e., $\beta_2 + \beta_3 = 1$.

(b) Does it make any difference whether we divide (1) by L rahter than by K?

6. ps4.dta 取自 1980, 1990 和 2000 年的「人力資源調查」中 19-22 歲人口及其父母的資訊, 其中七個變數, 變數內容如 label 所示。使用 ps4.dta 回答以下問題。

(a) 1980, 1990 和 2000 年時, 各有多少比例的 19-22 歲人口就讀於大學? 有何長期趨勢?

(b) 1980, 1990 和 2000 年時, 男女性就讀大學的比例各爲多少? 男女性就讀大學比例的差異各爲多少? 有何長期趨勢?

(c) 對 1980、1990 和 2000 年, 分別估計下列迴歸模型:

$$\text{c} = \beta_1 + \beta_2 \text{sex} + u$$

此時 sex 的係數 β_2 代表的是什麼? (提示: 與 (b) 作一比較。)

(d) 對 1980、1990 和 2000 年, 分別估計下列迴歸模型:

$$\text{c} = \beta_1 + \beta_2 \text{ sex} + \beta_3 \text{ fschyr} + \beta_4 \text{ mschyr} + \beta_5 \text{ taipei} + \beta_6 \text{ tpc} + u$$

說明當就讀大學比例持續增加時, 各個解釋變數對是否就讀大學影響的長期趨勢。