## Econometrics I

Problem Set 1
Sep. 27, 2002
Due: Oct. 4, 2002

1. The joint density function of two continuous random variables $X$ and $Y$ is as follows

$$
\begin{aligned}
f(X, Y) & =\frac{1}{3}(4-X-Y), \quad 0 \leq X \leq 1, \quad 0 \leq Y \leq 1 \\
& =0, \quad \text { otherwise } .
\end{aligned}
$$

(a) Find the marginal density functions, $f(X)$ and $f(Y)$.
(b) Find the conditional density functions, $f(X \mid Y)$ and $f(Y \mid X)$.
(c) Find $\mathrm{E}(X)$ and $\mathrm{E}(Y)$.
(d) Find $\mathrm{E}(X \mid Y=0.4)$.
2. Suppose that the joint probability distribution of $X$ and $Y$ is given by the following table:

| $(\mathrm{x}, \mathrm{y})$ | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: |
| 1 | 0.2 | 0 | 0.2 |
| 2 | 0 | 0.2 | 0 |
| 3 | 0.2 | 0 | 0.2 |

(a) Are $X$ and $Y$ independent? Explain.
(b) Find the marginal distributions of $X$ and $Y$.
(c) Find the conditional distribution of $Y$ given $X=1$ and hence $\mathrm{E}(Y \mid X=1)$ and $\operatorname{Var}(Y \mid X=1)$.
3. Are the following models linear regression model? Why or why not?
(a) $Y_{i}=e^{\beta_{1}+\beta_{2} X_{i}+u_{i}}$
(b) $Y_{i}=\frac{1}{1+e^{\beta_{1}+\beta_{2} X_{i}+u_{i}}}$
(c) $\ln Y_{i}=\beta_{1}+\beta_{2} \frac{1}{X_{i}}+u_{i}$
(d) $Y_{i}=\beta_{1}+\left(0.75-\beta_{1}\right) e^{-\beta_{2}\left(X_{i}-2\right)}+u_{i}$
(e) $Y_{i}=\beta_{1}+\beta_{2}^{3} X_{i}+u_{i}$

