Suggested Answers for Midterm Exam. II

Dec. 14, 2001

1a True. $(AB)' = B'A' = BA = AB$. Therefore, AB is symmetric.

1b False. $\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-K}$, then $\bar{R}^2 < R^2$ for $K > 1$.

1c True. There are more explanatory variables in (2), therefore it has smaller residual sum of squares.

1d False. Since $|A| = 0$, the inverse of A does not exist.

2a $\sum u_i^2 = u'u = (y - X\beta)'(y - X\beta)$.

2b

$$\begin{aligned} u'u &= (y - X\beta)'(y - X\beta) = (y' - \beta'X')(y - X\beta) \\ &= y'y - \beta'X'y - y'X\beta + \beta'X'X\beta = y'y - 2\beta'X'y + \beta'X'X\beta \end{aligned}$$

2c The first order conditon is

$$\begin{aligned} \frac{\partial u'u}{\partial \beta} &= -2X'y + 2X'X\beta = 0 \\ X'X\beta &= X'y \\ \hat{\beta} &= (X'X)^{-1}X'y \end{aligned}$$

2d

$$\begin{aligned} \beta &= (X'X)^{-1}X'y = (X'X)^{-1}X'(X\beta + u) = \beta + (X'X)^{-1}X'u \\ \mathrm{E}(\hat{\beta}) &= \beta + (X'X)^{-1}X'\mathrm{E}(u) = \beta \end{aligned}$$

2e

$$\begin{aligned} \mathrm{Var}(\hat{\beta}) &= \mathrm{E}[(\hat{\beta} - \mathrm{E}(\hat{\beta}))(\hat{\beta} - \mathrm{E}(\hat{\beta}))'] \\ &= \mathrm{E}[(X'X)^{-1}X'uu'X(X'X)^{-1}] \\ &= (X'X)^{-1}X'\mathrm{E}(uu')X(X'X)^{-1} \\ &= (X'X)^{-1}X'\sigma^2 IX(X'X)^{-1} = \sigma^2(X'X)^{-1} \end{aligned}$$

3 We have $\beta_2 = \alpha - \beta_1$ and $\beta_3 = -\alpha - \beta_1$. Substituting these in the given equation we get

$$\begin{aligned} Y_i &= \beta_1 X_{1i} + (\alpha - \beta_1)X_{2i} + (-\alpha - \beta_1)X_{3i} + u_i \\ &= \beta_1(X_{1i} - X_{2i} - X_{3i}) + \alpha(X_{2i} - X_{3i}) + u_i \end{aligned}$$

Define $Z_{1i} = X_{1i} - X_{2i} - X_{3i}$ and $Z_{2i} = X_{2i} - X_{3i}$ and estimate a regression of Y_i on Z_{1i} amd Z_{2i} without constant term. The estimated coefficient of Z_{2i} gives an estimate of α. Its variance can be obtained as usual.

4a

$$\begin{aligned} E(Y_i|D_i = 1) &= \alpha_1 + \alpha_2 + \beta X_i \\ E(Y_i|D_i = 0) &= \alpha_1 + \beta X_i \\ E(Y_i|D_1 = 1) - E(Y_i|D_i = 0) &= \alpha_2 \end{aligned}$$

4b

$$\begin{aligned} E(Y_i|D_i = 1) &= \alpha_1 + \alpha_2 + \beta X_i \\ E(Y_i|D_i = 2) &= \alpha_1 + 2\alpha_2 + \beta X_i \\ E(Y_i|D_1 = 2) - E(Y_i|D_i = 1) &= \alpha_2 \end{aligned}$$

α_2 is still the difference of male and female average salaries.

4c

$$\begin{aligned} E(Y_i|D_i = 1) &= \alpha_1 + \alpha_2 + \beta X_i \\ E(Y_i|D_i = -1) &= \alpha_1 - \alpha_2 + \beta X_i \\ E(Y_i|D_1 = -1) - E(Y_i|D_i = 1) &= -2\alpha_2 \end{aligned}$$

Therefore, the difference of male female avaerage salaries is $-2\alpha_2$.

4d

$$\begin{aligned} E(Y_i|D_i = 5) &= \alpha_1 + 5\alpha_2 + \beta X_i \\ E(Y_i|D_i = 0) &= \alpha_1 + \beta X_i \\ E(Y_i|D_1 = 5) - E(Y_i|D_i = 0) &= 5\alpha_2 \end{aligned}$$

The difference of male and female average salaries is $5\alpha_2$.

5a

	men	women
intercept	9.35	8.96
slope	0.0656	0.0744

5b Men: 6.56%, Women: 7.44%.

5c Lower since the coefficient of sex*s is significantly negative.

5d The coefficient will be -0.3840 and the standard error will be 0.0174.