Suggested Answers for Midterm Exam. II

Dec. 20, 2002

1a True. Since A is symmetric, then $A' = A$. $(A^{-1})' = (A')^{-1} = A^{-1}$, so A^{-1} is also symmetric.

1b False. R^2 will definitely decrease but $\bar{R}^2$ will not necessarily decrease since $\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-K}$, $\frac{\partial \bar{R}^2}{\partial K} = \frac{n-1}{n-K}\frac{\partial R^2}{\partial K} - (1 - R^2)\frac{n-1}{(n-K)^2} \gtrless 0$

1c True. To minimize $\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n}(Y_i - \hat{\beta}X_i)^2$, the first order condition is $\sum_{i=1}^{n}(-2)(Y_i - \hat{\beta}X_i)X_i = 0$. Therefore, the OLS estimator of β is $\hat{\beta} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2}$.

1d False. In general $AB \neq BA$.

2a $\mathrm{Var}(\beta_K - \beta_L) = \mathrm{Var}(\hat{\beta}_K) + \mathrm{Var}(\hat{\beta}_L) - 2\mathrm{Cov}(\beta_K, \beta_L) = 0.16 + 0.045 - 2 * 0.0225 = 0.16$. $t = \frac{0.632 - 0.452}{\sqrt{0.16}} = \frac{0.18}{0.4} = 0.45$, insignificant.

2b $\mathrm{Var}(\beta_K + \beta_L) = \mathrm{Var}(\hat{\beta}_K) + \mathrm{Var}(\hat{\beta}_L) + 2\mathrm{Cov}(\beta_K, \beta_L) = 0.16 + 0.045 + 2 * 0.0225 = 0.25$. $t = \frac{0.632 + 0.452 - 1}{\sqrt{0.25}} = \frac{0.084}{0.5} = 0.168$, insignificant.

3a $\mathrm{E}[\hat{\beta}^*] = \mathrm{E}[\hat{\beta} + CX\beta + u] = \beta + CX\beta$. For $\hat{\beta}^*$ to be unbiased, we need $CX = 0$.

3b

$$
\begin{aligned}
\hat{\beta}^* &= \beta + (X'X)^{-1}X'u + Cu \\
\mathrm{Var}(\hat{\beta}^*) &= \mathrm{E}[(\hat{\beta}^* - \beta)(\hat{\beta}^* - \beta)'] \\
&= \mathrm{E}\{[(X'X)^{-1}X'u + Cu][(X'X)^{-1}X'u + Cu]'\} \\
&= \mathrm{E}\{[(X'X)^{-1}X'u + Cu][u'XX'X)^{-1} + u'C']\} \\
&= \mathrm{E}[(X'X)^{-1}X'uu'XX'X)^{-1} + Cuu'XX'X)^{-1} + (X'X)^{-1}X'uu'C' + Cuu'C'] \\
&= \sigma^2(X'X)^{-1} + 0 + 0 + \sigma^2CC' \geq \mathrm{Var}(\hat{\beta})
\end{aligned}
$$

3c $\mathrm{Var}(\hat{\beta}^*) \geq \mathrm{Var}(\hat{\beta})$. This is the Gauss-Markov theorem, the OLS estimator of β–$\hat{\beta}$ is the most efficient estimator among the class of the linear unbiased estimators.

4a

$$
\begin{aligned}
\mathrm{E}(Y_i|D_i = 1) &= \beta_1 + \beta_2 + \beta_3X_i + \beta_4X_i \\
\mathrm{E}(Y_i|D_i = 0) &= \beta_1 + \beta_3X_i \\
\mathrm{E}(Y_i|D_1 = 1) - \mathrm{E}(Y_i|D_i = 0) &= \beta_2 + \beta_4X_i
\end{aligned}
$$

4b

$$
\begin{aligned}
\mathrm{E}(Y_i|D_i = -1, X_i = 10) &= \beta_1 - \beta_2 + \beta_3 * 10 - \beta_4 * 10 \\
\mathrm{E}(Y_i|D_i = 1, X_i = 10) &= \beta_1 + \beta_2 + \beta_3 * 10 + \beta_4 * 10 \\
\mathrm{E}(Y_i|D_1 = -1, X_i = 10) - \mathrm{E}(Y_i|D_i = 1, X_i = 10) &= -2\beta_2 - 20\beta_4
\end{aligned}
$$

5a generate c=1 if schyr$>=$ 16
replace c=0 if schyr==.

5b regress c sex

5c male: 26.2-3.5=22.7%, female: 26.2%.

5d $t = \frac{-0.0345699}{0.0099553} = -3.47$. Man is significantly less likely to have a college education.

5e

	son	daughter
intercept	-0.083	-0.086
slope	0.035	0.039

5f

Since the coefficient of $sex * fschyr$ is not significant, the effect of father schooling on c is not significant different for son and daughter.