Econometrics I
Midterm Exam. II
Dec. 7, 2001

1. (20%) State with brief reasons whether the following statements are true, false, or uncertain.

   (a) Suppose A and B are symmetric matrices, and AB are commutative, i.e. $AB = BA$, then AB is also symmetric.

   (b) In a computer report of the OLS regression with 10 explanatory variables, we have two versions of R^2, 0.345 and 0.340, then 0.345 is the adjusted R^2 and 0.340 is the regular R^2.

   (c) A researcher tried two specifications of a regression equation,

   $$Y_i = \alpha_1 + \beta_1 X_i + u_{1i} \quad (1)$$
   $$Y_i = \alpha_2 + \beta_2 X_i + \gamma_2 Z_i + u_{2i} \quad (2)$$

   then $\sum \hat{u}_{1i}^2 \geq \sum \hat{u}_{2i}^2$.

   (d) Let $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$, then the inverse of A is $\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}$

2. (25%) Let the multiple regression model be written as

   $$\underset{n \times 1}{y} = \underset{n \times K}{X} \underset{K \times 1}{\beta} + \underset{n \times 1}{u}$$

   where y is the column vector of dependent variable, X is the matrix of explanatory variables, β is the column vector of the K coefficients and u is the column vector of error terms. Assume that $\mathrm{E}(u) = 0$ and the covariance matrix of u is $\mathrm{E}(uu') = \sigma^2 I$.

   (a) Write $\sum u_i^2$ in the matrix form of u.

   (b) Simplify (a) in terms of y, X and β.

   (c) Find the OLS estimator of β, $\hat{\beta}$.

   (d) What is the mean of $\hat{\beta}$, $\mathrm{E}(\hat{\beta})$?

   (e) Derive the covariance matrix of $\hat{\beta}$, $\mathrm{Var}(\hat{\beta})$.

3. (15%) In the model

   $$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

   the coefficients are known to be related to a more basic economic parameter α according to the equations

   $$\beta_1 + \beta_2 = \alpha$$
   $$\beta_1 + \beta_3 = -\alpha$$

   Explain how you would estimate α and the variance of $\hat{\alpha}$.

4. (20%) Suppose we have the following model,

$$Y_i = \alpha_1 + \alpha_2 D_i + \beta X_i + u_i$$

where Y_i is the annual salary of a college professor, X_i is the years of teaching experience, and D_i is dummy variable with

$$\begin{aligned} D_i &= 1 \text{ if male} \\ &= 0 \text{ if female} \end{aligned}$$

   (a) What is the difference of male and female average salaries in terms of the regression coefficient?

   (b) Suppose the dummy variable is defined as $D_i = 1$ if female and $D_i = 2$ if male. Interpret the coefficient of D_i, α_2.

   (c) Suppose the dummy variable is defined as $D_i = 1$ if female and $D_i = -1$ if male, then what is the difference of male and female average salaries.

   (d) Suppose the dummy variable is defined as $D_i = 5$ if male and $D_i = 0$ if female, then what is the difference of male and female average salaries.

5. (20%) The following table is the OLS result of a wage equation.

Source	SS	df	MS
Model	1.5297859	1	1.5297859
Residual	26.5958015	344	.077313376
Total	28.1255874	345	.081523442

Number of obs	=	25556
F(5, 25550)	=	1962.81
Prob> F	=	0.0000
R-squared	=	0.2775
Adj R-squared	=	0.2774
Root MSE	=	.42892

$\ln y$	Coef.	Std. Err.	t	$P > \|t\|$	[95% Conf.	Interval]
ex	.0323744	.0009177	35.278	0.000	.0305756	.0341731
exsq	−.0004809	.0000166	−29.026	0.000	−.0005134	−.0004484
s	.0743663	.0014142	52.584	0.000	.0715943	.0771383
sex	.3899519	.0174496	22.347	0.000	.3557496	.4241541
sex*s	−.0087273	.0015540	−5.616	0.000	−.0117732	−.0056814
cons.	8.9566050	.0216440	413.815	0.000	8.9141810	8.9990280

The dependent variable is $\ln y$, log of monthly wage, the explanatory variables are (1) ex, experience (2) exsq, experience squared (3) s, years of schooling (4) sex, dummy variable for sex with male=1 (5) sex*s, interaction between sex and s.

   (a) Let experience be zero, draw the regression lines of log monthly wage, $\ln y$, on years of schooling, s, for men and women separately. (s on the horizontal axis, label the intercept and slope for the two regression lines.)

   (b) What are the rates of return of schooling for men and women?

   (c) Is men's rate of return higher or lower than women's rate of return?

   (d) Suppose we change the definition of sex so that 1 indicates female instead of male. What will be the estimated coefficient and standard error of sex?