Econometrics I
Midterm Exam. I
Oct. 26, 2001

1. (20%) State with brief reasons whether the following statements are true, false, or uncertain.

   (a) (5%) If $\hat{\beta}_2$ is statistically significant at a 99% level of confidence, then it must be also practically significant.

   (b) (5%) Gauss-Markov theorem states that given the assumptions of the classical linear regression model, the least-squares estimators are unbiased.

   (c) (5%) OLS estimates of the slope are more precisely estimated if the X-values are closer to their sample mean.

   (d) (5%) When an intercept is included in a classical linear regression model, the sum of residuals equals to zero, $\sum \hat{u}_i = 0$.

2. (15%) In deriving the least-squares estimator for σ^2, we have

$$\begin{aligned} \sum \hat{u}_i^2 &= (\hat{\beta}_2 - \beta_2)^2 \sum x_i^2 + \sum (u_i - \bar{u})^2 - 2(\hat{\beta}_2 - \beta_2) \sum x_i (u_i - \bar{u}) \\ &= A + B + C \end{aligned}$$

   where A, B, C correspond to the three terms of $\sum \hat{u}_i^2$. Calculate E(A), E(B) and E(C) respectively. Therefore, we can show that $\mathrm{E}(\sum \hat{u}_i^2) = (n-2)\sigma^2$. (Note that $\hat{\beta}_2$ can be written as $\hat{\beta}_2 = \beta_2 + \sum k_i u_i$, where $k_i = \frac{x_i}{\sum x_i^2}$, $\sum k_i = 0$ and $\sum k_i x_i = \sum k_i X_i = 1$.)

3. (15%) A random variable X follows the exponential distribution and has the following probability density function:

$$\begin{aligned} f(X) &= \frac{1}{\theta} e^{-\frac{X}{\theta}} \text{ for } X > 0 \\ &= 0 \text{ elsewhere} \end{aligned}$$

   where $\theta > 0$ is the parameter of the distribution. Derive the Maximum-Likelihood estimator of θ.

4. (20%) Consider a classical linear regression model of consumption function, $Y_i = \beta_1 + \beta_2 X_i + u_i$, where Y_i is monthly consumption and X_i is monthly income, both are measured in dollars. Suppose the OLS estimators are $\hat{\beta}_1$ and $\hat{\beta}_2$, and variance for $\hat{\beta}_2$ is $\mathrm{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{\sum \hat{u}_i^2/(n-2)}{\sum x_i^2}$.

(a) (10%) If we change the unit of income to 1,000 dollars and leave the unit of consumption unchanged, what are the estimators of β_1, β_2 and the variance of the estimator of β_2, compare to $\hat{\beta}_1$, $\hat{\beta}_2$ and $\text{Var}(\hat{\beta}_2)$?

(b) (10%) If we change the unit of both income and consumption to 1,000 dollars, what are the estimators of β_1, β_2 and the variance of the estimator of β_2, compare to $\hat{\beta}_1$, $\hat{\beta}_2$ and $\text{Var}(\hat{\beta}_2)$?

5. (20%) Suppose we have observations of monthly wage for 346 pairs of father and son, and run a classical normal linear regression model,

$$\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

Where Y_i is son's monthly wage while X_i is father's monthly wage. The resulting ANOVA table and coefficients are as follows.

Source	SS	df	MS
Model	1.5297859	1	1.5297859
Residual	26.5958015	344	.077313376
Total	28.1255874	345	.081523442

$\ln Y$	Coef.	Std. Err.	t	$P > \lvert t \rvert$	[95% Conf.	Interval]
$\ln X$	.1558127	.035028	4.45	0.000	.0869167	.2247086
cons	8.775105	.3671243	23.90	0.000	8.053014	9.497196

(a) (5%) What is the R^2, coefficient of determination, of the regression?

(b) (5%) Is β_2 significant at a 99% level of confidence?

(c) (5%) What is the F statistic for testing $H_0 : \beta_2 = 0$?

(d) (5%) From the regression results, how will the son's wage change if the father's wage has a 10% increase?

6. (10%) This question is an exercise for data collection. Your answers will not affect the score you get as long as you answer the questions truthfully.

(a) How many hours do you usually spend on studying for this class every week?

(b) How many hours do you spend for preparing this midterm examination?

(c) What is the average score of the two-semester Statistics classes you took?

(d) Where do you live? At home with your family, in school's dormitory or rent a room from your landlord?

(e) Do you have any suggestions about this class such as teaching style, materials to be covered or whatever you want to say to the teacher?