Econometrics I
Midterm Exam. I
Oct. 25, 2002

1. (25%) State with brief reasons whether the following statements are true, false, or uncertain.
   (a) (5%) If the disturbance term in the CLRM is not normally distributed, then the OLS estimators are biased.
   (b) (5%) In the simple regression model, since the variance of the regression coefficient $\hat{\beta}_2$ varies inversally with the variance of X, we should drop all the observations in the middle range of X and use only the extreme observations on X in the calculation of $\hat{\beta}_2$.
   (c) (5%) If $\hat{\beta}_2$ is statistically significant at a 99% level of confidence, then it must also be practically significant.
   (d) (5%) A high p-value means that the coefficient is significantly different from zero.
   (e) (5%) For a simple regression model, $Y_i = \beta_1 + \beta_2 X_i + u_i$. Instead of estimating β_1 and β_2 by OLS, if we force $\hat{\beta}_2$ to be zero and estimate β_1 only, the resulting R^2 will be greater.

2. (15%) The density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx(2-x) & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (5%) Find the value of k.
   (b) (10%) What is E$[X]$ and Var$[X]$?

3. (15%) A random variable X that has the following probability density function:

$$\begin{aligned} f(X) &= \theta X^{\theta-1}, 0 < X < 1 \\ &= 0 \text{ elsewhere} \end{aligned}$$

   where $\theta > 0$ is the parameter of the distribution.

   (a) (5%) Show that $f(X)$ is a proper probability density function.
   (b) (5%) Write down the log-likelihood function for a sample of n observations.
   (c) (5%) Derive the Maximum-Likelihood estimator of θ.

4. (25%) Consider the following classical linear regression model:

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

We have two estimators of β_2,

$$\begin{aligned} \tilde{\beta}_2 &= \frac{1}{n-1}\sum_{i=2}^{n}\frac{Y_i - Y_{i-1}}{X_i - X_{i-1}} \\ \hat{\beta}_2 &= \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}. \end{aligned}$$

(a) (5%) Give a geometric interpretation of $\tilde{\beta}_2$.

(b) (5%) Is $\tilde{\beta}_2$ unbiased?

(c) (7%) Is $\tilde{\beta}_2$ consistent?

(d) (8%) Is $\tilde{\beta}_2$ more efficient than $\hat{\beta}_2$?

5. (20%) Suppose we have observations of monthly wage for 102 pairs of father and son, and run a classical normal linear regression model,

$$\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

Where Y_i is son's monthly wage while X_i is father's monthly wage. The resulting ANOVA table and coefficients are as follows.

Source	SS	df	MSS
Model	1.0188	1	1.0188
Residual	25.47	100	0.2547
Total	26.4888	101	0.2622

(a) (5%) What is the R^2 of the regression?

(b) (5%) What is the F statistic for testing $H_0 : \beta_2 = 0$?

(c) (5%) Is β_2 significantly different from zero at a 95% level of confidence?($t_{0.025} \doteq 1.96$)

(d) (5%) Suppose the standard error of $\hat{\beta}_2$ is 0.1, how will the son's wage change if the father's wage has a 10% increase?