Suggested Answers for Final Exam.

2002.1.18

1a False. The least squares estimators are still unbiased. It is only the variances that are affected. They will be high.

1b False. Even with hetoroscedastic errors, the OLS estimators are unbiased. However, the standard errors are biased.

1c False. The estimators will be unbiased.

1d False. The assumption is that $\rho = 1$.

1e True.

2a $\text{Var}(u_i^*) = \text{Var}(\frac{u_i}{X_i}) = \frac{\text{Var}(u_i)}{X_i^2} = \frac{\sigma^2 X_i^2}{X_i^2} = \sigma^2$, u_i^* is homoscedastic.

2b The equation of transformed variable has only constant term, therefore GLS estimator of β_2 is $\bar{Z}$, the average of $\frac{Y_i}{X_i}$.

2c

$$\begin{aligned}
\hat{\beta}_2^{GLS} &= \frac{\sum(\frac{Y_i}{X_i})}{n} = \frac{\sum(\beta_2 + u_i^*)}{n} = \beta_2 + \frac{\sum \frac{u_i}{X_i}}{n} \\
\text{E}(\hat{\beta}_2^{GLS}) &= \beta_2 + \frac{\sum \frac{\text{E}(u_i)}{X_i}}{n} = \beta_2
\end{aligned}$$

2d

$$\begin{aligned}
\text{Var}(\hat{\beta}_2^{GLS}) &= \text{E}[(\hat{\beta}_2^{GLS} - \beta_2)^2] = \text{E}[(\frac{\sum u_i^*}{n})^2] \\
&= \frac{1}{n^2} \sum \text{Var}(u_i^*) = \frac{1}{n^2} \sum \sigma^2 = \frac{\sigma^2}{n}
\end{aligned}$$

2e Since there is no intercept, the OLS estimate of β_2 is

$$\begin{aligned}
\hat{\beta}_2 &= \frac{X_i Y_i}{\sum X_i^2} = \frac{\sum X_i(\beta_2 X_i + u_i)}{\sum X_i^2} = \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2} \\
\text{Var}(\hat{\beta}_2) &= \text{E}[(\hat{\beta}_2 - \beta_2)^2] = \text{E}\left[\left(\frac{\sum X_i u_i}{\sum X_i^2}\right)^2\right] = \frac{\sum X_i^2 \text{E}(u_i^2)}{(\sum X_i^2)^2} \\
&= \frac{\sum X_i^2 \sigma^2 X_i^2}{(\sum X_i^2)^2} = \frac{\sigma^2 \sum X_i^4}{(\sum X_i^2)^2}
\end{aligned}$$

3a

$$\begin{aligned}
Y_i &= 3.803 + 0.121 X_i \\
s.e. &= (4.570)(0.009)
\end{aligned}$$

3b No. The reason behind using the regression deflated by X is that the errors are heteroscedastic. The fact that R^2 is lower is an irrelevant issue.

3c The equation estimated will be

$$\frac{Y_i}{\sqrt{X_i}} = \frac{\beta_1}{\sqrt{X_i}} + \beta_2\sqrt{X_i} + \frac{u_i}{\sqrt{X_i}}$$

Estimate this equation with no constant term.

3d Higher R^2.

4a Since $0.9108 < d_L$, there is positive autocorrelation.

4b $\hat{\rho} \doteq 1 - \frac{d}{2} = 1 - \frac{0.9108}{2} = 1 - 0.4554 = 0.5446$.

4c $Y_1^* = \sqrt{1-\hat{\rho}^2}Y_1$ and $X_1^* = \sqrt{1-\hat{\rho}^2}X_1$.

4d $Y_t^* = Y_t - \hat{\rho}Y_{t-1}$ and $X_t^* = X_t - \hat{\rho}X_{t-1}, t = 2, \cdots, 24$.

4e Since $d_U = 1.45 < d = 1.83 > 4 - d_L = 2.73$, there is no autocorrelation anymore.