Suggested Answers for Problem Set 6
Jan. 11, 2002

1a All the regression results can be found in the log file, ps6q1.log.
The OLS regression, the result is

$$\begin{aligned} Y &= 1906.16 + 0.241X, \quad R^2 = 0.463 \\ s.e. &= (920.66) \ \ (0.098) \\ t &= \ \ 2.07 \qquad 2.46 \end{aligned}$$

1b The coefficient of $\ln X$ is -2.477 with standard error 4.14 and t-value 0.60. The result indicate that there is no heteroscedasticity according to the Park test.

1c The results are

$$\begin{aligned} |\hat{u}_i| &= 383.36 - 0.018X_i \\ s.e. &= (629.85) \ \ (0.067) \\ t &= \ \ 0.61 \qquad -0.27 \\ |\hat{u}_i| &= 530.67 - 3.31\sqrt{X_i} \\ s.e. &= (1277.4) \ \ (13.24) \\ t &= \ \ 0.42 \qquad -0.25 \end{aligned}$$

Both are insignificant, there is no hoteroscedasticity according to the specifications of Glejser test.

1d The Spearman's rank correlation coefficient is -0.4 and the t statistic is -1.155 which is insignificant with $df = 7$. Therefore, we can not reject the null hypothesis that the model is homoscedastic.

2a All the regression results can be found in the log file, ps6q2.log.
The OLS regression is

$$\begin{aligned} Y &= 193.06 + 0.032X \\ s.e. &= (990.97) \ \ (0.008) \\ t &= \ \ 0.19 \qquad 3.83 \end{aligned}$$

2b The explained sum of squares of regressing p_i on X_i is 17.82. Therefore, the χ^2 statistic is $\frac{1}{2}17.82 = 8.91$ with degree of freedom 1. It is significant at a 95% level of confidence. The null hypothesis that the error term is homoscedastic can be rejected.

2c The White $n \cdot R^2 = 18 \cdot 0.2896 = 5.213$ with d.f.=2. Therefore, at the 95do not reject the null that the model is homoscedastic. However, at the 90% level, one would reject the null that the model is homoscedastic.

2d The White's heteroscedasticity-consistent standard error is 0.010, which is greater than the standard error in (a).

3a

$$\begin{aligned}
\mathrm{Var}(u_t) &= \rho^2\mathrm{Var}(u_{t-1}) + \mathrm{Var}(\epsilon_t) = \rho^2\mathrm{Var}(u_t) + \mathrm{Var}(\epsilon_t) \\
(1-\rho^2)\mathrm{Var}(u_t) &= \sigma^2 \\
\mathrm{Var}(u_t) &= \frac{\sigma^2}{1-\rho^2}
\end{aligned}$$

3b

$$\begin{aligned}
\mathrm{Cov}(u_t, u_{t-1}) &= \mathrm{E}[(\rho u_{t-1} + \epsilon_t)u_{t-1}] = \rho\mathrm{Var}(u_{t-1}) = \rho\frac{\sigma^2}{1-\rho^2} \\
\mathrm{Cov}(u_t, u_{t-2}) &= \mathrm{E}[(\rho u_{t-1} + \epsilon_t)u_{t-2}] = \mathrm{E}[(\rho^2 u_{t-2} + \rho\epsilon_{t-1} + \epsilon_t)u_{t-2}] \\
&= \rho^2\mathrm{Var}(u_{t-2}) = \rho^2\frac{\sigma^2}{1-\rho^2}
\end{aligned}$$

In general, the covariance between u_t and u_{t-s} is $\rho^s\mathrm{Var}(u_{t-s}) = \rho^s\frac{\sigma^2}{1-\rho^2}$.

3c

$$\begin{aligned}
\mathrm{Var}(u) &= \mathrm{E}(uu') = \begin{bmatrix} \mathrm{Var}(u_1) & \mathrm{Cov}(u_1, u_2) & \cdots & \mathrm{Cov}(u_1, u_T) \\ \mathrm{Cov}(u_2, u_1) & \mathrm{Var}(u_2) & \cdots & \mathrm{Cov}(u_2, u_T) \\ \vdots & \vdots & \cdots & \vdots \\ \mathrm{Cov}(u_1, u_T) & \mathrm{Cov}(u_2, u_T) & \cdots & \mathrm{Var}(u_T) \end{bmatrix} \\
&= \begin{bmatrix} \frac{\sigma^2}{1-\rho^2} & \rho\frac{\sigma^2}{1-\rho^2} & \cdots & \rho^{T-1}\frac{\sigma^2}{1-\rho^2} \\ \rho\frac{\sigma^2}{1-\rho^2} & \frac{\sigma^2}{1-\rho^2} & \cdots & \rho^{T-2}\frac{\sigma^2}{1-\rho^2} \\ \vdots & \vdots & \cdots & \vdots \\ \rho^{T-1}\frac{\sigma^2}{1-\rho^2} & \rho^{T-2}\frac{\sigma^2}{1-\rho^2} & \cdots & \frac{\sigma^2}{1-\rho^2} \end{bmatrix} \\
&= \frac{\sigma^2}{1-\rho^2}\begin{bmatrix} 1 & \rho & \cdots & \rho^{T-1} \\ \rho & 1 & \cdots & \rho^{T-2} \\ \vdots & \vdots & \cdots & \vdots \\ \rho^{T-1} & \rho^{t-2} & \cdots & 1 \end{bmatrix}
\end{aligned}$$

4 For $n = 50$ and $k = 4$,

$$d_L = 1.378 \qquad 4 - d_L = 2.622$$
$$d_U = 1.721 \qquad 4 - d_U = 2.279$$

1. $d < d_L$, positive autocorrelation.
2. $d_L < d < d_U$, inconclusive.
3. $4 - d_U < d < 4 - d_L$, inconclusive.
4. $d > 4 - d_L$, negative autocorrelation.