Suggested Answers for Problem Set 5

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1a If there is perfect collinearity, the $X'X$ matrix can not be inverted because the determinant of $X'X$ is zero. The vector of coefficients is unidentified.

1b A test would be to examine the determinant of $X'X$, if it is zero, perfect collinearity exists.

2a If there is perfect multicollinearity, the $X'X$ matrix can not be inverted because the determinant of $X'X$ is zero. The covariance matrix is undefined.

2b If the collinearity is high, the covariance matrix is defined, but the variances will tend to be very large as the determinant of $X'X$ approaches zero as the collinearity gets stronger.

3a Given the relatively high R^2, 0.97, the significant F, and the improperly signed, insignificant coefficient on $\ln K$, it appears there may be multicollinearity in the model.

3b One would expect the sign on $\ln K$ to be positive. It is not, probably due to the collinearity.

4 Since there is no intercept, the OLS estimate of β_2 is

$$\begin{aligned}
\hat{\beta}_2 &= \frac{X_i Y_i}{\sum X_i^2} = \frac{\sum X_i(\beta_2 X_i + u_i)}{\sum X_i^2} = \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2} \\
\mathrm{Var}(\hat{\beta}_2) &= \mathrm{E}[(\hat{\beta}_2 - \beta_2)^2] = \mathrm{E}\left[\left(\frac{\sum X_i u_i}{\sum X_i^2}\right)^2\right] = \frac{\sum X_i^2 \mathrm{E}(u_i^2)}{(\sum X_i^2)^2} \\
&= \frac{\sum X_i^2 \sigma^2 X_i^2}{(\sum X_i^2)^2} = \frac{\sigma^2 \sum X_i^4}{(\sum X_i^2)^2}
\end{aligned}$$