Econometrics I
Problem Set 5
Dec. 21, 2001
Due: Dec. 28, 2001

1. In matrix notation, we have the OLS estimate of β as

$$\hat{\beta} = (X'X)^{-1}X'y$$

   (a) What happens to $\hat{\beta}$ when there is a perfect collinearity among the X's?
   (b) How would you know if perfect collinearity exists?

2. Using matrix notation, we have

$$\text{Cov}(\hat{\beta}) = \sigma^2(X'X)^{-1}$$

   What happens to this covariance matrix

   (a) when there is perfect multicollinearity.
   (b) when collinearity is high but not perfect.

3. Based on the annual data for the U.S. manufacturing sector for 1899-1922, Dougherty obtained the following regression results,

$$\begin{aligned}
\widehat{\ln Y} &= 2.81 - 0.53 \ln K + 0.91 \ln L + 0.047t \qquad (1)\\
s.e. &= (1.38) \quad (0.34) \qquad (0.14) \qquad (0.021)\\
R^2 &= 0.97\\
F &= 189.8
\end{aligned}$$

   where Y = index of real output, K = index of real capital input, L = index of real labor output, t = time or trend.

   (a) Is there multicollinearity in regression (1)? How do you know?
   (b) In regression (1), what is the *a priori* sign of $\ln K$? Does the results conform to this expectation? Why or why not?

4. In the model

$$Y_i = \beta_2 X_i + u_i$$

   Note that there is no intercept in the model. You are told that $\text{Var}(u_i) = \sigma^2 X_i^2$. Show that

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma^2 \sum X_i^4}{(\sum X_i^2)^2}$$