Econometrics I
Problem Set 4
Nov. 23, 2001
Due: Nov. 30, 2001

1. Show that $A(B+C) = AB + AC$, where A is a $n \times K$ matrix, B and C are both $K \times T$ matrices.

2. In an application of the Cobb-Douglas production function the following results were obtained:

$$\ln \hat{Y}_i = 2.3542 + \underset{(0.3022)}{0.9576} \ln X_{2i} + \underset{(0.3571)}{0.8242} \ln X_{3i}$$
$$R^2 = 0.8432, \ df = 12$$

where Y = output, X_2 = labor input, and X_3 = capital input, and where the figures in parentheses are the estimated standard errors.

   (a) The coefficients of the labor and capital inputs give the elasticities of output with respect to labor and capita. Test the hypotheis that these elasticities are *individually* equal to unity.

   (b) Test the hypotheis that the labor and capital elasticities are equal, assuming (i) the covariance between the estimated labor and capital coefficients is zero, and (ii) it is -0.0972.

3. Consider the following models.

$$\text{Model A} : Y_t = \alpha_1 + \alpha_2 X_{2t} + \alpha_3 X_{3t} + u_{1t}$$
$$\text{Model B} : Y_t - X_{2t} = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_{2t}$$

   (a) Will OLS estimates of α_1 and β_1 be the same? Why?

   (b) Will OLS estimates of α_3 and β_3 be the same? Why?

   (c) What is the relationship between α_2 and β_2?

   (d) Can we compare the R^2 of the two models? Why or why not?

4. Consider the Cobb-Douglas production function

$$Y = \beta_1 L^{\beta_2} K^{\beta_3} \tag{1}$$

where Y = output, L = labor input, and K = capital input. Dividing (1) through by K, we get

$$\frac{Y}{K} = \beta_1 \left(\frac{L}{K}\right)^{\beta_2} K^{\beta_2+\beta_3-1}$$

Takeing the natural log, we obtain

$$\ln\left(\frac{Y}{K}\right) = \beta_0 + \beta_2 \ln\left(\frac{L}{K}\right) + (\beta_2 + \beta_3 - 1)\ln K \tag{2}$$

where $\beta_0 = \ln \beta_1$.

(a) Suppose you had data to run regression (2). How would you test the hyothesis that there are constant returns to scale, i.e., $\beta_2 + \beta_3 = 1$.

(b) Does it make any difference whether we divide (1) by L rahter than by K?

5. This is a practice for running regressions by STATA. The data are individuals reporting positive earnings from the 1998 Labor Force Survey. There are 5 variables in the data file earn.dat, they are "monthly earning (denoted as Y)," "number of schooling year (S)," "age (AGE)," "married (MAR, 1 is married, 0 is unmarried)," and "sex (SEX, 1 is male, 0 is female)". Answer the following questions.

   (a) How many obervations are there in the data set, how many of them are male? female?

   (b) One version of the earnings equation is

   $$\ln Y = \beta_0 + \beta_1 S + \beta_2 EX + \beta_3 EX^2 + \beta_4 MAR + \epsilon \tag{3}$$

   where EX represents years of working experience which is defined as $Age - S - 6$. Run an OLS regression of this simple model for men and women seperately. How many percentage points will one's earnings increase if one have one more year of schooling? (This is usually referred as the rate of return for education.) Is it significant, under what significance level?

   (c) What is the effect of being married on $\ln y$ for men and women?

   (d) Creat a set of dummy variables indicating level of education. Group those with less than or equal to 6 years of schooling as "Primary School," those with $S = 9$ as "Junior High,", $S = 12$ as 'High School," $S = 14$ as "Junior College," and $S >= 16$ as "College,". Replace S in model (1) with those dummy variables and let "High School" be the omitted group. What is the rate of return of a college education for men and women?