Econometrics I
Final Exam.
2002.1.18

1. (25%) State with brief reasons whether the following statements are true, false, or uncertain.

   (a) (5%) In multiple regression, a high correlation in the sample among the regressors (multicollinearity) implies that the least squares estimators of the coefficients are biased.

   (b) (5%) Heteroscedasticity in the errors leads to biased estimates of the regression coefficients and their standard errors.

   (c) (5%) When autocorrelation is present, OLS estimators are biased as well as inefficient.

   (d) (5%) The first-difference transformation to eliminate autocorrelation assumes that the coefficient of autocorrelation ρ is -1.

   (e) (5%) When there is perfect collinearity among explanatory variables, the inverse of $X'X$ does not exist.

2. (30%) In the model

$$Y_i = \beta_2 X_i + u_i$$

   Note that there is no intercept in the model. You are told that $\text{Var}(u_i) = \sigma^2 X_i^2$. To obtain the efficient estimator of β_2, we transform the variables by dividing both sides by X_i,

$$\frac{Y_i}{X_i} = \beta_2 + \frac{u_i}{X_i} \equiv \beta_2 + u_i^*$$

   (a) (5%) Show that u_i^* is homoscedastic.

   (b) (5%) Let $Z_i = \frac{Y_i}{X_i}$, what is the GLS estmiator of β_2, $\hat{\beta}_2^{GLS}$?

   (c) (5%) What is the mean of $\hat{\beta}_2^{GLS}$?

   (d) (5%) What is the variance of $\hat{\beta}_2^{GLS}$?

   (e) (10%) Let $\hat{\beta}_2$ be the OLS estimator of the original model, show that

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma^2 \sum X_i^4}{(\sum X_i^2)^2}$$

3. (20%) In a study of 27 industrial establishments of varying size, $Y =$ the number of supervisors and $X =$ the number of supervised workers. The OLS results obtained were as follows.

$$\begin{aligned} Y_i &= 14.448 + 0.115 X_i && (1) \\ s.e. &= (9.562) \ (0.011), \quad n = 27, \ R^2 = 0.776 \end{aligned}$$

After the estimation of the equation and plotting the residuals against X, it was found that the variance of the residuals increased with X. Plotting the residuals against $\frac{1}{X}$ showed that there was no such relationship. Hence the assumption made was

$$\text{Var}(u_i) = \sigma^2 X_i^2$$

The estimated equation was

$$\begin{aligned} \frac{Y_i}{X_i} &= 0.121 + 3.803 \left(\frac{1}{X_i}\right) \qquad (2) \\ s.e. &= (0.009)\ (4.570) \end{aligned}$$

(a) (5%) In terms of the original equation, what are the corresponding coefficients and standard errors of $\hat{\beta}_1$ and $\hat{\beta}_2$?($\hat{\beta}_1$ denotes the coefficient of the constant term while $\hat{\beta}_2$ denotes the coefficient of X.)

(b) (5%) It can be calculated that R^2 of the regression after variables are transformed is 0.7587, a drop from 0.776 in (1), can we conclude that equation (1) is better?

(c) (5%) How would the equation be estimated if $\text{Var}(u_i) = \sigma^2 X_i$, instead of $\sigma^2 X_i^2$?

(d) (5%) To choose between the two specifications, $\text{Var}(u_i) = \sigma^2 X_i^2$ and $\text{Var}(u_i) = \sigma^2 X_i$, obtain the OLS residuals in (1), $\hat{u}_i$. Regress $\hat{u}_i^2$ on X_i^2 and X_i separately, then choose the specification that gives a *higher* or *lower* R^2?

4. (25%) Suppose we have the following model,

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

where Y_t is the log of help-wanted index, and X_t is the log of unemployment rate. OLS regressin results are

$$\begin{aligned} Y_t &= 7.3084 - 1.5375\ X_t \\ t &= (65.825)\ (-21.612),\ N = 24,\ R^2 = 0.9550,\ d = 0.9108 \end{aligned}$$

where d is the Durbin-Watson statistic. For 24 observations and 1 explanatory variable the 5% Durbin-Watson table shows that $d_L = 1.27$ and $d_U = 1.45$.

(a) (5%) Is there authcorrelation in u_t?

(b) (5%) Estimate ρ based on the Durbin-Watson d statistic?

(c) (5%) To obtain the feasbile GLS estimates of β_1 and β_2, how would you transform the first observation by $\hat{\rho}$ from (b)?

(d) (5%) How would you transform all the other observations?

(e) (5%) The results of running OLS regression on the transformed variables are

$$\begin{aligned} Y_t^* &= 3.1361 - 1.4800 X_t^* \\ t &= (38.583)\ (-12.351),\ N = 24,\ R^2 = 0.9685,\ d = 1.83 \end{aligned}$$

Is there still autocorrelation after the feasible GLS estimation?