Measuring the Fiscal Stance

By

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Abstract

In this paper we propose an index of the fiscal stance suitable for practical use in short-term policy making. The index is based on a comparison of a target level of the debt-GDP ratio for a given finite horizon with a forecast of the debt-GDP ratio based on a VAR formed from the government budget constraint. This approach to measuring the fiscal stance is different from the literature on fiscal sustainability. We emphasise the importance of having a forward-looking measure of the fiscal stance for the immediate future rather than a test for fiscal sustainability that is backward-looking, or based just on past behaviour which may not be closely related to the current fiscal position. We use our methodology to construct a time series of the indices of the fiscal stances of the US, the UK and Germany over the last 25 or more years. We find that both the US and UK fiscal stances have deteriorated considerably since 2000 and Germany’s has been steadily deteriorating since unification in 1989, and worsened again on joining EMU.

Keywords: Budget deficits, government debt, fiscal sustainability, VAR analysis.

JEL Classification: C22,C53,E62,E63
1 Introduction

Recent concerns in 2004 and 2005 about the fiscal stances of the US, France and Germany and of possible reforms to the EU’s Stability and Growth Pact (largely due to the errant fiscal positions of France and Germany) have renewed interest in the issue of how to measure the fiscal stance. In this paper we propose an index of the fiscal stance suitable for practical use in short-term policy making. We take a very different approach from the literature on fiscal sustainability even though, like this literature, it is based on the government inter-temporal budget constraint. We emphasise the importance of having a forward-looking measure of the fiscal stance that focuses on the implications of the current fiscal stance for the immediate future. We argue against focusing on formal tests of the stationarity of debts and deficits as they are backward-looking and not necessarily a good guide to the current stance of fiscal policy.

The index is based on a comparison of a target level of the debt-GDP ratio for a given finite horizon with a forecast of the debt-GDP ratio based on a VAR formed from the government budget constraint. By using a VAR forecasting model we avoid basing the index on a particular theoretical model of the economy, and the index is simple to compute and readily automated.

We use our methodology to examine the fiscal stances of the US, the UK and Germany over the last 25 or more years. We find that both the US and UK fiscal stances have deteriorated considerably since 2000 and Germany’s has been steadily deteriorating since unification in 1989 and worsened again on joining EMU.

The emphasis on the fiscal stance, as opposed to fiscal sustainability, is a key feature of this paper. Determining whether the current fiscal stance is sustainable has proved difficult and controversial, and has limited applicability in evaluating fiscal policy in the short run. Typically, tests for fiscal sustainability focus on the dynamic properties of past debts and deficits and assume that these processes will continue into the infinite future with a view to establishing whether the present value of future primary surpluses are sufficient to meet current government debt oblig-
ations. There are obvious problems with this approach. First, a failure to satisfy a test for fiscal sustainability does not necessarily have any implications for the current fiscal stance. A government could argue that fiscal sustainability can be achieved by changing future fiscal policy so that sufficient surpluses would be generated. Or, it may be that rejection of fiscal sustainability was due to past fiscal policy and that subsequent changes had removed the problem. In both cases, the time series properties of past debts and deficits would no longer be relevant for current policy.

Second, a failure to satisfy a test for fiscal sustainability has little immediate relevance if financial markets are still willing to hold government debt, perhaps in the belief that governments will make the appropriate changes to fiscal policy in the future. Third, a test statistic is not a user-friendly way of representing fiscal policy. Something more transparent is required such as an index series that can capture changes in the fiscal stance over time. Fourth, in the related literature on inter-temporal current account sustainability, the outcome of the test for sustainability depends on whether consumption is modelled correctly. We seek a measure of the fiscal stance based on the government constraint that is theory free.

Although the outcome of tests for fiscal sustainability have not played much of a role in discussions on fiscal policy, a measure of the current fiscal stance would still be helpful. Such a measure should be easy to represent and compute and not depend on a particular theoretical model of the economy. Governments need to know the likely consequences of their current fiscal stance for their debt obligations and the costs of borrowing and of servicing the debt. Markets need to know the risks associated with the fiscal stance in order to price government debt. The Maastricht Treaty was an attempt to ensure that fiscal policy was set appropriately in the run-up to EMU so that the temptation to inflate away debts was avoided. Its successor, the Stability and Growth Pact, seeks to avoid fiscal spillovers from one country to another which might affect monetary policy or euro-debt obligations. It is increasingly recognised, however, that such fiscal rules are neither necessary nor sufficient. Whatever the fiscal framework, a crucial ingredient is an appropriate measure of the current fiscal stance.
The index we propose is concerned with forecasting whether the debt-GDP ratio is likely to exceed or fall below a pre-specified target over a pre-specified time horizon. Given the time horizon and the target level of the debt-GDP ratio at the end of that horizon, the index is based on a comparison of the desired change in the debt-GDP ratio and a forecast of the present value of the current level of the debt-GDP ratio over the horizon derived from a simple VAR forecasting model of the economy. If the index exceeds unity then the current fiscal stance is said to be inconsistent with the debt objective over the horizon in the sense that debt is forecast to rise above target; if the index is less than unity then the fiscal stance is said to be consistent with the debt objective.

The choice of a VAR model is to avoid taking a particular view of the economy and to permit the method to be easily automated. The VAR is based on a log-linear approximation to the government’s inter-temporal budget constraint in order that interest rates, inflation and growth are allowed to be time varying. This approach is in contrast to much of the literature on fiscal sustainability where interest rates, inflation and growth are held constant over the forecast horizon in order to eliminate the non-linearities that their time variation would introduce into the inter-temporal budget constraint.

The paper is set out as follows. In Section 2 we examine a number of different ways of writing the government budget constraint and establish our notation. In Section 3 we present an analysis of fiscal sustainability with a view to showing its limitations in providing a useful measure of the current fiscal stance. We provide an intuitive rationale for the various tests for fiscal sustainability that have been proposed in the literature and discuss the technical problems in implementing these tests. We then show how, by using a log-linear approximation to the government budget constraint, fiscal sustainability can be tested in a way that permits the discount rate to be time-varying and enables linear methods of analysis to be used once more. We also comment on the implications of this analysis of fiscal sustainability for the debt and deficit limits of the EU’s Stability and Growth Pact. We derive our proposed fiscal index in Section 4 and show how it can implemented using VAR analysis. In Section 5 we calculate the index for the US, the UK and
Germany over the period from the 1970’s to 2005. Our findings are summarized in Section 6.

2 The government budget constraint

We begin by considering the nominal government budget constraint (GBC), the sustainability of fiscal policy and the implications of various fiscal rules, such as the EU’s Stability and Growth Pact.\(^1\) The nominal GBC can be written

\[
P_t g_t + (1 + R_t)B_{t-1} = B_t + \Delta M_t + P_t T_t \tag{1}
\]

where \(g_t\) is real government expenditure including real transfers to households, \(T_t\) is total real taxes and \(M_t\) is the stock of outside nominal, non-interest bearing money in circulation that is supplied by the government (the central bank) at the start of period \(t\), \(B_t\) is the nominal value of government bonds issued at the end of period \(t\), \(R_t\) is the average interest rate on bonds issued at the end of period \(t-1\) and \(R_t B_{t-1}\) is total interest payments made in period \(t\).\(^2\) Thus the left-hand side of equation (1) is total nominal expenditures in period \(t\) and the right-hand side is total revenues plus additions to government current financial resources.

The equivalent real GBC can be derived from the nominal GBC by dividing through the nominal GBC by the general price level \(P_t\). This gives

\[
g_t + (1 + R_t) \frac{P_{t-1}B_{t-1}}{P_t} = T_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{P_{t-1}M_{t-1}}{P_{t-1}}\]


\(^2\) In practice governments issue bonds at a discount and redeem them at par. Thus if all bonds were for one period, then \(B_t = B_t^{BG} \frac{P_t}{P_{t-1}}\) where \(B_t^{BG}\) is the number of bonds issued in period \(t\) each with price \(P_t^{BG} = \frac{1}{1+R_{t-1}}\) and \(B_{t-1}^{BG} = (1 + R_t)B_{t-1}\).
or

\[ g_t + (1 + r_t)b_{t-1} = T_t + b_t + m_t - \frac{1}{1 + \pi_t}m_{t-1} \]  

(2)

where \( \pi_t = \frac{\Delta P_t}{P_{t-1}} \) is the rate of inflation, \( b_t \) is the real stock of government debt, \( m_t \) is the real stock of money and \( r_t \) is the real rate of interest which, in view of our dating convention, is defined by

\[ 1 + r_t = \frac{1 + R_t}{1 + \pi_t} \]

Thus, approximately, \( r_t \approx R_t - \pi_t \).

The GBC can also be expressed in terms of proportions of nominal or real GDP by dividing through the nominal GBC by \( P_t y_t \), nominal GDP, where \( y_t \) is real GDP. We obtain

\[ \frac{g_t}{y_t} + \frac{1 + R_t}{(1 + \pi_t)(1 + \gamma_t)} \frac{b_{t-1}}{y_{t-1}} = \frac{T_t}{y_t} + \frac{b_t}{y_t} + \frac{m_t}{y_t} + \frac{1}{(1 + \pi_t)(1 + \gamma_t)} \frac{m_{t-1}}{y_{t-1}} \]  

(3)

where \( \gamma_t \) is the rate of growth of GDP and \( \frac{\Delta y_t}{y_t} \) is the average tax rate.

The total nominal government deficit (or public sector borrowing requirement, PSBR) is defined as

\[ P_t D_t = P_t g_t + R_t B_{t-1} - P_t T_t - \Delta M_t \]

Hence \( \frac{D_t}{y_t} \), the real government deficit as a proportion of GDP, is

\[ \frac{D_t}{y_t} = \frac{g_t}{y_t} + \frac{R_t}{(1 + \pi_t)(1 + \gamma_t)} \frac{b_{t-1}}{y_{t-1}} - \frac{T_t}{y_t} + \frac{b_t}{y_t} + \frac{m_t}{y_t} + \frac{1}{(1 + \pi_t)(1 + \gamma_t)} \frac{m_{t-1}}{y_{t-1}} \]

The right-hand side shows the net borrowing required to fund the deficit expressed as a proportion of GDP.

We also define the nominal primary deficit \( P_t d_t \) (the total deficit less debt interest payments) as

\[ P_t d_t = P_t D_t - R_t B_{t-1} \]

which implies that

\[ \frac{d_t}{y_t} = \frac{D_t}{y_t} - \frac{R_t}{(1 + \pi_t)(1 + \gamma_t)} \frac{b_{t-1}}{y_{t-1}} \]
Hence the ratio of the primary deficit to GDP is

\[
\frac{\Delta t}{y_t} = \frac{g_t}{y_t} - \frac{T_t}{y_t} - \frac{m_t}{y_t} + \frac{1}{(1 + \pi_t)(1 + \gamma_t)} \frac{m_{t-1}}{y_{t-1}}
\]

This is a non-linear difference equation in \( \frac{b_t}{y_t} \). If we define

\[
1 + \rho_t = \frac{1 + R_t}{(1 + \pi_t)(1 + \gamma_t)}
\]

where approximately, \( \rho_t = R_t - \pi_t - \gamma_t = r_t - \gamma_t \), is the real interest rate adjusted for economic growth, then equation (4) can be written as

\[
\frac{b_t}{y_t} = \frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t}
\]

This is the key equation for determining the sustainability of fiscal policy. The stability of the equation depends on the sign of \( \rho_t \).

We note that the evolution of \( \frac{b_t}{y_t} \) can also be written in terms of the total deficit since

\[
\frac{b_t}{y_t} = \frac{1}{(1 + \pi_t)(1 + \gamma_t)} \frac{b_{t-1}}{y_{t-1}} + \frac{D_t}{y_t}
\]

For positive inflation and growth this is a stable difference equation

### 3 Fiscal sustainability

Fiscal sustainability concerns the evolution of \( \frac{b_t}{y_t} \) and whether it remains finite or explodes. In this and subsequent sections we adopt the common terminology that the fiscal stance is said to be sustainable if \( \frac{b_t}{y_t} \) is finite - and if financial markets are willing to hold the level of debt that emerges. Before describing our proposed new procedure for determining whether the fiscal stance is consistent with given debt objectives, we review the principal methods in the literature for testing what is referred to as fiscal sustainability. All take equation (5) as their starting point.

In discussing sustainability it is convenient to distinguish between two cases: where the discount
rate $\rho_t$ (and hence $R_t, \pi_t$ and $\gamma_t$) is assumed to be constant and where it is allowed to be time varying.\(^3\)

### 3.1 Constant discount rate

If $\rho_t$ is assumed to be constant then, from equation (5), \(\frac{b_t}{y_t}\) evolves according to the difference equation

\[
\frac{b_t}{y_t} = (1 + \rho)\frac{b_{t-1}}{y_{t-1}} + \frac{d_t}{y_t}
\]

where $1 + \rho = \frac{1 + R}{(1 + \pi)(1 + \gamma)}$ or, approximately, $\rho = R - \pi - \gamma$. The solution for \(\frac{b_t}{y_t}\) depends on whether the equation (7) is stable or unstable. We consider both cases.

**Case1: $\rho < 0$ (stable case)**

In this case $\frac{1 + R}{(1 + \pi)(1 + \gamma)} < 1$ and equation (7) is a stable difference equation, and hence can be solved *backwards* by successive substitution. The expected value of the debt-GDP ratio in $n$ period’s time conditional on information at time $t$ is

\[
E_t(\frac{b_{t+n}}{y_{t+n}}) = (1 + \rho)^n \frac{b_t}{y_t} + \sum_{s=0}^{n-1} (1 + \rho)^{n-s} E_t(\frac{d_{t+s}}{y_{t+s}})
\]

Taking the limit as $n \to \infty$ gives

\[
\lim_{n \to \infty} (1 + \rho)^n \frac{b_t}{y_t} = 0
\]

implying that the current level of debt has no bearing on debt in the infinite future.

If (9) holds then we obtain

\[
\lim_{n \to \infty} E_t(\frac{b_{t+n}}{y_{t+n}}) = \lim_{n \to \infty} \sum_{s=1}^{n} (1 + \rho)^{n-s} E_t(\frac{d_{t+s}}{y_{t+s}})
\]

The evolution of the debt-GDP ratio depends on that of $\frac{d_t}{y_t}$. Suppose that $\frac{d_t}{y_t}$ may be stochastic but is expected to grow at the rate $\lambda$ such that

\[
E_t(\frac{d_{t+s}}{y_{t+s}}) = (1 + \lambda)^s \frac{d_t}{y_t}
\]

---

\(^3\) Ahmed and Rogers (1995) and Bohn (1995, 2005) argue that the appropriate discount rate to use for discounting future primary surpluses is the inter-temporal marginal rate of substitution and not the real interest rate. In a complete markets full general equilibrium model this would be the real rate of return used here.
It follows that

$$\lim_{n \to \infty} E_t \left( \frac{b_{t+n}}{y_{t+n}} \right) = \lim_{n \to \infty} \sum_{s=1}^{n} (1 + \rho)^{n-s} (1 + \lambda)^s \frac{d_t}{y_t}$$

$$= \lim_{n \to \infty} (1 + \lambda) \left( \frac{(1 + \lambda)^n - (1 + \rho)^n}{\lambda - \rho} \right) \frac{d_t}{y_t}$$

$$= - \frac{1}{\rho \frac{d_t}{y_t}} \quad \text{if } \lambda = 0$$

(12)

If $\rho, \lambda < 0$ then $\lim_{n \to \infty} E_t \left( \frac{b_{t+n}}{y_{t+n}} \right) = 0$. If $\lambda > 0$ then it will explode. Thus, the debt-GDP ratio will remain finite and positive if $(-\frac{d_t}{y_t})$, the ratio of the primary surplus to GDP, does not explode.

We note that if $\lambda < 0$ then $\frac{d_t}{y_t}$ is a stationary I(0) process and the expected, or long-run, value of the debt-GDP ratio is zero. And if $\lambda = 0$, then $\frac{d_t}{y_t}$ is a non-stationary I(1) process, and hence $\frac{b_t}{y_t}$ will also be I(1). Moreover, $\frac{b_t}{y_t}$ and $\frac{d_t}{y_t}$ will be cointegrated with cointegrating vector $(1, \frac{1}{\rho})$. Fiscal policy is therefore sustainable provided $\frac{b_t}{y_t}$ does not grow over time.

**Case 2: $\rho > 0$ (unstable case)**

In this case $0 < \frac{(1+\gamma)(1+\gamma)}{1+R} < 1$. Equation (7) is therefore an unstable difference equation and must be solved forwards, not backwards, as follows:

$$\frac{b_t}{y_t} = \frac{1}{1 + \rho} E_t \left( \frac{b_{t+1}}{y_{t+1}} - \frac{d_{t+1}}{y_{t+1}} \right)$$

$$= (1 + \rho)^{-n} E_t \left( \frac{b_{t+n}}{y_{t+n}} \right) - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t \left( \frac{d_{t+s}}{y_{t+s}} \right)$$

(13)

Taking limits as $n \to \infty$ gives the transversality condition

$$\lim_{n \to \infty} (1 + \rho)^{-n} E_t \left( \frac{b_{t+n}}{y_{t+n}} \right) = 0$$

(14)

If this holds then

$$\frac{b_t}{y_t} = \sum_{s=1}^{\infty} (1 + \rho)^{-s} E_t \left( \frac{-d_{t+s}}{y_{t+s}} \right)$$

(15)

implying that the expected present value of current and future primary surpluses expressed as a proportion of GDP (the right-hand side of equation (15)) must be sufficient to pay-off current debt.
Suppose once more that $\frac{dt}{yt}$ is expected to evolve according to equation (11) then

$$\frac{bt}{yt} = \sum_{s=1}^{\infty} (1 + \rho)^{-s} (1 + \lambda)^s \left( -\frac{dt}{yt} \right)$$

$$= \frac{1 + \lambda}{\rho - \lambda} \left( -\frac{dt}{yt} \right) \quad \text{if} \quad 1 < \lambda < \rho, \ \rho > 0$$

(16)

Thus, if $-1 < \lambda < \rho$ and $\rho > 0$, the present value of primary surpluses will meet current debt obligations. However, the debt-GDP ratio will grow at the rate $\lambda$, the same rate as $-\frac{dt}{yt}$.

If $-1 < \lambda < 0$ then $-\frac{dt}{yt}$ is stationary and $\frac{bt}{yt}$ will also be stationary and finite. If $\lambda = 0$, so that $-\frac{dt}{yt}$ is I(1), then we obtain the same condition as equation (12), namely,

$$\frac{bt}{yt} = \frac{1}{\rho} \left( -\frac{dt}{yt} \right)$$

(17)

implying that $\frac{bt}{yt}$ will be I(1) and will be cointegrated with $-\frac{dt}{yt}$ with cointegrating vector $(1, -\frac{1}{\rho})$.

These results provide an insight into the rationale behind a number of well-known empirical tests for fiscal sustainability. The test of Hamilton and Flavin (1986) is based on the following version of equation (13)

$$\frac{bt}{yt} = A_0 (1 + \rho)^{-t} - \sum_{s=1}^{\infty} (1 + \rho)^{-s} E_t \left( \frac{dt+s}{yt+s} \right)$$

except that real debt and the real primary deficit is used rather than $\frac{bt}{yt}$ and $\frac{dt}{yt}$. The transversality condition holds on the null hypothesis that $A_0 = 0$.

Trehan and Walsh (1988) propose a cointegration test for fiscal sustainability. They measure debt and the primary deficit in real terms rather than as proportions of GDP, but Hakkio and Rush (1991) employ the test expressing the variables as proportions of GDP. We have already seen from equations (12) and (17) that if the variables have unit roots and are cointegrated with cointegrating vector $(\rho, 1)$ then fiscal policy is sustainable. (Or, if the primary deficit is decomposed into government expenditures and revenues and both are I(1), then the cointegrating vector with debt must be $(\rho, 1, -1)$.)

Alternatively, if there is a cointegrating relation between debt and the primary deficit given by

$$\frac{dt}{yt} + \frac{bt}{yt} = ut$$
for some $\alpha$ and stationary $u_t$, then from equation (7),

$$(1 + \alpha) \frac{b_t}{y_t} = (1 + \rho) \frac{b_{t-1}}{y_{t-1}} + u_t$$

It follows that $\frac{b_t}{y_t}$ has a unit root if $\alpha = \rho$.

### 3.2 Time-varying discount rate

In practice, $\rho_t$ will be time-varying, not constant, and so these tests will in general be invalid. If $\rho_t < 0$ then the budget constraint, equation (5), will be stable and the debt-GDP ratio will remain finite if $\frac{d_t}{y_t}$ is stationary.

If $\rho_t > 0$ then we solve the budget constraint forwards to obtain

$$\frac{b_t}{y_t} = E_t[\prod_{s=1}^{n} \frac{1}{1 + \rho_{t+s}} \frac{b_{t+n}}{y_{t+n}} - E_t[\sum_{s=1}^{n} (\prod_{i=1}^{s} \frac{1}{1 + \rho_{t+i}}) \frac{d_{t+s}}{y_{t+s}}]]$$  \hspace{1cm} (18)

if

$$\delta_{t,s} = \prod_{i=1}^{s} \frac{1}{1 + \rho_{t+i}} \leq 1 \text{ for all } s \geq 1$$

Hence fiscal solvency depends on the transversality condition

$$\lim_{n \to \infty} E_t[\prod_{s=1}^{n} \frac{1}{1 + \rho_{t+s}} \frac{b_{t+n}}{y_{t+n}}] = 0$$  \hspace{1cm} (19)

If this holds then

$$\frac{b_t}{y_t} = E_t[\sum_{s=1}^{\infty} (\prod_{i=1}^{s} \frac{1}{1 + \rho_{t+i}})(-\frac{d_{t+s}}{y_{t+s}})]$$  \hspace{1cm} (20)

Like equation (15), equation (20) says that the present value of current and future primary surpluses must be sufficient to offset current debt liabilities. The difference is that the discount rate is compounded from time-varying rates.

In order to analyse sustainability we define the variables

$$x_t = \delta_{t,n} \frac{b_t}{y_t}$$

$$z_t = \delta_{t,n} \frac{d_t}{y_t}$$
We may now write equation (5) as
\[ \Delta x_t = z_t \]

Fiscal sustainability now requires the transversality condition
\[ \lim_{n \to \infty} E_t(x_{t+n}) = 0 \]

and implies that
\[ x_t = -\lim_{n \to \infty} E_t[\sum_{s=1}^{n} z_{t+s}] \]

Wilcox (1989) shows that fiscal sustainability is satisfied if \( x_t \) is a zero-mean stationary process. Uctum and Wickens (2000) prove a more general result that does not require \( x_t \) to be stationary. They show that fiscal sustainability is satisfied if \( z_t \) is a zero-mean stationary process when it follows that \( x_t \) will be an I(1) process.

3.3 Fiscal sustainability and the total deficit

Another approach to fiscal sustainability is to focus on the relation between the debt and the total deficit, rather than the primary deficit. This is given by equation (6) which is a stable difference equation if \( \pi_t + \gamma_t \), the rate of growth of nominal GDP, is positive. If, in addition, \( \frac{D}{y_t} \) is stationary then \( \frac{b}{y_t} \) will be stationary and hence remain finite.

Trehan and Walsh (1991) therefore argue that fiscal policy is sustainable with a variable discount rate if the total deficit is stationary. We also note from previous results that the stationarity of \( \frac{D}{y_t} \) is a consequence of the cointegration of \( \frac{b}{y_t} \) and \( \frac{d}{y_t} \) and vice-versa.

3.4 Stability and Growth Pact (SGP)

The SGP was based on the original Maastricht conditions that \( \frac{b}{y_t} \) must be less than 0.6 and \( \frac{D}{y_t} \) must be less than 0.03. For given values of \( \frac{b}{y_t} \) and \( \frac{D}{y_t} \) bounded above by \( \frac{b}{y_t} \) and \( \frac{D}{y} \) and for a
constant nominal growth rate the long-run solution to equation (6) is

\[ \frac{b}{y} = \frac{(1 + \pi)(1 + \gamma)}{(1 + \pi)(1 + \gamma) - 1} \frac{D}{y} \]

\[ \leq \frac{1}{\pi + \gamma} \frac{D}{y} \]

It follows that nominal growth must satisfy

\[ \pi + \gamma \leq \frac{D}{y} \]

Hence, given the limits on debt and deficits specified under the SGP, the nominal rate of growth must not be less than \( \frac{0.03}{0.02} \equiv 5\% \). If nominal growth were less than this then debt would rise above 60% even if the deficit limit were satisfied. Although the debt-GDP ratio would exceed the SGP limit, it would still satisfy the condition for fiscal sustainability.

Now suppose that the deficit exceeds the 3% limit. Whether or not the debt-GDP ratio exceeds 60% depends on the rate of nominal growth. The higher the rate of nominal growth, the less likely is the debt-GDP ratio to exceed its limit. Once again, this does not affect fiscal sustainability.

It follows that the SGP is neither necessary nor sufficient for fiscal sustainability in the long run. This is because fiscal sustainability may be satisfied even if the SGP limits are breeched and because it is also necessary that the rate of nominal growth is appropriate.

### 3.5 A log-linear approach to fiscal sustainability

To complete our discussion of fiscal sustainability, we propose an alternative way to deal with a time-varying discount rate which we make use of later. This is to use a log-linear approximation to the government budget constraint taken about the steady-state solution (assuming it exists). As the primary deficit can take negative values, it is necessary to write the GBC in terms of total expenditures \( g_t \) and total revenues \( v_t \) both of which are strictly positive. We therefore re-write the GBC, equation (3), as

\[ \frac{b_t}{y_t} = \frac{g_t}{y_t} - \frac{v_t}{y_t} + (1 + \rho_t) \frac{b_{t-1}}{y_{t-1}} \]
where

\[
\frac{v_t}{y_t} = \frac{T_t}{y_t} + \frac{m_t}{y_t} - \frac{1}{(1 + \sigma_t)(1 + \gamma_t)} \frac{m_{t-1}}{y_{t-1}}
\]

Next we approximate the GBC about the steady-state solution in which we assume that all variables are constant. The steady-state solution to the GBC is

\[
\frac{\rho b}{y} = -\frac{g}{y} + \frac{v}{y}
\]

The GBC may be re-written as

\[
f(x_t) = \exp[\ln b_t - \exp[\ln g_t] + \exp[\ln v_t] - \exp[\ln (1 + \rho_t) + \ln \frac{b_{t-1}}{y_{t-1}}] = 0
\]

Noting that a first-order Taylor series approximation to \(h(x_t) = \exp[\ln x_t]\) about \(\ln x\) is

\[
h(x_t) \approx x_t \approx [\ln x_t - \ln x]
\]

a log-linear approximation to the GBC about the steady-state is given by

\[
\ln \frac{b_t}{y_t} \approx c + \frac{g}{b} \ln \frac{g_t}{y_t} - \frac{v}{b} \ln \frac{v_t}{y_t} + (1 + \rho) \ln (1 + \rho_t) + (1 + \rho) \ln \frac{b_{t-1}}{y_{t-1}}
\]

\[c = -\rho \ln \frac{b}{y} - \frac{g}{b} \ln \frac{g}{y} + \frac{v}{b} \ln \frac{v}{y} - (1 + \rho) \ln (1 + \rho)
\]

As \(\ln (1 + \rho_t) \approx \rho_t\), in effect, the discount rate is an additional variable in the equation.

Using equation (21), fiscal sustainability may be analysed with a linear model even though the discount rate is time-varying. The stability of the log-linearized GBC depends on the sign of \(\rho\).

Assuming that \(\rho > 0\), we solve the equation forwards to obtain

\[
\ln \frac{b_t}{y_t} = (1 + \rho)^{-n} E_t(\ln \frac{b_{t+n}}{y_{t+n}}) - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

\[k_t = c + \frac{g}{b} \ln \frac{g_t}{y_t} - \frac{v}{b} \ln \frac{v_t}{y_t} + (1 + \rho) \ln (1 + \rho_t)
\]

where \(k_t\) is, in effect, the logarithmic equivalent of the primary deficit. The transversality condition is therefore

\[
\lim_{n \to \infty} (1 + \rho)^{-n} E_t(\ln \frac{b_{t+n}}{y_{t+n}}) = 0
\]
which implies that

\[ \ln \frac{b_t}{y_t} = - \sum_{s=1}^{\infty} (1 + \rho)^{-s} E_t (k_{t+s}) \]  

(25)

If \( k_t \) is stationary then \( \ln \frac{b_t}{y_t} \), and hence \( \frac{b_t}{y_t} \), remains stationary and finite. This may occur due to the individual terms of \( k_t \) being stationary, or due to some terms being I(1) but being cointegrated. From equation (21), the cointegrating equation is

\[ \ln \frac{b_t}{y_t} \sim - \frac{c}{\rho} - \frac{g}{\rho b} \ln \frac{g_t}{y_t} + \frac{v}{\rho b} \ln \frac{v_t}{y_t} - \frac{1 + \rho}{\rho} \ln (1 + \rho) \]

4 An index of the fiscal stance

4.1 An assessment of the tests for fiscal sustainability

All of these tests of fiscal sustainability, including the new log-linear test that we propose, are of limited practicality. The main problem is that the tests are based on the past behaviour of debts and deficits whereas the sustainability of current fiscal stance is related to their future behaviour. The test outcome could be dominated by an influential, but anomalous, period in the distant past yet the current fiscal stance may still be sustainable. Even if the current fiscal stance is not sustainable, governments could claim that a policy change planned for the future would make it sustainable. As a result, the tests provide an ineffective constraint on fiscal policy, especially in the near future.

This suggests that we need a more forward-looking approach that focuses on the short-term implications of the current fiscal stance. As the fiscal position varies over time, it would be helpful to have a measure that reflects this and enables historical comparisons to be made. We therefore propose constructing an index number series of the current fiscal stance.

The index is based on the inter-temporal government budget constraint. The index measures the ratio of the desired change in the discounted debt-GDP ratio over a given time horizon relative to the forecast change. The target debt-GDP ratio at the end of the horizon could be, for example,
a particular number such as the 60% SGP limit, a percentage reduction or the maintenance of the current level of debt.

The forecast change in the debt-GDP ratio is, in effect, the present value of current and future primary surpluses. Future primary surpluses and discount rates are forecast using a VAR based on the variables in the government budget constraint. Any other forecasting model could be used instead, including a structural model of the whole economy. The reasons for choosing a such a VAR are its simplicity and its ease of replication and automation for any economy. We also wish to try to avoid taking a particular view on macroeconomic theory and on the structural of the economy. Since time variation in the future discount rate may be of importance, we base the VAR on our log-linear approximation to the government budget constraint.

The use of an index of sustainability was initially proposed by Blanchard, Chouraqui, Hagemann, and Sartor (1990) and Buiter, Corsetti and Roubini (1993). Their indices are based on a comparison of the current debt-GDP ratio and that periods ahead with given fixed values of the deficit and discount rate. By allowing the deficit and discount rate to be time-varying and endogenous, and the target level of the debt-GDP ratio to be a choice variable, we generalize these indices.

### 4.2 Constructing the index

The basis of our proposed index is the inter-temporal log-linearized budget constraint equation (22). This can be re-written as

\[
(1 + \rho)^{-n} E_t (\ln b_{t+n} \frac{b_t}{y_{t+n}}) - \ln b_t \frac{b_t}{y_t} = \sum_{s=1}^{n} (1 + \rho)^{-s} E_t (k_{t+s})
\]

If we replace \( E_t (\ln b_{t+n} \frac{b_t}{y_{t+n}}) \) by the target \( \ln (\frac{b_{t+n}}{y_{t+n}}) \) we obtain

\[
(1 + \rho)^{-n} \ln (\frac{b_{t+n}}{y_{t+n}}) - \ln b_t \frac{b_t}{y_t} = \sum_{s=1}^{n} (1 + \rho)^{-s} E_t (k_{t+s})
\]

The left-hand side of equation (26) can be interpreted as the desired change in discounted debt between periods \( t \) and \( t + n \). The right-hand side is the logarithmic equivalent of the present
value of the primary surpluses required to achieve this desired change in discounted debt. We replace \( E_t(k_{t+s}) \) by forecasts of the future values of \( k_t \) based on the information available at time \( t \), including the current fiscal stance.

A measure of whether the current fiscal stance is likely to achieve the debt objective is obtained by comparing the two sides of equation (26). If, for example, the aim is to decrease discounted debt then the left-hand side will be negative and the right-hand side gives the present value of the primary surplus required to achieve this reduction in debt. We therefore base our measure of the consistency of the current fiscal stance with the \( n-\)period debt objective on the gap between the objective and the forecast outcome:

\[
FS(t, n) = [(1 + \rho)^{-n} \ln\left(\frac{b_{t+n}}{y_{t+n}}\right) - \ln\left(\frac{b_t}{y_t}\right) - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})] \geq 0
\]

Our index is

\[
FSI(t, n) = \exp[FS(t, n)]
\]

\[
K_{t,n} = \frac{b_t}{b/y_t}
\]

\[
\ln K_{t,n} = (1 + \rho)^{-n} \ln\left(\frac{b_{t+n}}{y_{t+n}}\right) - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

\[
k_t = c + \frac{\rho}{b} \ln \frac{g}{y_t} - \frac{\rho}{b} \ln \frac{v}{y_t} + (1 + \rho) \ln(1 + \rho)
\]

\[
c = -\rho \ln \frac{b}{y} - \frac{\rho}{b} \ln \frac{g}{y} + \frac{\rho}{b} \ln \frac{v}{y} - (1 + \rho) \ln(1 + \rho)
\]

As \( n \to \infty \) the first term in \( \ln K_{t,n} \) tends to zero and the index can be interpreted as comparing the existing level of the debt-GDP ratio with the resources to pay it off.

The index may be interpreted as follows:

(i) if \( FSI(t, n) = 1 \) the debt-GDP ratio in period \( t+n \) is forecast to be on target

(ii) if \( FSI(t, n) > 1 \) the debt-GDP ratio is forecast to be below target

(iii) if \( FSI(t, n) < 1 \) the debt-GDP ratio is forecast to be above target.

Only in case (iii) is the forecasted present value of the primary surplus insufficient to achieve the desired change in the debt-GDP ratio. In this sense the current fiscal stance would not be
In practice, the special case considered by Buiter and Blanchard of maintaining a constant debt-GDP ratio over the planning horizon will often be of most interest. In this case

\[
FS(t, n) = [(1 + \rho)^{-n} - 1] \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s}) \geq 0
\]

The index then becomes

\[
FSI(t, n) = \exp[FS(t, n)]
\]

\[
= \frac{K_{t,n}}{b_t/y_t}
\]

\[
\ln K_{t,n} = (1 + \rho)^{-n} \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

Since in this case

\[
\ln \frac{b_t}{y_t} = (1 + \rho)^{-n} \ln \frac{b_t}{y_t} - \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

\[
= -\frac{1}{1 - (1 + \rho)^{-n}} \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

\[
= -\frac{1}{n\rho} \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

the index could also be calculated as

\[
FSI(t, n) = \frac{K_{t,n}^*}{b_t/y_t}
\]

\[
\ln K_{t,n}^* = -\frac{1}{1 - (1 + \rho)^{-n}} \sum_{s=1}^{n} (1 + \rho)^{-s} E_t(k_{t+s})
\]

where the numerator of the index is now proportional to the present value of primary surpluses.

We consider this case in our empirical examples below.

### 4.3 Forecasting the fiscal variables

In order to compute the index we require forecasts of the variables of the following vector \(z_t\)

\[
z_t = \left( \ln \frac{b_t}{y_t}, \ln \frac{g_t}{y_t}, \ln \frac{\pi_t}{y_t}, \ln (1 + \rho_t), \ln (1 + \gamma_t), \ln (1 + \pi_t) \right)
\]
For the reasons given above, we use of a VAR(p) to obtain these forecasts. This is a simple forecasting scheme that is easily implemented and is theory free. We denote the VAR by

$$z_t = A_0 + \sum_{i=1}^{p} A_i z_{t-i} + e_t,$$  \hspace{1cm} (29)

where $e_t \sim i.i.d. [0, \Sigma]$. The vector of variables $z_t$ may be I(0) or I(1). For forecasting purposes it is unnecessary to take account any non-stationarity or cointegration among the variables. Equally, if cointegration exists, a cointegrated VAR could be estimated instead of a levels VAR and the cointegrated VAR could then be written in levels to obtain equation (29). We also note that to improve the forecasts, $z_t$ could contain additional variables to those that appear in the budget constraint.

$n$-period ahead forecasts may be obtained using the companion form

$$Z_t = B_0 + BZ_{t-1} + u_t,$$

where $Z_t' = [z'_t, z'_{t-1}, ..., z'_{t-p+1}]$, $u_t' = [e'_t, 0, ..., 0]$, $B_0 = [A_0', 0, ..., 0]$ and

$$B = \begin{bmatrix}
A_1 & A_2 & \cdots & A_{p-1} \\
0 & I & 0 & \cdots \\
0 & & I & 0 \\
& & & \ddots \\
& & & & 0 & I & 0
\end{bmatrix}$$

The forecast of $Z_{t+n}$ is therefore

$$E_t[Z_{t+n}] = \sum_{i=0}^{n-1} B^i B_0 + B^n Z_t$$

Defining the selection matrix $S = [I, 0, 0, ..., 0]$ such that

$$z_t = SZ_t$$

and expressing $k_t$ as the following linear function of $z_t$

$$k_t = a + \beta' z_t$$
we obtain

\[ FS(t, n) = \ln K_{t,n} - \ln \frac{b_t}{y_t} = \frac{-1}{1 - (1 + \rho)^{-n}} \sum_{s=1}^{n} \{(1 + \rho)^{-s} [a + \beta'(S_{i=0}^{n-1} B' B_0 + B' Z_t)] \} - \ln \frac{b_t}{y_t} \]

As the last term \( \ln \frac{b_t}{y_t} \) is another linear function of \( Z_t \), \( FS(t, n) \) could also be written as

\[ FS(t, n) = a_n + b_n' Z_t \]

where \( a_n \) is a scalar dependent on the time horizon and \( b_n \) is a vector. This emphasises that \( FS(t, n) \) is based on information available at time \( t \) and, in particular, the current fiscal stance. Increasing the forecast horizon alters \( a_n \) and \( b_n \) but not \( Z_t \).

To implement this in practice it will be necessary to derive \( a_n \) and \( b_n \) from the VAR estimates. The choice of \( \rho \) and \( c \) may be based, for example, on the average values in the sample, their average values over the forecast period or their time \( t \) values. A time series for \( FS(t, n) \) may be calculated from the sample either using all of the sample observations to estimate the VAR, or recursively using only observations up to period \( t \).

5 Indices of the fiscal stances of the US, the UK and Germany

We now construct a time series of the index of the fiscal stance for the US, the UK and Germany.

For the US we consider three horizons: one-year, two-years and five-years ahead. For the UK and Germany we use just a one-year horizon. We assume that the aim in each period is to maintain the current level of the debt-GDP ratio. Hence, we use the version of the index given by equation (28). The data are quarterly from 1960-2005 for the US and from 1970-2005 for the UK, but are annual from 1977-2005 for Germany. The data sources and the construction of the variables are described in the Appendix. There are minor differences in definitions for the different countries. For example, the debt data for the US are measured as net liabilities. This is different from the
Maastricht definition of debt but, given the definitions of the other variables, is consistent with the government budget constraint.

In calculating the present values we require values for $v$, $g$ and $\rho$. We estimate $b$, $g$ and $\rho$ using their sample averages. Table 1 gives the average values for Germany, the UK and the US.

<table>
<thead>
<tr>
<th></th>
<th>$b$</th>
<th>$g$</th>
<th>$v$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.290</td>
<td>0.447</td>
<td>0.459</td>
<td>0.041</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.352</td>
<td>0.405</td>
<td>0.435</td>
<td>0.086</td>
</tr>
<tr>
<td>United States</td>
<td>0.423</td>
<td>0.308</td>
<td>0.331</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Note: $b$, $g$ and $\rho$ are sample averages, $v$ is constructed from the steady-state equation $v = g + \rho b$

5.1 The United States

Figure 1 gives a plot of eight key variables for the US: $b_t$, $g_t$, $v_t$, $(g_v - v_t)/g_t$, $R_t$, $\pi_t$, $\gamma_t$, $\rho_t$. The first four variables are expressed as percentages of GDP and the last four are annualised percentages.

In Table 1 we report Augmented Dickey-Fuller tests for these variables using up to 6 lags. We conclude that we cannot reject a unit root for any variable other than the real growth rate.
Table 1
Augmented Dickey-Fuller tests

<table>
<thead>
<tr>
<th>D-Lag</th>
<th>ln $\frac{b}{y}$</th>
<th>ln $\frac{g}{y}$</th>
<th>ln $\frac{v}{y}$</th>
<th>ln (1 + $R$)</th>
<th>ln (1 + $\pi$)</th>
<th>$\rho$</th>
<th>$\Delta \ln y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-1.280</td>
<td>-2.305</td>
<td>-2.127</td>
<td>-1.656</td>
<td>-1.984</td>
<td>-1.515</td>
<td>-5.347**</td>
</tr>
<tr>
<td>5</td>
<td>-1.734</td>
<td>-2.341</td>
<td>-2.089</td>
<td>-1.749</td>
<td>-2.031</td>
<td>-1.633</td>
<td>-5.415**</td>
</tr>
<tr>
<td>4</td>
<td>-2.156</td>
<td>-2.462</td>
<td>-2.076</td>
<td>-1.750</td>
<td>-1.927</td>
<td>-1.774</td>
<td>-5.947**</td>
</tr>
<tr>
<td>3</td>
<td>-1.197</td>
<td>-2.836</td>
<td>-2.062</td>
<td>-1.496</td>
<td>-1.704</td>
<td>-1.638</td>
<td>-5.481**</td>
</tr>
<tr>
<td>2</td>
<td>-0.7304</td>
<td>-2.511</td>
<td>-2.102</td>
<td>-1.324</td>
<td>-1.896</td>
<td>-2.082</td>
<td>-6.262**</td>
</tr>
<tr>
<td>1</td>
<td>-0.8553</td>
<td>-1.996</td>
<td>-1.952</td>
<td>-1.312</td>
<td>-2.375</td>
<td>-2.628</td>
<td>-6.882**</td>
</tr>
<tr>
<td>0</td>
<td>-0.4945</td>
<td>-1.824</td>
<td>-2.316</td>
<td>-1.270</td>
<td>-3.098*</td>
<td>-3.825*</td>
<td>-9.950**</td>
</tr>
</tbody>
</table>

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

As we are using the VAR only for forecasting we estimate a VAR in levels of the variables and ignore any possible cointegration arising from the variables that have unit roots. For space reasons we do not report the VAR estimates, but we note that a lag of 6 produces serially uncorrelated residuals.

We examine fiscal sustainability based a constant target debt-GDP ratio for three horizons: one-year, two-years and five-years ahead. For each horizon we present four figures. Figures 2.n are plots of $FSI(n)$, the index of the fiscal stance. We recall that $FSI(n) < 1$ implies that the debt-GDP ratio is forecast to be above target. The forecasts are based on estimates of the VAR for the whole sample.

Figures 3-5 give various breakdowns of the index into its component parts. Thus, Figures 3.n are plots of $ln\frac{b}{y}$ and the forecast logarithm of the present value of current and future primary surpluses, $ln K_t,n$, which we denote in the graph by $EPVBCG(n)$. There are three components to $FS(t,n)$: the desired change in discounted debt $PVdb(n)$, the present value of the primary surplus.
$PVs(n)$ and the term for the discount factor, $PV\rho(n)$. These are plotted in Figures 4.\textit{n}. An indication of the benefit of using a log-linear model is given by the extent to which $PV\rho(n)$ differs from unity. Finally, in Figures 5.\textit{n} we plot the two components of $PVs(n)$. These are the present value of revenues $PVv(n)$ and of expenditures $PVg(n)$.

(i) One-year horizon

![Figure 2.1: US FSI(1).](image1)

![Figure 3.1: US b/y and exp[PVGBC(1)].](image2)
Figure 4.1: US PVs(1), PVdb(1) and PVrho(1).

Figure 5.1: US PVv(1) and PVg(1).

(ii) Two year horizon
Figure 2.2: US FSI(2).

Figure 3.2: US b/y and exp[PVGC(2)].
(iii) Five-year horizon
Figure 2.5: US FSI(5).

Figure 3.5: US b/y and exp[PVGBK(5)].
We observe that $FSI(n)$, the index of the fiscal stance, exceeds unity for any length of time only during 1990’s. In the other periods it is either roughly equal to unity (implying that the fiscal stance is compatible with a non-rising debt-GDP ratio) or less than unity (implying that the debt-GDP ratio is rising). From 2001 the $FSI$ strongly indicates a rising level of the debt-GDP ratio at each horizon. The $FSI$ is also less than unity for the period ending in 1989. The start date of this period depends on the time horizon. For one-year and two-year horizons it is similar, consisting of most of the 1980’s, but for the five-year horizon it extends back through the 1970’s,
almost to 1965. Thus the 1990’s marked a period of US fiscal recovery which ended in around 2000.

Decomposing the index into its components, we find that $FSI < 1$ for the period 1979-1994 when the debt-GDP ratio rose substantially. We also find that variations in the present value of forecast primary surpluses are the main determinant of fluctuations in the index. The change in debt target and the discount factor nearly offset each other. This is because we have assumed a constant discounted debt target and so the discount factor is the variable causing the change in discounted debt term to fluctuate.

The present values for expenditures and revenues are similar before 1995 but are different thereafter. In the period 1995-2001 the present value of revenues exceed those of expenditures thereby producing a fiscal recovery. After 2001 the present value of expenditures exceed those of revenues. This fiscal deterioration was due to a combination of rising expenditures and sharply falling revenues. Fluctuations in the discount rate make an additional, but not large, contribution.

To summarize, there is clear evidence of a break in US fiscal policy from 2001 that has resulted in a rising debt-GDP ratio no matter the horizon over which we look. This fiscal stance would be unsustainable if maintained. The cause is a combination of a rising present value of expenditures and of sharply falling revenues. There have been previous periods when the fiscal stance also led to a rising debt-GDP ratio, most notably from 1979-1994. This was not fully corrected until the period 1995-2000 when the present value of expenditures was reduced and was much lower than that of revenues.

### 5.2 The United Kingdom

The data for the UK are plotted in Figure 6.
Augmented Dickey-Fuller tests are reported in Table 3. We conclude from these results that $\ln \frac{g}{y}$ and the real growth rate are stationary variables.

<table>
<thead>
<tr>
<th>D-Lag</th>
<th>Variable</th>
<th>$\ln \frac{b}{y}$</th>
<th>$\ln \frac{g}{y}$</th>
<th>$\ln \frac{v}{y}$</th>
<th>$\ln (1 + R)$</th>
<th>$\ln (1 + pi)$</th>
<th>$\rho$</th>
<th>$\Delta \ln y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\ln \frac{b}{y}$</td>
<td>-2.349</td>
<td>-4.184**</td>
<td>-2.416</td>
<td>-1.503</td>
<td>-1.267</td>
<td>-1.620</td>
<td>-3.600*</td>
</tr>
<tr>
<td></td>
<td>$\ln \frac{g}{y}$</td>
<td>-2.432</td>
<td>-3.390*</td>
<td>-3.194*</td>
<td>-1.362</td>
<td>-1.768</td>
<td>-1.582</td>
<td>-4.595**</td>
</tr>
<tr>
<td>1</td>
<td>$\ln \frac{v}{y}$</td>
<td>-1.400</td>
<td>-1.996</td>
<td>-2.250</td>
<td>-0.9936</td>
<td>-1.691</td>
<td>-1.757</td>
<td>-3.981**</td>
</tr>
</tbody>
</table>

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

Based once again on a levels VAR(6), but considering only a one-year horizon, we obtain the measures of the index reported in Figures 7-10.
Figure 7: UK FSI(1).

Figure 8: UK b/y and exp[PVGC(1)].
We observe only two brief periods where $FSI > 1$. These are 1986-1988 and 1997-2000. From 1971-1984 and after 2000 $FSI < 1$ often by a considerable margin. The period 1984-2005 has four clear episodes. From 1984-1989 there were falls in the debt-GDP ratio and in both revenues and expenditures in present value terms resulting in an improving fiscal position. This was a period where privatization receipts were used to pay off debt, even though the assets were not included in our measure of debt, namely, net government liabilities. From 1989-1992, when sterling left the ERM, the fiscal position deteriorated sharply due to rising expenditures. This may even have
been a contributory factor in the speculation against sterling in 1992. After 1992 the debt-GDP rose steadily as it did in the US, but expenditures, after continuing to rise, turned down, which caused an improvement in the fiscal stance. From 1996-2001 there was a marked improvement in the fiscal position mainly due to rising revenues from the upturn in economic activity. From 2001 the fiscal stance deteriorated again due to expenditures (which started to increase in 1998) rising much more than revenues. The Chancellor of the Exchequer has said throughout his tenure that the UK is meeting its fiscal targets, but this evidence indicates that this has not precluded an obvious decline in the sustainability of the UK’s fiscal stance.

5.3 Germany

The data are annual for the period 1970 to 2005 and are plotted in Figure 11.

![Figure 11: Germany data](image)

The augmented Dickey-Fuller tests reported in Table 4 do not allow us to reject a unit root for any of the variables.

Table 4
Augmented Dickey-Fuller tests (sub-sample 1976-2005)

<table>
<thead>
<tr>
<th>D-Lag</th>
<th>( \ln \frac{b}{y} )</th>
<th>( \ln \frac{g}{y} )</th>
<th>( \ln \frac{v}{y} )</th>
<th>( \ln (1 + R) )</th>
<th>( \ln (1 + \pi_t) )</th>
<th>( \rho )</th>
<th>( \Delta \ln y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-1.315</td>
<td>-2.102</td>
<td>-1.653</td>
<td>-2.016</td>
<td>-1.355</td>
<td>-2.176</td>
<td>-2.515</td>
</tr>
<tr>
<td>1</td>
<td>-1.918</td>
<td>-2.080</td>
<td>-1.382</td>
<td>-1.645</td>
<td>-1.635</td>
<td>-2.850</td>
<td>-3.472*</td>
</tr>
<tr>
<td>0</td>
<td>-3.582*</td>
<td>-2.017</td>
<td>-1.422</td>
<td>-5.303**</td>
<td>-2.125</td>
<td>-3.431*</td>
<td>-3.680*</td>
</tr>
</tbody>
</table>

Note: * denotes significance at the 5% level and ** denotes significance at the 10% level.

The results on the index of the fiscal stance for the period from 1977 are reported in Figures 11-15 for a one-year horizon. The reason for starting in 1977 is that prior to this the debt-GDP ratio was negative.

![Figure 12: Germany FSI (1).](image-url)
Figure 13: Germany b/y and exp[PVGBE(1)].

Figure 14: Germany PVs(1), PVdb(1) and PVRho(1).
There has been a steady deterioration in the FSI over the whole period since 1977. There were two occasions when the index worsened sharply. They are in 1989 on German unification, and again in 1999 shortly after EMU began. Both events seem to have been very harmful to the fiscal stance. Throughout the period the debt-GDP ratio has risen and, with the exception of the period 1992-1999, the fiscal position has gradually deteriorated. The improvement during the period 1992-1999 coincides with improvements in the US and UK and is due to sustained economic growth causing a rise in tax revenues. But since expenditures also increased during this period, the improvement in the German fiscal stance was less marked that for those of the US and UK. Since 1999 the fiscal stance has continued to worsen as expenditures, although falling over the period, have exceeded revenues which have also decreased. The observed secular decline in the German fiscal stance reflects and supports the widespread perception that Germany may need structural reform.

6 Conclusions

In this paper we have proposed the construction of an index to measure the current fiscal stance. We have distinguished this from existing measures of the sustainability of the fiscal stance and
argued that such tests, which focus on the past, may not be a helpful guide to the current stance of fiscal policy. Like the tests for fiscal sustainability, this index is based on the government inter-temporal budget constraint. The main differences are that the index is forward looking, it applies to a finite time horizon, and it uses a log-linear approximation to the government budget constraint which enables the inflation, economic growth and interest rates to be time varying rather than constant. In effect, the index is based on a comparison of the forecast and the desired debt-GDP ratio over that horizon where the forecast is constrained to satisfy the government budget constraint. We propose the use of a VAR forecasting model based on the government budget constraint as this is simple to compute and easily automated. We have shown how to identify individual components of the index that may be causing problems for the fiscal stance.

We have applied this methodology to three countries: the US, the UK and Germany. In the UK and US the index of fiscal sustainability has fluctuated considerably with periods when the debt-GDP ratio has risen followed by periods when it has fallen. During the period of strong economic growth in the 1990’s the fiscal positions of all three countries improved considerably, but in recent years the fiscal stance in all three countries has been steadily deteriorating. Our index indicates that a continuation of the present fiscal stances is leading to fiscal unsustainability in the three countries. We have shown that the German fiscal position has worsened steadily over the last thirty years with only a brief respite in the mid 1990’s. A sharp deterioration occurred after unification and again on joining EMU.
7 References


Data appendix

The US data are quarterly for the period 1960.1 to 2005.4 and are taken from the OECD Economic Outlook database and are described in the OECD Economic Outlook Database Inventory and on the Annex Tables session of the Sources and Methods.

\( GDP \), Value, at market prices, of gross domestic product;

\( GNFL \), Value of government net financial liabilities\(^4\);

\( PGDP \), deflator of \( GDP \) at market prices;

\( GGINTP \), Value of gross government interest payments;

\( GGINTR \), Value of gross government interest receipts;

\( GNINTP \), Value of net government interest payments\(^5\);

\( YPGT \), Value of government total disbursement;

\( YRGT \), Value of government total receipts;

\( IRS \), Short-term nominal interest rate (in percentages)\(^6\);

\( IRL \), Long-term interest rate (in percentages)\(^7\).

The variables used in this study are then calculated as follows:

1. \( \frac{b_t}{y_t} \) is \( GNFL \) deflated by \( GDP \).

2. \( \frac{v_t}{y_t} \) is \( YRGT \) minus \( GGINTR \) and deflated by \( GDP \).

3. \( \frac{g_t}{y_t} \) is \( YPGT \) minus \( GGINTP \) deflated by \( GDP \).

4. \( R_t \) is \( GNINTP \) deflated by the \( GNFL \) in the previous period value

5. \( \pi_t \) is the quarterly rate of change in the natural logarithm of \( PGDP \).

\(^4\) This variable refers to the consolidated gross financial liabilities of the government sector net of short-term financial assets, such as cash, bank deposits, loans to the private sector etc.

\(^5\) \( GGINTP = GNINTP - GNINTR \)

\(^6\) U.S. rates refer to interest rates on United States dollar three-month deposits in London, UK interest rates are 3-month rates on interbank loans, while Germany interest rates refer to the 3-month FIBOR rate.

\(^7\) Rates refer to the ten-year government bond yield for the US and the UK, while they refer to the federal bond yield in the case of Germany.
6. $r_{st}$ is $IRS$ divided by 100

7. $r_{lt}$ is $IRL$ divided by 100