Neoclassical vs. Endogenous Growth Analysis: An Overview

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After a long period of quiescence, growth economics has in the last decade (1986–1995) become an extremely active area of research—both theoretical and empirical.[1] To appreciate recent developments and understand associated controversies, it is necessary to place them in context, i.e., in relation to the corpus of growth theory that existed prior to this current burst of activity. This article's exposition will begin, then, by reviewing in Sections 1–4 the *neoclassical* growth model that prevailed as of 1985. Once that has been accomplished, in Section 5 we shall compare some crucial implications of the neoclassical model with empirical evidence. After tentatively concluding that the neoclassical setup is unsatisfactory in several important respects, we shall then briefly describe a family of "endogenous growth" models and consider controversies regarding these two classes of theories. Much of this exposition, which is presented in Sections 6–8, will be conducted in the context of a special-case example that permits an exact analytical solution so that explicit

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[1] Notable contributions have been made by Romer (1986, 1989, 1990), Lucas (1988), Rebelo (1991), King and Rebelo (1990), Levine and Renelt (1992), Barro and Sala-i-Martin (1992, 1995), Summers and Heston (1988), and Mankiw, Romer, and Weil (1992), among others. A comprehensive and authoritative treatment of growth analysis has recently been provided by Barro and Sala-i-Martin (1995).

comparisons can be made. Finally, some overall conclusions are tentatively put forth in Section 9. These conclusions, it can be said in advance, are broadly supportive of the endogenous growth approach. Although the article contends that this approach does not strictly justify the conversion of "level effects" into "rate of growth effects," which some writers take to be the hallmark of endogenous growth theory, it finds that the quantitative predictions of such a conversion may provide good approximations to those strictly implied.

1. BASIC NEOCLASSICAL SETUP

Consider an economy populated by a large (but constant) number of separate households, each of which seeks at an arbitrary time denoted $t = 1$ to maximize

$$u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots, \tag{1}$$

where c_t is the per capita consumption of a typical household member during period t and where $\beta = 1/(1+\rho)$ with $\rho > 0$ the rate of time preference. The instantaneous utility function u is assumed to be well behaved, i.e., to have the properties $u' > 0,\ u'' < 0,\ u'(0) = \infty,\ u'(\infty) = 0$. The analysis would not be appreciably altered if leisure time were included as a second argument, but to keep matters simple, leisure will not be recognized in what follows. Instead, it will be presumed that each household member inelastically supplies one unit of labor each period.

It is assumed that the number of individuals in each household grows at the rate ν; thus each period the number of members is $1+\nu$ times the number of the previous period. In light of this population growth, some analysts postulate a household utility function that weights each period's $u(c_t)$ value by the number of household members, a specification that is effected by setting $\psi = 1$ in the following more general expression:

$$u(c_1) + (1+\nu)^{\psi}\beta u(c_2) + (1+\nu)^{2\psi}\beta^2 u(c_3) + \ \ldots. \tag{1$'$}$$

With $\psi = 0$, expression (1′) reduces to (1) whereas ψ values between 0 and 1 provide intermediate assumptions about this aspect of the setup. Most of what follows will presume $\psi = 0$, but the more general formulation (1′) will be referred to occasionally.

Each household operates a production facility with input-output possibilities described by a production function $Y_t = F(K_t, N_t)$, where N_t and K_t are the household's quantities of labor and capital inputs with Y_t denoting output during t. The function F is presumed to be homogeneous of degree one so, by letting y_t and k_t denote per capita values of Y_t and K_t, we can write

$$y_t = f(k_t), \tag{2}$$

where $f(k_t) \equiv F(k_t, 1)$. It is assumed that f is well behaved (as defined above).

Letting v_t denote the per capita value of (lump-sum) government transfers (so $-v_t$ = net taxes), the household's budget constraint for period t can be written in per capita terms as

$$f(k_t) + v_t = c_t + (1+\nu)k_{t+1} - (1-\delta)k_t. \quad (3)$$

Here δ is the rate of depreciation of capital. As of time 1, then, the household chooses values of $c_1, c_2, \ldots$ and $k_2, k_3, \ldots$ to maximize (1) subject to (3) and the given value of k_1. The first-order condition necessary for optimality can easily be shown to be

$$(1+\nu)u'(c_t) = \beta u'(c_{t+1})[f'(k_{t+1}) + 1 - \delta], \quad (4)$$

and the relevant transversality condition is[2]

$$\lim_{t\to\infty} k_{t+1}\beta^{t-1}u'(c_t) = 0. \quad (5)$$

The latter provides the additional side condition needed, since only one initial condition is present, for (3) and (4) to determine a unique time path for c_t and $k_{t+1.}$ Satisfaction of conditions (3), (4), and (5) is necessary and sufficient for household optimality.[3]

To describe this economy's competitive equilibrium, we assume that all households are alike so that the behavior of each is given by (3), (4), and (5).[4] The government consumes output during t in the amount g_t (per person), the value of which is determined exogenously. For some purposes one might want to permit government borrowing, but here we assume a balanced budget. Expressing that condition in per capita terms, we have

$$g_t + v_t = 0. \quad (6)$$

For general competitive equilibrium (CE), then, the time paths of c_t, k_t, and v_t are given by (3), (4), and (6), plus the transversality condition (5). In most of what follows, it will be assumed that $g_t = v_t = 0$, in which case the CE values

[2] The role of this condition is outlined in Appendix A.

[3] Some references to proofs, in the context of a version that includes stochastic technology shocks, are given in McCallum (1989).

[4] Actually, the general equilibrium character of the analysis would be more apparent if we were to distinguish between quantities of capital supplied and demanded (per capita) by household h, writing them as $k_t(h)$ and $k_t^d(h)$. Then market clearing for period t would be represented by the condition $\Sigma k_t(h) = \Sigma k_t^d(h)$, where the summation is over all households. But with all households being treated as alike, which we do for simplicity, that condition reduces to $k_t(h) = k_t^d(h)$, so nothing is lost by failing to make the distinction. A similar conclusion is applicable to labor demand and supply, so the economy under discussion should be thought of as one with markets for capital and labor, even though these do not appear explicitly in the discussion. Also, the presence of a loan market is implicitly assumed, with one-period loans and capital serving a household equally well as stores of value.

of c_t and k_t are given by (4) and

$$f(k_t) = c_t + (1+\nu)k_{t+1} - (1-\delta)k_t, \tag{7}$$

provided that they satisfy (5).

Much interest centers on CE paths that are *steady states*, i.e., paths along which every variable grows at some constant rate.[5] It can be shown that in the present setup, with no technical progress, any steady state is characterized by stationary (i.e., constant) values of c_t and k_t.[6] (These constant values imply growth of economy-wide aggregates at the rate ν, of course.) Thus from (4) we see that the CE steady state is characterized by $f'(k) + 1 - \delta = (1+\nu)(1+\rho)$ or

$$f'(k) - \delta = \nu + \rho + \nu\rho. \tag{8}$$

This says that the net marginal product of capital is approximately (i.e., neglecting the interaction term $\nu\rho$) equal to $\nu + \rho$, a condition that should be kept in mind. If the more general utility function ($1'$) is adopted, the corresponding result is $f'(k) + 1 - \delta = (1+\rho)(1+\nu)^{1-\psi}$. Thus with $\psi = 1$, i.e., when household utility is $u(c)$ times household size, we have $f'(k) - \delta = \rho$.

It can be shown that, in the model at hand, the CE path approaches the CE steady state as time passes. Given an arbitrary k_1, in other words, k_t approaches the value k^* that satisfies (8) as $t \to \infty$. This result can be clearly and easily illustrated in the special case in which $u(c_t) = \log c_t$, $f(k_t) = Ak_t^\alpha$, and $\delta = 1$.[7] (Below we shall refer to these as the "LCD assumptions," L standing for log and CD standing for both Cobb-Douglas and complete depreciation.) In this case, equations (4) and (7) become

$$\frac{(1+\nu)}{c_t} = \frac{\beta\alpha Ak_{t+1}^{\alpha-1}}{c_{t+1}} \tag{9}$$

and

$$Ak_t^\alpha = c_t + (1+\nu)k_{t+1}. \tag{10}$$

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[5] Some authors use the term "balanced growth" for such paths. To me it seems preferable to use "steady state" so as to suggest a generalization of the concept of a stationary state, in which case every variable must grow at the constant rate of zero.

[6] That conclusion can be justified as follows. In (4), $u'(c_t) \equiv \lambda_t$ is an important variable. For it to grow at a constant rate, λ_{t+1}/λ_t must be constant through time. But by (4) that implies that $f'(k_{t+1})$ must be constant and so the properties of f imply that k is constant. Then we draw upon the algebraic requirement that for any three variables related as $y_t = x_t + z_t$, all three can grow at constant rates only if the rates are equal. (This can be seen by writing $y_t - y_{t-1} = x_t - x_{t-1} + z_t - z_{t-1}$ and then dividing by y_{t-1} : $(y_t - y_{t-1})/y_{t-1} = (x_t - x_{t-1})/y_{t-1} + (z_t - z_{t-1})/y_{t-1} = (x_{t-1}/y_{t-1})(x_t - x_{t-1})/x_{t-1} + (z_{t-1}/y_{t-1})(z_t - z_{t-1})/z_{t-1}$. Then if the growth rate of x, $(x_t - x_{t-1})/x_{t-1}$, exceeds that of z, the growth rate of y will increase as time passes—so the rates must be the same for x and z, and thus for y.) But then the budget constraint (3) implies, by repeated application of the foregoing requirement, that c_t, y_t, and v_t must all grow at the same rate as k_t, i.e., zero.

[7] Throughout, $\log x$ denotes the natural logarithm of x.

Since the value of k_t^α summarizes the state of the economy at time t, it is a reasonable conjecture that k_{t+1} and c_t will each be proportional to k_t^α. Substitution into (9) and (10) shows that this guess is correct and that the constants of proportionality are such that $k_{t+1} = \alpha\beta(1+\nu)^{-1}Ak_t^\alpha$ and $c_t = (1-\alpha\beta)Ak_t^\alpha$. These solutions in fact satisfy the transversality condition (TC) given by (5), so they define the CE path. The k_t solution can then be expressed in terms of the first-order linear difference equation

$$\log k_{t+1} = \log[\alpha\beta A/(1+\nu)] + \alpha \log k_t, \tag{11}$$

which can be seen to be dynamically stable since $|\,\alpha\,| < 1$. Thus $\log k_t$ converges to $(1-\alpha)^{-1}\log[\alpha\beta A/(1+\nu)]$. For reference below, we note that subtraction of $\log k_t$ from each side of (11) yields

$$\log k_{t+1} - \log k_t = (1-\alpha)[\log k^* - \log k_t], \tag{12}$$

where $k^* = [\alpha\beta A/(1+\nu)]^{1/(1-\alpha)}$, so $1-\alpha$ is in this special case a measure of the speed of convergence of k_t to k^*.

It might be thought that the complete-depreciation assumption $\delta = 1$ renders this special case unusable for practical or empirical analysis. But such a conclusion is not inevitable. What is needed for useful application, evidently, is to interpret the model's time periods as pertaining to a span of calendar time long enough to make $\delta = 1$ a plausible specification—say, 25 or 30 years. Then the parameters A, β, and ν must be interpreted in a corresponding manner. Suppose, for example, that the model's time period is 30 years in length. Thus if a value of 0.98 was believed to be appropriate for the discount factor with a period length of one year, the appropriate value for β with 30-year periods would be $\beta = (0.98)^{30} = 0.545$. Similarly, if the population growth parameter is believed to be about one percent on an annual basis, then we would have $1+\nu = (1.01)^{30} = 1.348$. Also, a realistic value for A would be about $10k^{(1-\alpha)}$, since it makes $k/y = 3/30 = 0.1$. So the LCD assumptions could apparently be considered for realistic analysis, provided that one's interest is in long-term rather than cyclical issues.[8]

2. TECHNICAL PROGRESS

Since the foregoing model approaches a steady state in which per capita values are constant over time, it may seem to be a strange framework for the purpose of *growth* analysis. But in the neoclassical tradition, growth in per capita values is provided by assuming that steady technical progress occurs, continually shifting the production frontier as time passes. With technical

[8] It is true, of course, that $\delta = 1$ implies a qualitatively different time profile for the depreciation of capital than when $\delta < 1$, since in the latter case a given stock of capital will never disappear entirely in finite time. But the usual assumption is made more for tractability than because of any evidence that it is truly representative of actual physical decay processes.

progress proceeding at the rate γ, the production function would in general be written as $Y_t = F(K_t, N_t, (1+\gamma)^t)$.[9] It transpires, however, that steady-state growth is only possible when technical progress occurs in a "labor-augmenting" fashion, i.e., when

$$Y_t = F(K_t, (1+\gamma)^t N_t).^{10} \tag{13}$$

But then with F homogeneous of degree one, we have

$$\hat{y}_t = f(\hat{k}_t), \tag{14}$$

where $y_t \equiv Y_t/(1+\gamma)^t N_t$ and $\hat{k}_t \equiv K_t/(1+\gamma)^t N_t$ are values of output and capital per "efficiency unit" of labor. Alternatively, $y_t = y_1(1+\gamma)^t f(k_t/k_1(1+\gamma)^t)$. The household's budget constraint, when expressed in terms of these variables (and with $v_t \equiv 0$), becomes

$$f(\hat{k}_t) = c_t/(1+\gamma)^t + (1+\nu)(1+\gamma)\hat{k}_{t+1} - (1-\delta)\hat{k}_t. \tag{15}$$

Maximizing (1) subject to (15) gives rise to the following first-order condition, analogous to (4):

$$(1+\nu)(1+\gamma)u'(c_t)(1+\gamma)^t = \beta u'(c_{t+1})(1+\gamma)^{t+1}[f'(\hat{k}_{t+1}) + 1 - \delta]. \tag{16}$$

In addition, we have the transversality condition

$$\lim_{t\to\infty} \hat{k}_{t+1}\beta^{t-1}u'(c_t)(1+\gamma)^t = 0. \tag{17}$$

Since there are no additional equilibrium conditions, presuming that $g_t = v_t = 0$, competitive equilibrium time paths of c_t and k_t are determined by (15) and (16), given the initial value of k, and the limiting condition (17).

Now, in order for steady growth of both c_t and $u'(c_t)$ to be possible, it will be assumed that agents' preferences are such that the function $u'(c_t)$ has a constant elasticity.[11] For reasons of symmetry, the function is usually written as

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta} \qquad \theta > 0, \tag{18}$$

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[9] We assume that this rate γ satisfies $\gamma < \rho$. If instead we had $\gamma \geq \rho$, then the value of c_t would grow rapidly enough that in a steady state the infinite series (1) would not be convergent. Such a situation gives rise to mathematical complexities that are beyond the scope of the present exposition.

[10] For a proof that this form of technical progress is necessary for steady-state growth, see Appendix B. Of course, it is true that if the production function is Cobb-Douglas, then both Hicks-neutral and capital-augmenting technical progress are equivalent to the labor-augmenting type.

[11] Equation (16) implies that, if $\hat{k}_t$ is to be constant as it must be in a steady state, we must have $u'(c_{t+1}) = \psi u'(c_t)$ where ψ is a constant. Let $c_{t+1} = (1+\gamma)c_t$ and differentiate the previous expression with respect to c_t, obtaining $u''(c_{t+1})c_{t+1}/u'(c_{t+1}) = u''(c_t)c_t/u'(c_t)$, which implies that $u'(c)$ has the same elasticity for all values of c.

which has an elasticity of marginal utility of $-\theta$ and reduces to $u(c_t) = \log c_t$ in the special limiting case in which $\theta \to 1$.[12] Using (18), then, we rewrite (16) as

$$(1+\rho)(1+\nu)(c_t/c_{t+1})^{-\theta} = f'(\hat{k}_{t+1}) + 1 - \delta. \qquad (19)$$

Finally, we define $\hat{c}_t = c_t/(1+\gamma)^t$, which implies that $c_t/c_{t+1} = \hat{c}_t/\hat{c}_{t+1}(1+\gamma)$, so we can rewrite (19) once more as

$$(1+\rho)(1+\nu)(\hat{c}_{t+1}/\hat{c}_t)^{\theta}(1+\gamma)^{\theta} = f'(\hat{k}_{t+1}) + 1 - \delta. \qquad (20)$$

The latter shows that k_t will be constant in the CE steady state, and (15) then implies that the same will be true for $\hat{c}_t$. Thus we see that the per capita variables k_t, c_t, and y_t will all grow at the rate γ. Thus equation (20) shows that the (constant) value of $f'(\hat{k})$, which equals the marginal product of capital in unadjusted units, will satisfy

$$f'(\hat{k}) - \delta = (1+\rho)(1+\nu)(1+\gamma)^{\theta} - 1. \qquad (21)$$

We can approximate $(1+\gamma)^{\theta}$ with $1+\gamma\theta$, assuming γ is small in relation to 1.0, so dropping cross-product terms we have the approximation

$$f'(\hat{k}) - \delta = \rho + \nu + \gamma\theta. \qquad (22)$$

In the special case with $u(c_t) = \log c_t$, i.e., with $\theta = 1$, the right-hand side of (22) becomes $\rho + \nu + \gamma$. Furthermore, with the other LCD assumptions it can easily be verified that $\hat{k}_t$ behaves as k_t does in Section 2. In particular, with $\hat{y}_t = A\hat{k}_t^{\alpha}$ we have

$$\log \hat{k}_{t+1} - \log \hat{k}_t = (1-\alpha)[\log \hat{k}^* - \log \hat{k}_t], \qquad (23)$$

where $\log \hat{k}^* = (1-\alpha)^{-1} \log[\alpha\beta A/(1+\nu)(1+\gamma)]$, implying that $\hat{k}_t$ approaches $\hat{k}^*$ as time passes with $1-\alpha$ being the rate of convergence.

3. OPTIMALITY

For social optimality, one would want to maximize the typical household's utility subject to the economy's overall resource constraint. But in the case in which $g_t = v_t = 0$, this constraint is exactly the same as the household's budget constraint, if each is expressed in per capita terms. So the social optimization problem becomes formally indistinguishable from the one solved by a typical household. Accordingly, the CE path will satisfy all the conditions for social optimality. This result would not hold, however, if $g_t > 0$ were financed by

[12] To find the limit as $\theta \to 1$ of $(c^{1-\theta} - 1)/(1-\theta)$, we use l'Hopital's rule to express it as the ratio of the limits of $d(c^{1-\theta} - 1)/d\theta = -c^{(1-\theta)} \log c$ and $d(1-\theta)/d\theta = -1$. Thus for $\theta \to 1$, we have $-c^0 \log c/(-1) = \log c$.

taxes that are distorting or if the model were modified so as to reflect some sort of externality.[13]

Now suppose that we ask the following question: In the model with technical progress, what $\hat{k}$ will yield the highest value of $\hat{c}$ that can be permanently sustained? In other words, among steady states that are not necessarily CE paths, which one yields the highest value of $\hat{c}$? Clearly, the budget constraint (15) implies

$$\hat{c} = f(\hat{k}) - (1+\nu)(1+\gamma)\hat{k} + (1-\delta)\hat{k} \tag{24}$$

for *any* steady state, so one can maximize $\hat{c}$ by differentiating the right-hand side with respect to $\hat{k}$ and setting the result equal to zero. We find that

$$0 = f'(\hat{k}) - (1+\nu)(1+\gamma) + (1-\delta) \tag{25}$$

is the implied condition on $\hat{k}$. Approximately, then,

$$f'(\hat{k}) - \delta = \nu + \gamma, \tag{26}$$

where the cross-product term $\nu\gamma$ has been dropped.

It will be noticed immediately that this implied "golden rule" value of $\hat{k}$, which we call $\hat{k}^{+}$, does not agree with the one given by (22) as the steady-state value $\hat{k}^{*}$ that is approached by the CE path.[14] Also, we see that, with $f' < 0, \hat{k}^{+}$ will normally exceed $\hat{k}^{*}$ since $\nu + \gamma$ will normally be smaller than $\nu + \rho + \theta\gamma$. (The latter will clearly be true under our assumption of $\gamma < \rho$ that guarantees convergence of [1].) Here the main point is that $\hat{k}^{*}$ has been found to be socially optimal, since it is the value approached by the CE under conditions that make CE paths satisfy all the requirements for social optimality. This fact sometimes generates confusion, since the steps leading to (26) seem to make $\hat{k}^{+}$ optimal from a steady-state perspective, as it gives a value of $\hat{c}$ larger than with $\hat{k}^{*}$. But because of time preference—i.e., $\rho > 0$ or $\beta < 1$—an economy in a steady state with $\hat{k}_t = \hat{k}^{+}$ could increase the value of (1) by immediately consuming slightly more than the golden rule amount, given by (24) with $\hat{k}^{+}$, and moving to a steady state with $\hat{k}$ somewhat smaller than $\hat{k}^{+}$. And so long as $\hat{k} > \hat{k}^{*}$, this same possibility remains open. Thus we conclude again that the optimal path beginning at any time would be given by (23), which implies that k_t will approach $\hat{k}^{*}$ as time passes.

[13] Suppose, for example, that $g_t > 0$ and is financed by a tax on production at the rate τ so that the (per capita) government budget constraint is $g_t = \tau f(k_t)$. Then a typical household's condition analogous to (4) becomes $(1+\upsilon)u'(c_t) = \beta u'(c_{t+1})[(1-\tau)f'(k_{t+1}) + 1 - \delta]$, but the counterpart for social optimality does not include the $(1-\tau)$ term. For a steady state with no technical progress, then, we would have $(1-\tau)f'(k) - \delta = (1+\upsilon)(1+\rho) - 1$ in CE which makes $f'(k)$ larger than optimal—that is, too little capital is accumulated (even conditional on g_t). An externality example will be considered below, in Section 6.

[14] The value $\hat{k}^{*}$ is frequently referred to as the "modified golden rule" value.

One way to understand the foregoing is to note that the golden rule path yields "steady-state optimality" only for the highly artificial problem of first imposing a steady-state restriction and then optimizing, instead of optimizing and then finding what conditions would be implied by an equilibrium that happens to be a steady state. The latter comes much closer to answering a relevant operational question. Or, to put the contrast in other words, k^{+} is the answer to the question "what capital stock would your economy like to be miraculously given under the condition that it be *required* to maintain that value forever?"[15] By contrast, $\hat{k}^{*}$ is the answer to the question, "Among capital stocks that your economy would *willingly* maintain forever, which one is most desirable?"

It might be noted, incidentally, that the importance of this point regarding steady-state optimality would not be diminished by the presence of money—i.e., a transaction-facilitating medium of exchange—in the economy under consideration. Thus it remains true in monetary models that optimal steady-state paths are those found by conducting optimality analysis prior to the imposition of steady-state conditions. Failure to proceed in this manner has led to misleading conclusions or suggestions by several analysts and even mars the widely used graduate textbook of Blanchard and Fischer (1989, Chapter 4).[16]

4. HISTORY OF THOUGHT

Before continuing, let us pause to note that development of the neoclassical growth model is frequently attributed to Ramsey (1928), Cass (1965), and Koopmans (1965), with an extension to a stochastic environment provided by Brock and Mirman (1972). These papers were all concerned, however, only with the social planning problem, not with market outcomes. Recognition that the mathematical expressions could be reinterpreted so as to provide a positive theory of the behavior of a competitive market economy was first made in print—as far as I have been able to determine—by Brock (1974a). Extension to a monetary economy was accomplished by Brock (1974b).

A justly famous paper by Solow (1956) developed an analysis of growth that is in several ways closely related to the one provided by the neoclassical model. Solow's paper did not include dynamic optimizing analysis of households' saving behavior, however, but simply took the fraction of income saved to be a given constant. A contemporaneous paper by Swan (1956) developed

[15] In efficiency-adjusted per capita units, that is.

[16] Specifically, Blanchard and Fischer suggest on page 191 that the Chicago Rule optimal inflation result—i.e., that the optimal steady inflation rate equals the negative of the real rate of return on capital—depends upon the property of monetary superneutrality, and on page 181 they state that the Chicago Rule is not optimal in a monetary overlapping generations model. Both of these claims are overturned, however, by analysis that imposes steady-state restrictions after conducting a more general optimality analysis (see McCallum [1990], pp. 976–78, and 983).

a rather similar analysis in a fashion that was less mathematically explicit. Discussion of the "golden rule" condition can be found in papers by Phelps (1961) and Solow (1962).

Prior to publication of the Solow and Swan papers, considerable attention had been given to a result of Harrod (1939) and Domar (1947) to the effect that in a steady state the product of the saving/output ratio and the output/capital ratio must equal the rate of growth of capacity output. In other words, k_t and the capacity level of output $\bar{y}_t$ must grow at the same rate if $y_t/\bar{y}_t$ is to remain constant (as was taken to be necessary). Much was made of the idea that these three numbers might be determined by different aspects of economic behavior, and it was suggested that satisfaction of the condition might be unlikely to result in market economies without activist government policy. Solow (1956) cogently observed that the output/capital ratio could adjust endogenously, but—as Hahn (1987) has noted—this observation does not actually speak to the Harrod-Domar "problem." That is because Solow showed that k_t and y_t could grow at equal rates, but in doing so, he *assumed* that $y_t/\bar{y}_t$ was constant, which was actually the matter of concern to Harrod and Domar. Solow's contribution was great, nevertheless, because he (and Swan) developed something that might reasonably be called a *model*, whereas Harrod and Domar had only derived (via elementary algebra) a *condition* that needed to be satisfied for steady growth.[17]

The resurgence of growth theory that took place in the 1980s, and involved the development of endogenous growth models, arose in response to a perception that the neoclassical framework was severely inadequate for the analysis of actual growth experiences. To detail the perceived inadequacies and the subsequent response is the purpose of the next two sections.

5. WEAKNESSES OF THE NEOCLASSICAL MODEL

The evident trouble with the neoclassical growth model outlined above is that it fails to explain even the most basic facts of actual growth behavior. To a large extent, this failure stems directly from the model's prediction that output per person approaches a steady-state path along which it grows at a rate γ that is given exogenously. For this means that the rate of growth is determined outside the model and is independent of preferences, most aspects of the production function, and policy behavior. As a consequence, the model itself suggests either the same growth rate for all economies or, depending on one's interpretation, different values about which it has nothing to say. But in reality different nations have maintained different per capita growth rates over long periods of time—and these rates seem to be systematically related

[17] The Solow contribution is not a complete optimizing model, as has been mentioned, but is a model nevertheless, in the sense of a falsifiable depiction of some economic phenomena.

to various national features, e.g., to be higher in economies that devote large shares of their output to investment. These and other failings were stressed by Romer (1986, 1987, 1989) and Lucas (1988).

Of course, the neoclassical model does imply that *transitional* growth rates will differ across economies, being faster in those that have existing capital-to-effective-labor ratios relatively far below their CE steady-state values. This observation is what prevented fundamental dissatisfaction from being openly expressed before the appearance of the Romer and Lucas papers and is one of the two lines of defense recently mentioned in a lively discussion by Mankiw (1995, p. 281).[18] But transitional phenomena cannot provide a quantitative explanation of the magnitude of long-lasting growth rate differences under the standard neoclassical presumption that the production function is reasonably close to the Cobb-Douglas form with a capital elasticity of approximately one-third (roughly capital's share of national income).[19] One way to describe the problem is to consider a comparison in which one economy's per capita output increases by a factor of 2.9 relative to another's over a period of 30 years, which is the factor that would be relevant if the first economy's average growth rate exceeded the second's by about 3.6 percent per year. (This last figure is twice the standard deviation of per capita growth rates among 114 nations over the years 1960 to 1990, as reported by Barro and Sala-i-Martin [1995], p. 3, so a sizable fraction of all nation pairs have had differences exceeding that value.) Then, with a capital elasticity of one-third, the capital stock per capita would have to increase by a factor of $2.9^3 = 24.4$ relative to the second economy, if their rates of technical progress were the same. Thus the real rate of interest—i.e., the marginal product of capital—in the first economy would fall by a relative factor of $24.4^{2/3} = 8.4$. So if the two economies had similar real interest rates at the end of the 30-year period, the first economy's rate would have been 8.4 times as high as the second's at the start of the period! But of course we do not observe in actual data changes in capital/labor ratios or real interest rates that are anywhere near as large as those magnitudes, even though we observe many output growth differentials of 3.6 percent and more.[20] Some evidence that this argument is robust to production function assumptions, and a dramatic comparison involving Japan and the United States, is provided by King and Rebelo (1993).

[18] The second line of defense, that the neoclassical model may do a reasonable job of explaining cross-country differences in the *level* of income, will be discussed below.

[19] The following discussion is adapted from McCallum (1994).

[20] The interest rate portion of the foregoing argument counterfactually presumes two closed economies but so does the usual neoclassical growth analysis.

The same general type of calculation is also relevant for cross-country comparisons. The level of per capita incomes in the industrial nations of the world are easily 10 times as high as in many developing nations. With a production function of the type under discussion, this differential implies a capital per capita ratio of $10^3 = 1000$, and therefore a ratio of marginal products of capital of $1000^{-2/3} = 1/100 = 0.01$. In other words, the real rate of return to capital is predicted to be about 100 times as high in the developing nations as in those that are industrialized. But surely a differential of this magnitude would induce enormous capital flows from rich to poor countries, flows entirely unlike anything that is observed in actuality.[21]

Another perspective on the neoclassical vs. endogenous growth issue involves the question of "convergence," which has been much discussed in the literature. From equations (14) and (23) above we see that if all nations had the same taste and technology parameters, and the same population growth rate, then they should, according to the neoclassical model, have the same steady-state level of per capita income. Thus as time passes, per capita income levels in different countries should converge to a common value, with low income countries growing more rapidly than those in which beginning per capita income levels are high. Empirically, however, it is the case that growth rates over periods such as 1960 to 1985 are virtually uncorrelated with initial-year income levels. In fact, there is a small, positive coefficient in the Mankiw-Romer-Weil (1992) sample of 98 "non-oil" countries; their cross-section regression is

$$\begin{aligned} \log y_{1985} - \log y_{1960} &= \underset{(0.38)}{-0.27} + \underset{(0.050)}{0.094} \log y_{1960} \\ \bar{R}^2 = 0.03 &\qquad SE = 0.44. \end{aligned} \tag{27}$$

The neoclassical model does not actually require, however, that population growth values are equal in various countries and does not imply that taste and technology parameters must be the same. So convergence in the "unconditional" sense of the foregoing discussion is not, it can be argued, relevant to the performance of the neoclassical model. What that model does imply, according to authors including Barro and Sala-i-Martin (1992, 1995) and Mankiw, Romer, and Weil (1992), is a concept that has been termed "conditional convergence." It will be discussed below, in Section 8.

It should be noted that the foregoing discussion does not imply that the neoclassical analysis was unproductive. On the contrary, it played a major and essential role in the development of dynamic general equilibrium analysis, the basis for much of today's economic theory. It is only as a theory of growth that it is here being criticized.

[21] For more comparisons of this type and some discussion, see Lucas (1990).

6. ENDOGENOUS GROWTH MECHANISMS

In response to the various failures of the neoclassical model, Romer, Lucas, King and Rebelo, and other scholars have developed models in which steady growth can be generated endogenously—i.e., can occur without any exogenous technical progress—at rates that may depend upon taste and technology parameters and also tax policy. There are numerous variants of such models, but several important points can be developed by focusing on three basic mechanisms. Two of these, one involving a capital accumulation externality and the second relying upon the accumulation of human capital, will be discussed in this section, with the third following in Section 7.

Let us consider first the *externality* model. For its presentation we will modify the setup of Section 1, in which there is no exogenous technical progress. There is, however, an externality in production so that the typical household's per capita production function is

$$y_t = f(k_t, \bar{k}_t) \qquad f_2 > 0_1,\ f_{22} < 0, \tag{28}$$

where $\bar{k}_t$ is the *economy-wide average* capital stock per person. Quoting Romer (1989, p. 90), the "rationale for this formulation is based on the public good character of knowledge. Suppose that new physical capital and new knowledge or inventions are produced in fixed proportions so that [$\bar{k}_t$] is an index not only of the aggregate stock of physical capital but also of the aggregate stock of public knowledge that any firm can copy and take advantage of." But each firm or household is small, so it views $\bar{k}_t$ as given when making its choice of k_{t+1} and other decision variables.

So as to highlight the effect of the resulting externality, suppose that the production function is Cobb-Douglas,

$$y_t = Ak_t^{\alpha}\bar{k}_t^{\eta}, \tag{28$'$}$$

and that the other LCD assumptions hold as well (i.e., $u(c_t) = \log c_t$ and $\delta = 1$). Also, let $g_t = v_t = 0$. Then the household's budget constraint is

$$Ak_t^{\alpha}\bar{k}_t^{\eta} = c_t + (1+\nu)k_{t+1} \tag{29}$$

and its first-order optimality condition is

$$\frac{(1+\nu)}{c_t} = \frac{\beta\alpha Ak_{t+1}^{\alpha-1}\ \bar{k}_{t+1}^{\eta}}{c_{t+1}}. \tag{30}$$

In addition, for general competitive equilibrium the following condition must be satisfied, since households are alike:

$$k_t = \bar{k}_t. \tag{31}$$

Given these relations,[22] it is a reasonable conjecture that in a CE both k_{t+1} and c_t will be proportional to $k_t^{\alpha+\eta}$. Substitution into (29) and (30), using (31), shows this guess to be correct and that the resulting expression for k_{t+1} is

$$k_{t+1} = \alpha\beta(1+\nu)^{-1} A k_t^{\alpha+\eta}. \quad (32)$$

There are two interesting points relating to this solution. First, with $\eta > 0$ the CE path is not socially optimal. For social optimality, the problem is to maximize (1) subject not to (29), but to (29) with (31) imposed. In that case, the equation comparable to (32) that results is

$$k_{t+1} = (\alpha+\eta)\beta(1+\nu)^{-1}\ A k_t^{\alpha+\eta}. \quad (32')$$

Clearly, if $\alpha + \eta < 1$, then (32) implies that k_t approaches a constant value, but it is one that is smaller than the steady-state value implied by (32′)—an outcome that reflects the failure of individuals to take account of their own actions' effect on the economy-wide state of knowledge. Second, if by chance it happened that $\alpha + \eta = 1$, then k_t would grow forever at a constant rate equal to $\alpha\beta(1+\nu)^{-1}A - 1$.[23] Thus, it is possible, within this framework that excludes exogenous technical progress, for steady-state growth to be generated, in which case its rate will be dependent upon α, β, ν, and A. Admittedly, the case with $\alpha + \eta = 1$ exactly might be regarded as rather unlikely to prevail. That issue will be taken up below.

Now let us consider the second of the two basic endogenous growth mechanisms, this one involving the accumulation of human capital—in the sense of labor-force skills that can be enhanced by the application of valuable resources. One simple way to represent this phenomenon is to specify that physical output is accumulated according to

$$A k_t^{\alpha}(h_t n_t)^{1-\alpha} = c_t + (1+\nu)k_{t+1}, \quad (33)$$

where n_t is the fraction of the typical household's work time that is allocated to goods production and h_t is a measure of human capital—i.e., workplace skills—of a typical household member at time t.[24] These skills are produced by devoting the fraction $1 - n_t$ of working time to human capital accumulation. In general, physical capital would also be an important input to this process, but for simplicity let us initially assume that the accumulation of productive skills obeys the law of motion

$$h_{t+1} - h_t = B(1-n_t)h_t - \delta_h h_t, \quad (34)$$

[22] And also a transversality condition.

[23] In this case the transversality condition requires $\lim(1+\nu)k_{t+1}\beta^{t-1}/c_t = 0$. With k_{t+1} given by (32) and $c_t = (1-\alpha\beta)Ak_t^{\alpha+\eta}$, the relevant expression is $\beta^{t-1}\alpha\beta/(1-\alpha\beta)$, which does indeed approach zero as $t \to \infty$.

[24] In (33), the h_t human capital measure enters in the same way that the labor-augmenting technical progress term does in the neoclassical setup. So it can be seen that constant growth will occur if the overall model is such that h_t grows (endogenously) at a constant rate.

where the final term reflects depreciation of skills that occurs as time passes. In this expression, and for the rest of this example, we let $\nu = 0$.

Maximization of (1) subject to constraints (33) and (34) gives rise to the following first-order conditions:

$$c_t^{-1} = \beta c_{t+1}^{-1} \alpha A k_{t+1}^{\alpha-1} (h_{t+1} n_{t+1})^{1-\alpha} \tag{35a}$$

$$c_t^{-1} A k_t^{\alpha} h_t^{1-\alpha} (1-\alpha) n_t^{-\alpha} = \mu_t B h_t \tag{35b}$$

$$\mu_t = \beta \mu_{t+1} [B(1 - n_{t+1}) + 1 - \delta_h] + \beta c_{t+1}^{-1} A k_{t+1}^{\alpha} n_{t+1}^{1-\alpha} (1-\alpha) h_{t+1}^{-\alpha}. \tag{35c}$$

Here μ_t is the shadow price of human capital, i.e., the Lagrange multiplier attached to (34). With $g_t = v_t = 0$, the CE is given by the five equations (33), (34), and (35a)–(35c),[25] which determine time paths for c_t, k_t, h_t, n_t, and μ_t. Since (33) and (34) are the same from the private and social perspectives, there is no departure from social optimality implied by the CE.[26]

Now consider the possibility of steady-state growth in this system. Since n_t is limited to the interval [0,1], it must be constant in any steady state. If its value is n, then (34) shows that h_t will grow at the steady rate $B(1 - n) - \delta_h$, which we now denote as ξ. Then (35a) implies, since c_{t+1}/c_t must be constant, that k_t must also grow at the rate ξ—and by (33) the same must be true for c_t. Finally, (35b) shows that $1/\mu_t$ must grow like c_t—and these conclusions are consistent with (35c) having each term grow at the same rate. To find out what this growth rate will be, we can equate μ_t from (35b) and (35c), using $\mu_{t+1} = \mu_t/(1 + \xi)$ in the latter, and after some tedious simplification find that

$$\rho(1 + \xi) = Bn. \tag{36}$$

Since also $\xi = B(1 - n) - \delta_h$, we can solve for

$$n = \frac{\rho(1 + B - \delta_h)}{(1 + \rho)B} \tag{37}$$

in terms of basic parameters of the problem. Then ξ is found easily from expression (36).

An important property of (37) to be noted is that the steady-state value of n increases with ρ. Thus ξ, the growth rate, decreases with ρ, the rate of time preference. In other words, the more impatience is exhibited by the economy's individuals, the lower will be the steady-state growth rate. This is precisely the sort of result that some analysts have found highly plausible but is not generated by the neoclassical model. If $\nu \neq 0$ is assumed, moreover, the growth rate is negatively related to ν.

[25] Plus a pair of transversality conditions. The present model, it should be said, is the first of two in Lucas (1988), but here it is given without an externality, and it is very similar to one developed much earlier, by Uzawa (1965).

[26] This last statement presumes the absence of government spending and distortionary taxes.

An obvious objection to the model based on (33) and (34) is that production of h_t should be specified as dependent on the use of capital—i.e., physical goods—in that process. That extension has been studied by Rebelo (1991), who uses the following in place of (34):

$$h_{t+1} - h_t = B(m_t k_t)^a [h_t(1 - n_t)]^{1-a} - \delta_h h_t. \qquad (38)$$

Here m_t denotes the fraction of the capital stock that is devoted to production of human capital, so $(1 - m_t)k_t$ replaces k_t in (33) in this model.[27] Rebelo finds that the same conclusions involving steady growth and its dependence upon ρ hold with this extension. Furthermore, if production of physical output is taxed, say at the rate τ, then the steady-state growth rate will be negatively related to τ.

Of the two mechanisms considered, knowledge externalities and human capital, it is not obvious which is the more plausible as a source of major quantitative departure from the neoclassical model. But there is no reason to consider them on an either-or basis; both could be relevant simultaneously. Indeed, the Lucas (1988) model, of which our (33) and (34) are a special case, posits human capital accumulation as in (34) together with a production function for physical output in which there is an externality involving average economy-wide human—rather than physical—capital.

In what follows it will be useful to have at hand the full dynamic, period-by-period solution for a representative endogenous growth model. It is possible to derive such a solution for the Lucas model, even with the human capital production externality included, provided that we use the LCD version, which in this case requires that human capital fully depreciates in one period. Accordingly, let us now modify the model of equations (33), (34), and (35) by using $Ak_t^{\alpha}(h_t n_t)^{1-\alpha}\bar{h}_t^{\eta}$ as the production function and setting $\delta_h = 1$. Also, we shall permit population growth again, which implies that h_{t+1} in (34) and μ_t in (35c) are multiplied by $(1 + \nu)$. Then the household's optimality conditions, other than the transversality conditions, can be written as follows:

$$Ak_t^{\alpha}(h_t n_t)^{1-\alpha}\bar{h}_t^{\eta} = c_t + (1 + \nu)k_{t+1} \qquad (39a)$$

$$(1 + \nu)h_{t+1} = B(1 - n_t)h_t \qquad (39b)$$

$$(1 + \nu)c_{t+1} = c_t \beta\alpha A k_{t+1}^{\alpha-1}(h_{t+1}n_{t+1})^{1-\alpha}\bar{h}_{t+1}^{\eta} \qquad (39c)$$

$$c_t^{-1}Ak_t^{\alpha}h_t^{1-\alpha}(1 - \alpha)n_t^{-\alpha}\bar{h}_t^{\eta} = \mu_t B h_t \qquad (39d)$$

$$(1 + \nu)\mu_t = \beta\mu_{t+1}[B - (1 - n_t)] + \beta c_{t+1}^{-1}Ak_{t+1}^{\alpha}n_{t+1}^{1-\alpha}(1 - \alpha)h_{t+1}^{-\alpha}\bar{h}_{t+1}^{\eta}. \qquad (39e)$$

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[27] Rebelo also includes variable leisure in his setup.

In competitive equilibrium we will also have $h_t = \bar{h}_t$, so in what follows we assume that condition to hold. To solve these equations for $c_t, k_{t+1}, h_{t+1}, n_t$, and μ_t, we proceed by guessing—in analogy with the method of Section 2—that those five variables are determined in response to the state variables k_t and h_t by expressions of the form

$$c_t = \phi_{10} k_t^{\phi_{11}} h_t^{\phi_{12}} \tag{40a}$$

$$k_{t+1} = \phi_{20} k_t^{\phi_{21}} h_t^{\phi_{22}} \tag{40b}$$

$$h_{t+1} = \phi_{30} k_t^{\phi_{31}} h_t^{\phi_{32}} \tag{40c}$$

$$n_t = \phi_{40} k_t^{\phi_{41}} h_t^{\phi_{42}} \tag{40d}$$

$$\mu_t = \phi_{50} k_t^{\phi_{51}} h_t^{\phi_{52}}. \tag{40e}$$

If we can determine the implied values of the ϕ's, we will have substantiated this guess.

We begin by substituting (40c) and (40d) into (39b), obtaining

$$(1+\nu)\phi_{30} k_t^{\phi_{31}} h_t^{\phi_{32}} = Bh_t - B\phi_{40} k_t^{\phi_{41}} h_t^{\phi_{42}} h_t. \tag{41}$$

But then for (40) to be valid for all values of k_t and h_t, it must be that $\phi_{31} = \phi_{41} = \phi_{42} = 0$ and $\phi_{32} = 1$. Continuing in this manner of reasoning,[28] we end up with various sensible-looking results such as that h_t grows steadily at the rate $[B\beta/(1+\nu)] - 1$, the fraction of physical output saved is $\alpha\beta$, and especially that k_t evolves as

$$k_{t+1} = \frac{\alpha B(1-\beta)^{1-\alpha}}{(1+\nu)} A k_t^{\alpha} h_t^{1-\alpha+\eta}. \tag{42}$$

We shall make use of this last solution expression in Section 8.

An interesting and influential variant results when we again suppress the externality, by setting $\eta = 0$, but assume that human capital is produced by a production function of type (38) but with $a = \alpha$, i.e., with the same parameters as pertain to production of consumption (and physical capital) output. With log utility and Cobb-Douglas production functions, m_t and n_t will be constant over time; and with the production functions the same as well, the relative price of a unit of human capital in terms of output will be 1.0. Thus the sum of the two outputs is of the form (const.) $k_t^{\alpha} h_t^{1-\alpha}$ = (const.) $k_t(h_t/k_t)^{1-\alpha}$. But in this

[28] Specifically, we find that $n_t = \phi_{40} \equiv n$ and $h_{t+1} = \phi_{30} h_t$, with $\phi_{30} = B(1-n)/(1+\nu)$ and n yet to be determined. Next, we substitute into (39a) and in the same way find that $\phi_{11} = \phi_{21} = \alpha$ and $\phi_{12} = \phi_{22} = 1-\alpha+\eta$. Also, substitution of (40e) and $c_t = \phi_{10} k_t^{\alpha} h_t^{1-\alpha+\eta}$ into (39d) yields $\phi_{51} = 0$ and $\phi_{52} = -1$. Finding the ϕ_{j0} values is a bit more difficult. But the equations that result when the k_t and h_t terms are canceled out of (39) with (40) inserted imply that $n = \phi_{40} = 1-\beta$ and that $\phi_{10} = A(1-\alpha\beta)(1-\beta)^{1-\alpha}$, $\phi_{20} = A\alpha\beta(1-\beta)^{1-\alpha}/(1+\nu)$, $\phi_{30} = B\beta/(1+\nu)$, and $\phi_{50} = (1-\alpha)/B(1-\alpha\beta)(1-\beta)$.

special case it is also true that k_t/h_t is constant, so the foregoing expression reduces to a constant times k_t, often written as $y_t = Ak_t$. Hence, this is one case of the so-called "*AK*" model, which from a growth perspective is similar to an extreme special case of the neoclassical model—one in which the capital elasticity parameter α equals one. In this case, k_t and therefore output per person grows without limit at a constant rate, even with no technological progress, as inspection of equation (11) shows clearly. Furthermore, even if a and α differ so that k_t/h_t varies from period to period, the model works as indicated from a steady-state growth perspective. Consequently, the *AK* model—which may also be rationalized in other ways—has played a prominent role in the discussion of endogenous growth possibilities. We shall refer to it again shortly.

7. ISSUES CONCERNING ENDOGENOUS GROWTH ANALYSIS

Do models of the type outlined in Section 6 make more sense than the neoclassical construct that they were designed to replace? Clearly they have the virtue of at least attempting to explain growth endogenously, but are these attempts logically satisfactory and empirically plausible? In this writer's view, there are some highly attractive features of the models discussed above, including the possibility of knowledge externalities and the recognition that progress in terms of workforce skills relies in large part upon the allocation of resources to the production of such skills. But there are apparently two logical difficulties with these models that need to be considered before conclusions can be drawn.

The first of these difficulties is that in the Lucas or Lucas-Rebelo model, never-ending growth requires never-ending increases in human capital h_t, our measure of the productive skills of a typical worker. But for such a variable, never-ending growth is implausible because the skills in question are ones possessed by individual human beings and so are not automatically passed on to workers in succeeding generations. The son of a skilled craftsman is not born with dexterity and judgment but must start over again in developing them—again expending resources to do so—and has only a finite lifetime in which to do so. In this regard human capital is different from the stock of *knowledge,* which is possessed by society in general and is passed on from generation to generation, in the sense that it is available to those who wish to draw upon it.

Thus it is some form of knowledge, not human capital, that can plausibly provide the basis for never-ending growth.[29] But the development of knowledge also requires the expenditure of resources, so the question that arises is why rational private agents would devote resources to its development when the product will be possessed by society in general, rather than by themselves. A

[29] This point is stated clearly by Grossman and Helpman (1994, p. 35).

response is suggested, however, by consideration of our existing patent system. Formally, the answer in the literature is that the development of "designs" or "blueprints" that are privately profitable can have the by-product of adding to the stock of accessible productive knowledge. This type of process can continue without limit, so it can serve as the basis for never-ending growth. Several models expressing this notion have been developed;[30] let us briefly consider the most prominent of them, due to Romer (1990).

In Romer's (1990) setup the production function for (consumable) output can be written as

$$y_t = n_t^{1-\alpha} \sum_{i=1} x_{it}^{\alpha}, \tag{43}$$

where n_t is again the fraction of labor time devoted to the production of consumables and where x_{it} is the quantity of an intermediate good of type i used in period t.[31,32] The summation index i ranges from 1 to ∞ but in any period x_{it} will be zero for $i > A_t$, where A_t indicates the number of distinct intermediate goods in use. The technology for producing each intermediate good requires that ζ units of "capital" k_t must be used in the production of a unit of x_i, where capital is simply consumable output that is not consumed. Thus, if because of symmetry there is a common value $\bar{x}_t$ of x_{it} for those intermediate goods that are produced in t, we have

$$k_t = \zeta \bar{x}_t A_t. \tag{44}$$

But also $\Sigma x_{it}^{\alpha} = A_t \bar{x}_t^{\alpha}$, so substitution into (43) yields

$$y_t = n_t^{1-\alpha} A_t (k_t/\zeta A_t)^{\alpha} = (1/\zeta) n_t^{1-\alpha} k_t^{\alpha} A_t^{1-\alpha}. \tag{45}$$

From the latter it is clear that steady growth of consumable output will be possible if A_t grows exponentially without bound.

In addition to requiring ζ units of capital per unit produced, each intermediate good requires the use of one design. Designs, like output and intermediate goods, require resources in their production. Let $1 - n_t$ be the fraction of labor time devoted to the production of designs, an activity that will be called "research." Romer (1990) assumes that the research process is such that the creation of new designs by an individual is proportional to $(1 - n_t)A_t$, where A_t is the total number of intermediate good designs, not the per-person value. Thus

[30] Authors include Aghion and Howitt (1992), Grossman and Helpman (1991), King and Levine (1993), and Goodfriend and McDermott (1995).

[31] Romer's (1990) presentation pretends that there is a single price-taking producer of final output of consumables. Consequently, we shall not at this point distinguish between per-person and aggregative magnitudes.

[32] Actually, Romer (1990) has all labor allocated to the production of consumables and also includes a human capital measure, with some human capital used in the production of designs. Our specification is notationally simpler and basically equivalent.

Romer assumes that designs are non-rival in the research process, the activities of each researcher being enhanced by the entire stock of design knowledge accumulated to date. The evolution over time of A_t will therefore conform to

$$A_{t+1} - A_t = \sigma N(1 - n_t)A_t, \tag{46}$$

where σ is the constant of proportionality and N is the number of researchers each devoting $(1 - n_t)$ units of labor to research activity in t. Here the crucial allocation problem is the determination of $1 - n_t$, the fraction of time devoted to research instead of consumable output. In Romer's setup, this allocation depends upon the derived demand for research, which itself stems from the usefulness of intermediate goods in the production of output and the necessity of designs for these goods. Thus the evolution of A_t is determined by the optimizing choices of private individual agents (as well as technology). But in a steady state, which is shown to exist by Romer's careful analysis, n_t will be constant over time and A_t will grow at a constant rate, as indicated by (46).[33] Thus never-ending growth is generated in this model via endogenously rationalized, never-ending accumulation of knowledge.[34]

The second logical difficulty of the endogenous growth approach is the assumption of precisely constant returns to scale in the crucial production process. In the Lucas-Rebelo model, for instance, the sum of the exponents on physical and human capital in (33) and (38) must equal 1.00 exactly for steady-state growth to be implied; if this sum equals 0.99 instead, then the economy will approach a steady state in which there is no growth in the per capita quantities. Similarly, in the externality model $\alpha + \eta$ must equal 1.00 exactly in (28′) for steady growth to occur—this can be seen clearly in (32). And in the Romer (1990) model, the exponent on A_t on the right-hand side of (46) must be exactly 1.00. Consequently, the dramatically different properties of these models, as compared with the neoclassical construct, require very special parameter values that obtain only on measure-zero subsets of the relevant parameter spaces.[35] That must be regarded as implying that the endogenous growth approach does not actually generate steady, everlasting growth in the absence of exogenous technical progress.[36]

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[33] Romer (1990) emphasizes the monopoly power possessed by each design creator, market power that gives the individual an incentive to devote resources to the research activity (in our exposition, by hiring labor). It should be noted that this is the rather benign type of monopoly power that is granted by patent systems.

[34] Rivera-Batiz and Romer (1991) argue that an important application of this analysis is in the international context, where it implies that growth is fostered by economic integration and trade liberalization (for both goods and ideas).

[35] This conclusion does not seem to be inconsistent with the discussion of Romer (1994, pp. 17–18), despite the difference in tone.

[36] Some analysts argue that all production functions must, as a matter of logic, have input coefficients summing to precisely 1. But even if one ignores the presence of land, which is probably of some importance, this argument misses the issue, which is whether the coefficients

Nevertheless, the endogenous growth approach has been highly productive, because with returns to scale reasonably *close* to 1.00 in (33) and (34), the model will have very slow transition dynamics. The speed of convergence of k_t to k_t^*, given in (12) as $1 - \alpha$ for the LCD neoclassical model, will be more nearly equal to $1 - (\alpha_1 + \alpha_2)$, where $\alpha_1 + \alpha_2$ is the sum of capital and human capital coefficients. Therefore, if the human capital coefficient arises as in (33), via the effect of skill-adjusted labor, one would expect $1 - (\alpha_1 + \alpha_2)$ to be close to zero and convergence to be very slow. But with very slow transition dynamics, growth rate differences due to transitional movements toward the CE steady state will be prolonged—so observed growth rate differences might be sustained over very long time periods. So even if the approach of the endogenous-growth proponents fails to explain never-ending steady-state growth, it could plausibly explain many features of the empirical data and potentially provide the basis for useful policy analysis.

In this regard, it is notable that equation (11) indicates that there is no discontinuity involving the distinction between neoclassical models with α close to 1.0 and endogenous-growth *AK* models with $\alpha = 1.0$ exactly. Specifically, if at some point in time the efficiency parameter *A* were changed by the amount Δ (say), then the effect on $\log k_t$ after *T* periods will be $T \log \Delta$ according to the *AK* model or $\log \Delta(1-\alpha^T)/(1-\alpha)$ according to the neoclassical model. For α values close to 1.0, these magnitudes are similar (and are equal in the limit as $\alpha \rightarrow 1.0$). With $\alpha = 0.98$ for example, we have $(1 - 0.98^5)/(1 - 0.98) = 4.8$ when $T = 5$ and $(1 - 0.98^{20})/(1 - 0.98) = 16.6$ when $T = 20$. So the response of capital and output to changes in *A*—or another variable that affects the steady-state value of k_t—will be reasonably similar whether the counterpart of α is 0.98 or 1.00. Since time periods in our formulation are about 25 to 30 years in duration, the similarity holds for substantial spans of time.

Of course, strictly speaking, this sort of weakened version of the approach does not result in the conversion of "level effects" into "rate-of-growth effects" that some writers take to be the hallmark of endogenous growth analysis. But the difference is not too great, quantitatively. Furthermore, while that conclusion implies a less dramatic difference between neoclassical and endogenous growth models, it also rescues the latter from evidence suggesting apparent empirical rejections. For example, Jones (1995) points out that the U.S. growth rate has not risen over the last century despite increases in some variables (e.g., investment share, R&D share) that would bring about rate-of-growth effects in the standard endogenous growth models with 1.00 values for the relevant parameters.

on k_t and h_t in (33) sum to 1, not the coefficients on k_t and n_t. In other words, with h_t generated by (34) or (38), the issue is whether effective labor is $h_t n_t$ or n_t multiplied by some nonlinear function of h_t.

It has been argued by Mankiw, Romer, and Weil (1992) and Mankiw (1995) that, although it has some weaknesses, the neoclassical model's empirical performance is much better than is suggested by the discussion of its endogenous growth critics or that of Section 5 above.[37] In particular, the neoclassical model is fairly successful in explaining cross-country differences in income *levels* and is even more successful when the role of human capital is taken into account.[38] To understand this point, let us return to the LCD version of the neoclassical model with technical progress. From the definition of k_t and the relation $\hat{k}_t^* = [\alpha\beta A/(1+\nu)(1+\gamma)]^{1/(1-\alpha)}$, we see that the steady-state value of k_t at time t will be

$$k_t = k_0(1+\gamma)^t[\alpha\beta A/(1+\nu)(1+\gamma)]^{1/(1-\alpha)}. \tag{47}$$

Thus for steady-state $\log y_t$ we have, using $\log(1+\gamma) \doteq \gamma$,

$$\log y_t = const + \gamma t + [\alpha/(1-\alpha)]\log[\alpha\beta A/(1+\nu)(1+\gamma)]. \tag{48}$$

For any given value of t, accordingly, $\log y_t$ will be larger the larger is β and the smaller are ν and γ. In the special model at hand, it happens that $\alpha\beta$ equals the fraction of income s that is saved each period. Thus it accords with the Mankiw, Romer, Weil estimation of an equation analogous to (48) on various cross-section samples of national economies with data averaged over the period 1960 to 1985.[39] They assume the same γ for all nations and are therefore able to incorporate γt into the constant term.[40] They obtain estimates with $\log s$ and $\log[(1+\nu)(1+\gamma)]$ entered separately and test the hypothesis that the slope coefficients are equal in magnitude and opposite in sign. The striking result of this exercise is that for their sample of 98 non-oil nations, the variables $\log s$ and $\log(1+\nu)(1+\gamma)$ have a considerable amount of explanatory power for $\log y_t$, the adjusted R^2 value being 0.59. Furthermore, the slope coefficient hypothesis mentioned above cannot be rejected at conventional significance levels. The one serious flaw acknowledged by Mankiw, Romer, and Weil is that the implied value of α is about 0.6, much larger than the one-third value that is usually presumed (and that matches the capital share of income). But this failure can be largely overcome, they demonstrate, by including additional variables designed to proxy for the level of human capital or labor-force skill in

[37] Actually, these authors are concerned with the Solow model, i.e., the special case of the neoclassical model in which the saving rate is given exogenously. But that difference is unimportant for the issues at hand.

[38] Account is taken in a different way than in our discussion surrounding equation (34), however, since human capital enters the production function as an additional input rather than as an efficiency term attached to labor input.

[39] Of course they do not use the $\delta = 1$ assumption that permits us to derive the $s = \alpha\beta$ result.

[40] They treat cross-country differences in $\log A$ as a component of the regression's disturbance term. This is not innocuous, as will be seen momentarily.

the various countries. Thus they conclude that "the Solow model is consistent with the international evidence if one acknowledges the importance of human as well as physical capital" (1992, p. 433).[41]

This argument is ingenious and the finding is interesting, but the suggestion that it serves to rescue the neoclassical model from its critics seems inappropriate.[42] For the model was designed to provide understanding about *growth,* not about international differences in income levels. In support of this last assertion, it may be noted that there is no mention of using the model for the latter purpose in Solow (1956, 1970, 1994), or Meade (1962), or Hahn (1987). That use seems to have been discovered by Mankiw, Romer, and Weil (1992).[43]

In this regard, another objection to the Mankiw, Romer, and Weil analysis has been expressed by Grossman and Helpman (1994). As mentioned above, that analysis assumes that γ, the rate of technological progress, is the same for all countries in their cross-section regressions—which thereby pushes γt into the constant term in expressions like (48). But, as Grossman and Helpman (1994, p. 29) say, "if technological progress [actually] varies by country, and [variation in γt] is treated as part of the unobserved error term, then ordinary least squares estimates of the . . . equation will be biased when investment-GDP ratios are correlated with country-specific productivity growth. In particular, if investment rates are high where productivity grows fast, the coefficient on the investment [or saving] variable will pick up . . . part of the variation due to their different experiences with technological progress. . . . [Furthermore,] an economist would certainly expect investment to be highest where capital productivity is growing the fastest." Thus the Mankiw, Romer, and Weil (1992) estimate of the effect of the saving/investment variable is overstated and the slope-coefficient test is consequently biased. Whether this bias is large quantitatively has not yet been established, but in any event it pertains only to the neoclassical model's role of explaining cross-section income levels, which seems rather incidental.[44]

[41] It should be said, however, that the Mankiw, Romer, and Weil (1992) measure of human capital leaves much to be desired. In particular, they use an estimate of the fraction of the working age population that is currently enrolled in secondary school. A measure of the fraction of the current working age population that attended (in the past) secondary school would be much more appropriate.

[42] Mankiw (1995, p. 423) says that from the standpoint of enhancing understanding, "the neoclassical model is still the most useful theory of growth that we have."

[43] A word of explanation is needed here since Denison (1967) and others have in fact looked at cross-country differences in income levels. The point is that they have done so in a way that relies on transitional differences, whereas the Mankiw, Romer, and Weil equation (48) pertains to steady-state differences.

[44] Very recently, Islam (1995) has implemented a panel-data approach to conditional convergence regressions, thereby permitting some production function parameters to differ across countries. His approach retained the assumption that γ is the same for all countries, however, so it does not address the specific criticism stressed by Grossman and Helpman.

8. CONDITIONAL CONVERGENCE

Because conditional convergence has figured prominently in the literature's controversies, it should be useful to describe—as promised above—this concept. From equation (23) we have $\log \hat{k}_{t+1} = \alpha \log \hat{k}_t + (1-\alpha)\log \hat{k}^*$, which, since $\log \hat{y}_t = \log A + \alpha \log \hat{k}_t$, implies

$$\log \hat{y}_{t+1} = \alpha \log \hat{y}_t + (1-\alpha)\log \hat{y}^*. \tag{49}$$

Iteration then shows that

$$\log \hat{y}_{t+j} = \alpha^j \log \hat{y}_t + (1-\alpha^j)\log \hat{y}^*, \tag{50}$$

so that we have

$$\log y_{t+j} - \log y_t = (1-\alpha^j)[\log \hat{y}^* - \log y_t] + j\log(1+\gamma). \tag{51}$$

But $\log \hat{y}^* = \log A + \log[\alpha\beta A/(1+\nu)(1+\gamma)]$. Thus in a cross section of economies one needs to take account of potential cross-economy differences in taste, technology, and population-growth parameters even if it is assumed that γ is the same everywhere. But with proxies for these included in a regression with $(1/j)(\log y_{t+j} - \log y_t)$ on the left and $\log y_t$ on the right-hand side, the coefficient on the latter is predicted by this special case of the neoclassical model to be $-(1-\alpha^j)/j$. Some simple endogenous growth models such as (29) and (30) with $\alpha + \eta = 1$ suggest, by contrast, that the coefficient on $\log y_t$ should be zero. So a significant negative coefficient would constitute evidence against these rudimentary specifications. Other two-sector versions such as (33)–(38) feature transitional dynamic adjustments, however, that are not ruled out by findings of conditional convergence. That has been established by Mulligan and Sala-i-Martin (1993) and will be demonstrated below for our version of the Lucas model. Thus the fact that most existing studies of the type under discussion do find significant negative coefficients does not discriminate between endogenous and neoclassical specifications.

It should be mentioned explicitly that the foregoing exposition makes use of the LCD assumptions, which lead to the conclusion that the coefficient on $\log y_t$ is a function only of the capital-elasticity parameter α. More generally, without those assumptions this coefficient will depend also on other parameters including ν, γ, and the rate of depreciation—at least in the standard approximation that is typically used in the literature (see Barro and Sala-i-Martin [1995], p. 81, or Mankiw [1995], p. 310).

To see that conditional convergence is a property of the LCD version of the Lucas model, as stated above, rewrite the solution equation (42) as follows:

$$\log k_{t+1} - \log k_t = (1-\alpha)[\log h_t - \log k_t] + \eta \log h_t + \log[\alpha\beta(1-\beta)^{1-\alpha}A/(1+\nu)]. \tag{52}$$

This shows clearly that conditional convergence holds for $\log k_t$, i.e., that $\log k_t$ will have a negative slope coefficient in a regression with $\log k_{t+1} - \log k_t$ as the dependent variable, when $\log h_t, \log s_t, \log(1+\nu)$, and $\log(1-\beta)$ are included as regressors. Furthermore, the same is true for $\log y_t$, as can be seen by using the production function to eliminate k_t and k_{t+1} in favor of y_t and y_{t+1}.

Quite recently, an interesting implication of the literature's empirical findings on conditional convergence was pointed out by Cho and Graham (1996). The basic finding is that in cross-section regressions with large samples of heterogeneous countries, estimates of the slope coefficient b_1 in relations of the form

$$\log y_{t+j} - \log y_t = b_0 + b_1[\log y_t^* - \log y_t] \tag{53}$$

are positive, when expressions for y_t^* are ones suggested by the neoclassical model. But one of the main reasons that the conditional convergence formulation was invented is that in such samples of countries one frequently obtains positive estimates of b_3 in regressions of the form

$$\log y_{t+j} - \log y_t = b_2 + b_3 \log y_t. \tag{54}$$

Equating the right-hand sides of (53) and (54), however, we see that

$$b_0 + b_1[\log y_t^* - \log y_t] = b_2 + b_3 \log y_t, \tag{55}$$

plus regression residuals. But with $b_1 > 0$ and $b_3 > 0$, (55) indicates that output is smaller in relation to its steady-state value (of the current period) for high-income countries than for low-income countries. In other words, if low-income countries have less capital than in the CE steady state, then they are relatively closer to their steady-state positions than are rich nations and, in that sense, have less economic development yet to be accomplished, i.e., negative catching up! Admittedly, estimates of b_3 are quite unrealiable and often insignificantly positive. But suppose, then, that we take b_3 to be essentially zero. Then (55) suggests that low-income countries are on average neither closer to (proportionately) nor farther from their steady-state positions than are rich countries.

9. CONCLUSION

Let us conclude with a brief summary of the arguments developed above. Our review of the neoclassical model emphasizes that it is in fact not a model of ongoing growth, since it implies that per capita output rates will approach constant values in the absence of exogenous (therefore unexplained) technological progress. Several analytical results are exposited, including the distinction between golden rule and optimal steady states. Following this review, it is argued that the neoclassical approach not only fails to provide an explanation of everlasting steady-state growth, but also cannot plausibly explain actual observed cross-country growth rate differences by reference to transitional (i.e., non-steady-state) episodes. It can, with the inclusion of human capital inputs, explain a substantial portion of observed cross-country differences in income levels, but there are some questionable aspects of this accomplishment and, in any event, explaining levels is not the main task of a theory of growth.

The endogenous growth literature attempts to provide explanations for ongoing, steady-state growth in per capita output values and consequently for growth rate differences across countries. Three types of endogenous growth models are presented, featuring (i) externalities resulting from linked capital-and-knowledge accumulation, (ii) accumulation of human capital (i.e., individuals' workplace skills), and (iii) continuing growth in the stock of existing productive "designs," with the entire stock facilitating the creation of additional designs (that are produced in response to private rewards). The last of these types seems most plausible as a mechanism capable of generating long-lasting growth. The likelihood of obtaining steady-state (never-ending but non-explosive) growth from any of the models seems very small, however, since such a result would require highly special (zero measure) parameter values. The endogenous growth approach seems fruitful, nevertheless, as it can in principle rationalize long-lasting growth and growth rate differences across economies and will indicate with reasonable accuracy the effects of changes in policy, tastes, or technology that alter the steady-state capital/labor ratio.

APPENDIX A

The purpose here is not to furnish rigorous mathematical proofs but instead to provide some intuition concerning the role and nature of transversality conditions in infinite horizon optimization problems. Let us proceed in the context of the problem of Section 1, to maximize (1) subject to constraint (3). We begin with a T-period *finite* horizon version for which the Lagrangian expression is

$$\begin{aligned} L_1 = {} & u(c_1) + \beta u(c_2) + \ldots + \beta^{T-1}u(c_T) + \lambda_1[f(k_1) - c_1 - (1+\nu)k_2 \\ & + (1-\delta)k_1] + \beta\lambda_2[f(k_2) - c_2 - (1+\nu)k_3 + (1-\delta)k_2] \\ & + \ldots + \beta^{T-1}\lambda_T[f(k_T) - c_T - (1+\nu)k_{T+1} + (1+\delta)k_T]. \end{aligned} \tag{A1}$$

For $t = 1, 2, \ldots, T$ we have the first-order conditions

$$u'(c_t) - \lambda_t = 0 \tag{A2}$$

$$-(1+\nu)\lambda_t + \beta\lambda_{t+1}[f'(k_{t+1}) + 1 - \delta] = 0. \tag{A3}$$

In addition there is the derivative with respect to k_{T+1}, $\partial L_1/\partial k_{T+1} = -\lambda_T \beta^{T-1}(1+\nu)$. If the problem were such that one could be assured of a positive solution for k_{T+1}, as is the case for $c_1, \ldots, c_T$ and $k_2, \ldots, k_T$, then one might be inclined to set this partial equal to zero. But of course the household would like for k_{T+1} to be negative and very large, since that would permit c_T to be very large. Thus the inherent constraint $k_{T+1} \geq 0$ becomes relevant and leads to the two-part Kuhn-Tucker condition

$$-\lambda_T \beta^{T-1}(1+\nu) \leq 0 \qquad - k_{T+1}[\lambda_T \beta^{T-1}(1+\nu)] = 0. \tag{A4}$$

Since $\lambda_1, \ldots, \lambda_T$ will by (A2) be strictly positive, the first of these is irrelevant, and the second implies that $k_{T+1} = 0$.

Now consider the infinite horizon version of the same problem by letting $T \rightarrow \infty$. Heuristically we again have conditions (A2) and (A3), relevant for all $t = 1, 2, \ldots$. And in place of (A4) we now have the *TC*

$$\lim_{T \rightarrow \infty} k_{T+1} \beta^{T-1} \lambda_T = 0. \tag{A5}$$

Here the interpretation is that the present value of k_{T+1} in marginal utility units must approach zero as T grows without bound. Since $\beta^{T-1} \rightarrow 0$, this does not require that $k_{T+1} \rightarrow 0$.

Note that it is fortunate that the *TC* is available, for without it (or some replacement) the two difference equations (A2) and (A3) could not provide a well-defined path in the infinite horizon case for there is only one relevant initial condition present (i.e., the given value of k_1). This, then, is the role of the *TC* condition, to provide an additional side condition for starting up the solution sequence $c_1, c_2, \ldots, k_2, k_3, \ldots$. It serves to prevent the optimizing agent from starting on paths that satisfy (A2) and (A3) but lead to negative values of k_t or to wastefully large accumulations of assets that are *never* turned into consumption.

For the infinite horizon problem at hand, it is the case that (subject to the Kuhn-Tucker "constraint qualification") conditions (A2) and (A3) for $t = 1, 2, \ldots$ and condition (A5) are necessary and jointly sufficient for optimality. There are a few exceptional setups with concave objective functions and convex constraint sets, and lots of differentiability, for which the *TC* is not necessary for optimality. But in most infinite horizon problems, the *TC* is also necessary—as is shown by Weitzman (1973).

APPENDIX B

Here we show that if the production function in per capita terms is

$$y = f(n, k, t) \tag{B1}$$

and is homogeneous of degree one (HD1) in n and k, then steady-state growth is possible only when the technical progress term involving t is labor augmenting. To begin, let us assume that labor supply is inelastic so that $n = 1$. Then HD1 implies that $\lambda f(1, k, t) = f(\lambda, \lambda k, t)$ for any $\lambda > 0$. So if we define $x = k/y$, then $1 = f(1/y, x, t)$, which permits us to define the function ϕ such that $y = \phi(x, t)$. Calculating the partial derivative with respect to k then gives

$$\partial y/\partial k = \phi_1(x, t)[-ky^{-2}\partial y/\partial k + y^{-1}], \tag{B2}$$

which can be rearranged to yield

$$\partial y/\partial k = \frac{\phi_1(x, t)}{\phi(x, t) + x\phi_1(x, t)}. \tag{B3}$$

Thus for $\partial y/\partial k$ to be independent of t, we must be able to write the right-hand side of (B3) as $c(x)$, say, implying that $\phi_1(x, t) = c(x)[\phi(x, t) + x\phi_1(x, t)]$ or

$$\frac{\phi_1(x, t)}{\phi(x, t)} = \frac{c(x)}{1 - xc(x)} \tag{B4}$$

so that $\phi(x, t)/\phi_1(x, t)$ is independent of t. But that implies that $\phi(x, t)$ can be written as

$$\phi(x, t) = A(t)\psi(x), \tag{B5}$$

say, so $y = A(t)\psi(x)$ and $x = \psi^{-1}[y/A(t)]$. Then $k = xy = y\psi^{-1}[y/A(t)]$ and

$$k/A(t) = [y/A(t)]\psi^{-1}[y/A(t)] \equiv G[y/A(t)]. \tag{B6}$$

Finally, inversion of G yields

$$y/A(t) = G^{-1}[k/A(t)] = g[k/A(t)]. \tag{B7}$$

Thus it must be that $y = f(1, k, t)$ is of the form $y = \tilde{f}(A(t), k)$.

This proof has been adapted from Uzawa (1961). The statement involving (B5) is treated by Uzawa as obvious. Uzawa's proof pertains, it should be noted, to a proposition that is more general than the one proved by Barro and Sala-i-Martin (1995, pp. 54–55) or Solow (1970, pp. 35–37).

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