Tensile Strength of Lightly Cemented Sand through Indentation Tests

Résistance à la traction de sable légèrement cimenté par des tests d'indentation

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ABSTRACT: Compared to the compressive or shear strength of soil, its tensile strength is generally assumed to be zero, or insignificant, in geotechnical engineering practice because of its relatively small value and lack of a satisfying laboratory technique. The tensile strength of soil is, however, an important parameter in the design of geosystems, where tensile cracks contribute to progressive erosion or landslides in excavation, slopes, dams, highway embankments, riverbanks, hydraulic barriers, and other earth structures. In this paper, a recent development of the indentation test is presented. The theoretical framework is re-evaluated followed by a series of test results on lightly cemented sand.

RÉSUMÉ : Par rapport à la résistance à la compression ou de cisaillement du sol, sa résistance à la traction est généralement considérée comme nulle ou insignifiante, dans la pratique de la géotechnique en raison de sa valeur relativement faible et le manque d'une technique de laboratoire satisfaisant. La résistance à la traction de sol est, cependant, un paramètre important dans la conception des géosystèmes, où des fissures de traction contribuent à l'érosion progressive de sangles glissements de terrain dans l'excavation, les pistes, les barrages routiers, des remblais, des rives, des barrières hydrauliques, et d'autres structures terrestres. Dans cet article, un développement récent de l'essai d'indentation est présenté. Le cadre théorique est réévalué suivie d'une série de résultats d'essais sur le sable légèrement cimenté.

KEYWORDS: tensile strength, indentation, limit analysis.

1 INTRODUCTION

Determination of tensile strength of soil can be categorized to direct and indirect methods. In the direct method, the soil specimen is directly pulled apart and assumed to split in the middle (e.g. Das and Dass 1995, Tang and Graham 2000, Nahlawi et al. 2004, Tamrakar et al. 2007, Zeh and Witt 2007). Its tensile strength is computed as the measured maximum pull-up force divided by the cross sectional area. However, it is normally quite challenging to have the soil specimen clamped or glued in the split mold. In the indirect method, tensile failure is induced by compressive load. A well-known Brazilian test is of such kind. The indentation test, also called unconfined penetration tests (Fang and Chen 1972, Fang and Fernandez 1981, and Kim et al. 2012) is an indirect method for tensile strength determination. It uses a pair of cylindrical metal indenters, or punches, to compress a cylindrical soil specimen as shown in Figure 1. The tensile strength is computed through the equation developed from the limit analysis (e.g. Chen 1975). The test gives an applied compressive axial load and indenter displacement curve. The maximum load is identified for further data reduction to compute the tensile strength. The limitation of this test method is that a certain level of brittleness of the specimen is required so that a split tension failure would occur.

2 BACKGROUND

As described in Chen (1975), an ideal failure mechanism developed in the soil specimen can be presented in Figure 1. By equating the external work with the internal work, an upper bound solution can be obtained as follows

\[
p' = \frac{1 - \sin \phi}{2 \sin \alpha \cos (\alpha + \phi)} \left( q_u + \tan (\alpha + \phi) \frac{bH}{a} - \cot \alpha \right) q_t, \tag{1}
\]

where

- \( p' \) is the upper bound load that causes the split tension failure,
- \( \alpha \) is the radius of the indenter,
- \( b \) is the radius of the soil specimen,
- \( \phi \) is the internal frictional angle of the soil,
- \( a \) is the developed angle underneath the indenter when failure occurs,
- \( q_u \) is the unconfined compressive strength while \( q_t \) is the tensile strength to be determined.

The upper bound solution has a minimum value when \( \frac{\partial p'}{\partial \alpha} = 0 \), where

\[
p' = \left[ \frac{bH}{a} \tan (2\alpha + \phi) - 1 \right] q_u, \tag{2}
\]

By equating (1) and (2), the equation below can be obtained,

\[
\cot \alpha = \tan \phi + \sec \phi \left[ 1 + \frac{bH \cos \phi}{q_t} \left( \frac{1 - \sin \phi}{2} - \sin \phi \right) \right]^{1/2}, \tag{3}
\]

Substituting (3) into (1) and let \( p \) is the maximum compressive load applied from the test, which is less than the upper bound load, \( p' \). Therefore,

\[
p \leq p' = \frac{\pi bH \tan (2\alpha + \phi) - a^2}{q_t}, \tag{4}
\]

\[
q_t = \frac{p}{\pi (bHK - a^2)}, \tag{5}
\]

where \( K = \tan (2\alpha + \phi) \).
The value of $K$ is challenging to determine in laboratory testing as it is a function of the internal frictional angle of the soil, the unconfined compressive-tensile strength ratio, as well as the size of the indenter. The recommended values of $K$ based upon a given assumed materials properties, such as the brittleness ratio, $q_\alpha/q_t = 10$, $\phi = 30^\circ$, and $b/H = 1$ were suggested for practical use (Fang and Fernandez, 1981). However, they may not be suitable for other material setting. In this study, we carried out unconfined compression tests and direct shear tests to acquire information on $q_\alpha$, $q_t$, and $\phi$. Afterwards, from our indentation test results, we can iteratively compute both values of the $K$ and the tensile strength $q_t$.

To assess the theoretical framework for indentation test described above, nine different test configurations were presented in this paper. The diameter of each specimen, $2b$, was kept as 100 mm. The height of each specimen, $H$, was chosen as 50, 100, and 150 mm respectively. The ratio of the indenter diameter to specimen diameter, $2a/2b$, was selected as 0.225, 0.281, and 0.338 respectively. Replication for one test configuration indicated that the results of the indentation tests are repeatable within 5% variation. Therefore, only the results for each test configuration were shown here.

The specimens were compacted silica sand-Kaolinite mixture. As we would like to have a lightly cemented sandy specimen, which is brittle enough for the indentation tests, the combination of 15% dry weight of Kaolinite plus 85% dry weight of sand were selected after several trials. The water was then added to reach moisture content of 8.1% followed by sealing the soil mixture in a bag resting for 24 hours. After compaction, the dry density was about 1.62 g/cm$^3$. The specimen was mounted in the testing apparatus shown in Figure 2. An indentation rate of 0.5 mm/min was applied to each test. The indenter penetration and the corresponding compressive load were recorded for data reduction. An unconfined compression test on a specimen with 100 mm in diameter and 200 mm in height was conducted as well as direct shear tests under normal stress of 40, 80, and 160 kPa.

### Table 1. Summary of test results

<table>
<thead>
<tr>
<th>$H$ (mm)</th>
<th>$2a/2b$</th>
<th>$\alpha$ (°)</th>
<th>$p$ (N)</th>
<th>$K$ (N/m$^2$)</th>
<th>$q_t$ (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.225</td>
<td>15.9</td>
<td>15.7</td>
<td>1.93</td>
<td>1061.1</td>
</tr>
<tr>
<td>50</td>
<td>0.281</td>
<td>18.9</td>
<td>18.6</td>
<td>2.55</td>
<td>955.3</td>
</tr>
<tr>
<td>50</td>
<td>0.338</td>
<td>21.4</td>
<td>22.1</td>
<td>3.42</td>
<td>849.1</td>
</tr>
<tr>
<td>100</td>
<td>0.225</td>
<td>12.6</td>
<td>23.5</td>
<td>1.49</td>
<td>1020.2</td>
</tr>
<tr>
<td>100</td>
<td>0.281</td>
<td>14.9</td>
<td>27.7</td>
<td>1.77</td>
<td>1013.2</td>
</tr>
<tr>
<td>100</td>
<td>0.338</td>
<td>18.1</td>
<td>29.7</td>
<td>2.36</td>
<td>819.9</td>
</tr>
<tr>
<td>150</td>
<td>0.225</td>
<td>11.1</td>
<td>29.5</td>
<td>1.33</td>
<td>951.8</td>
</tr>
<tr>
<td>150</td>
<td>0.281</td>
<td>13.1</td>
<td>34.5</td>
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<td>961.0</td>
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<td>150</td>
<td>0.338</td>
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</tr>
</tbody>
</table>

All nine curves show softening after peak load, indicating split tension failure mode occurred. Before computing the tensile strength, unconfined compression tests were conducted with a specimen height-to-diameter ratio 2 followed the ASTM standards. The $q_t$ was found to be around 16336.5 N/m$^2$. A internal frictional angle $\phi = 30.8^\circ$ was determined through a series of direct shear tests.

With the information of peak axial force $p$, $q_u$, and $\phi$, tensile strength, $q_t$, and the corresponding $K$ value can be determined through the iterations using equations 3 and 5. Table 1 shows the iterated $q_t$ and $K$ associated with each test configuration.

From the test results show in Table 1, it is recognized that the tensile strength of the lightly cemented sand is specimen size dependent. For a given specimen height, the higher the indenter-to-specimen diameter ratio, $a/b$, the smaller the tensile strength is. The developed cone angle $\alpha$ is also found specimen and indenter size dependent. For a given specimen height, the $\alpha$ angle is in proportional to indenter size. For a given indenter size, higher specimen had smaller developed angle $\alpha$. The coefficient $K$ is also found dependent on test configuration. The trend for $K$ to specimen height and indenter size is similar to the trend for the $\alpha$ angle. The computed $K$ values fall between 1.5 and 3.5 depending on test configuration as displayed in Figure 6. This is not consistent with the recommended values, where 1.0 to 1.2 for soil was presented in Fang and Chen (1972). The major difference between Fang and Chen’s work and the current study lies in the brittleness ratio, $q_u/q_t$. In Fang and Cheng’s pioneer work, the brittleness ratio was assumed to be 10 for compacted soil. In the current study, the brittleness ratio was determined to be around 16 to 20, as shown in Figure 7. Looking back on equation 5, the tensile strength is dependent on $K$ and $q_u$, and the tensile strength is a function of the internal frictional angle $\phi$. Afterwards, $K$ was found to be around 16336.5 N/m$^2$. A internal frictional angle $\phi = 30.8^\circ$ was determined through a series of direct shear tests.

Also, the angle $\alpha$ is a function of brittleness ratio as shown in equation 3. As a result, using an iterated value for $K$ seems more attractive as the brittleness for soil could have a wide range.
3

CONCLUSION

An indirect method for determining tensile strength of lightly cemented sand was re-visited. The method is based upon an upper bound solution to a split tension failure in limit analysis. In order to assess the analytical solution, three different sizes of indenters were used along with three different specimen heights. As the coefficient $K$ is a function of the developed $\alpha$ angle and the brittleness ratio, evaluating $K$ through a numerical iteration is recommended.

ACKNOWLEDGEMENTS

The financial support from the National Science Council of Taiwan under the contract of 101-2218-E-002-005- is greatly appreciated.

REFERENCES


