Loss aversion and the term structure of interest rates

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This article studies how the loss averse behaviour affects the term structure of real interest rates. Since the pro-cyclical conditional expected marginal rate of substitution, implied from the US consumption data, is consistent with the proposition of loss aversion, we incorporate the loss averse behaviour of prospect theory into the consumption-based asset pricing model. Motivated by the similarity between habit formation and the prospect theory utility, habit formation is exploited to determine endogenously the reference point of this behavioural finance utility. The highly curved characteristic of the term structure of real interest rates can thus be captured by the additional consideration of loss aversion. This model also fits the downward sloping volatility of the real yield curve in the data of US Treasury Inflation-Protection Securities (TIPS). Moreover, depending on the effective risk attitude of the representative agent with the loss averse behaviour of prospect theory, our model is capable of generating a normal or an inverted yield curve.

I. Introduction

Due to the inability of the expected utility framework to explain the behaviour of asset returns, introducing findings in behavioural finance has been recognized as a possible alternative to improve the performance of asset pricing models.1 One of the most famous findings in behavioural finance is prospect theory, proposed first by Kahneman and Tversky (1979).2 Some psychological experiments have been conducted to determine how people make decisions when facing different types of gambles, the results of which show that the major factor affecting people’s decisions is not their wealth level after the gamble but the amount of gains or losses from the gamble. They also discovered that people are more sensitive about the losses than the gains and are more willing to take risks to avoid losses. This phenomenon is often termed loss aversion.

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1 Other alternatives include: (1) nonexpected utilities in Weil (1989) and Epstein and Zin (1990); (2) habit formation in Abel (1999), Constantinides (1990) and Campbell and Cochrane (1999); (3) some types of market incompleteness, such as Rietz (1988), the asymmetric underlying process in Hung (1994), the transaction cost in Aiyagari and Gertler (1991) and Heaton and Lucas (1996), heterogeneous agents in Mankiw (1986), Mankiw and Zeldes (1991), Weil (1992), Lucas (1994), Constantinides and Duffie (1996), etc.

2 Tversky and Kahneman (1992) further extended the original prospect theory to the cumulative prospect theory to solve a variety of experiment evidence inconsistent with standard expected utility theory. Afterwards many studies tried to analyse the characteristic of this theory, including Schmidt (2003), Law and Peel (2007), Schmidt and Zank (2008), Cain et al. (2008), etc.
Various asset pricing models combining the feature of loss aversion have been applied to many financial and economic studies. However, as pointed out in Campbell (2000), there are some unsettled issues when loss aversion is incorporated into asset pricing models. For instance, one issue is the argument of the objective function, and another is the determination and updating of the reference point. For the argument of the objective function, since individuals derive their utilities from the consumption rather than the increase of wealth, we use the consumption-based prospect theory utility instead of the wealth-based prospect theory utility, the latter of which is commonly adopted in the previous studies. In addition, it is generally believed that what people really care about is not the absolute value of their consumption level but the changes in their level of consumption. Namely, people exhibit the phenomenon of habit formation. 

Motivated by the similarity between habit formation and the prospect theory utility, habit formation is employed to determine and update endogenously the reference point in our model. Finally, the exponential utility-based prospect theory is adopted in our model to avoid the undesired characteristic that the first-order derivative of the power utility approaches infinity at the reference point.

The importance of combining loss aversion with the consumption-based asset pricing model is illustrated as follows. Mehra and Prescott (1985) demonstrate that the traditional consumption-based asset pricing model with a proper degree of risk aversion cannot generate a large enough equity premium. In their research, a two-state Markov process is proposed for the consumption growth rate, \( g_{t+1}^c = c_{t+1}/c_t \), where \( c_t \) and \( g_{t+1}^c \) denote the consumption level at \( t \) and the consumption growth rate from \( t \) to \( t+1 \) respectively. Matching the data of annual consumption growth in the US from 1889 to 1978, \( E[g_{t+1}^c] = 1.018 \), \( \text{var}[g_{t+1}^c] = 0.036 \), and the first-order serial correlation of the consumption growth, \( \text{corr}(g_{t+1}^c, g_{t+1}^c) \), equal to \(-0.14\), the consumption growth for each state is \( g_L = 0.982 \) and \( g_H = 1.054 \), and the transition probabilities are

\[
\begin{bmatrix}
\pi_{LL} & \pi_{LH} \\
\pi_{HL} & \pi_{HH}
\end{bmatrix} = \begin{bmatrix}
0.43 & 0.57 \\
0.57 & 0.43
\end{bmatrix}
\]

Along the line, Melino and Yang (2003) study the transition of the conditional marginal rate of substitution between the states \( g_L \) and \( g_H \). Calibrated with the historical means and variances of the real annual returns of the stock index and the risk-less assets from 1889 to 1978: \( E[R^s] = 1.07 \), \( \text{var}[R^s] = 0.165 \), \( E[R^f] = 1.008 \), \( \text{var}[R^f] = 0.056 \), and the Euler equation \( E[M_{t+1}^s R_{t+1}^f] = 1 \) for both these assets, where \( M_{t+1}^s \) is the marginal rate of substitution, the values of the conditional marginal rates of substitution are

\[
\begin{bmatrix}
M_{LL} & M_{LH} \\
M_{HL} & M_{HH}
\end{bmatrix} = \begin{bmatrix}
1.862 & 0.244 \\
1.127 & 0.949
\end{bmatrix}
\]

Our work is motivated by a further observation that the conditional expected marginal rate of substitution is pro-cyclical. Based on the aforementioned matrices of conditional marginal rates of substitution and transition probabilities, one can show that conditional on the recession, the expected marginal rate of substitution is \( E[M_{t+1}^s | g^c_{t-1} = g_L] = 0.43 \times 1.862 + 0.57 \times 0.244 = 0.939 \), and conditional on the boom, the expected marginal rate of substitution is \( E[M_{t+1}^s | g^c_{t-1} = g_H] = 0.57 \times 1.127 + 0.43 \times 0.949 = 1.0546 \).

The pro-cyclical conditional expected marginal rate of substitution may be attributed to loss aversion, which is elaborated by the following example. Consider a representative agent economy, and suppose the representative agent exhibits the loss averse attitude during the period of recession. To focus on the effect of loss aversion, we also assume that the representative agent is with a piecewise linear (risk-neutral) loss averse utility on consumption, of which the first-order derivative with respect to \( c_t \) is as follows:

\[
u(c_t) = \begin{cases}
1 & \text{if } g^c_{t-1} = g_H \\
\lambda_1 & \text{if } g^c_{t-1} = g_L
\end{cases}
\]

Based on the above utility and the two-state Markov process of the consumption growth rate in Mehra and Prescott (1985), the expectations of the marginal rates of substitution conditional on \( g^c_{t-1} = g_L \) and \( g^c_{t-1} = g_H \) are shown as follows:

\[
E_t[M_{t+1}^s | g^c_{t-1} = g_L] = E_t[\frac{\delta u'(c_t + 1)}{u'(c_t)} | g^c_{t-1} = g_L] = \frac{\delta}{\lambda_1} (0.43 \cdot 0.57 + 1 \cdot 0.57)
\]

\[
E_t[M_{t+1}^s | g^c_{t-1} = g_H] = E_t[\frac{\delta u'(c_{t+1})}{u'(c_t)} | g^c_{t-1} = g_H] = \frac{\delta}{\lambda_1} (0.57 \cdot 0.057 + 1 \cdot 0.43)
\]

For example, one of the pioneer articles to apply the loss aversion of the prospect theory to solving the equity premium puzzle is Benartzi and Thaler (1995), in which they consider a wealth-based loss averse utility for investors. In addition, in a more recent research, Lien (2001) study the effect of the loss aversion of the prospect theory on the optimal futures hedge ratio, in which the investor is assumed to maximize the expected utility on his period-end wealth.

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where $\delta_t$ is the subject discount factor at $t$. Based on the aforementioned results, the ratio of the above two equations equals $0.939/1.0546$, i.e.

$$
\frac{E_t[M_{t+1}^{g_t}] | g_{t-1} = g_t}{E_t[M_t^{g_t}] | g_{t-1} = g_t} = \frac{0.939}{1.0546} = \frac{0.431 + 1 \cdot 0.57}{0.57 + 1 \cdot 0.43}
$$

Solving the above equation, the value of $\lambda_1$ is $1.1072$. The value of $\lambda_1$ derived from the historical data is indeed larger than 1, which is consistent with the proposition that the representative agent is with the loss averse attitude.

Note that in the recession, the conditional expected marginal rate of substitution, $E_t[M_{t+1}^{g_t}] | g_{t-1} = g_t = \delta(0.43 + 0.57/\lambda_1)$, declines with the increase of $\lambda_1$. On the other hand, in the boom, the conditional expected marginal rate of substitution, $E_t[M_{t+1}^{g_t}] | g_{t-1} = g_t = \delta(\lambda_1 + 0.57 + 0.43)$, increases with the increase of $\lambda_1$. Based on the above reasoning, it is believed that if the representative agent is with a higher degree of loss aversion, i.e. if $\lambda_1$ increases, the conditional expected marginal rate of substitution is more inclined to exhibit the pro-cyclical characteristic. This finding that US consumption data exhibits the implications of the loss averse behaviour motivates us to combine the prospect theory utility and the consumption-based asset pricing model.

To explain the determinants of the shape of the term structure of interest rates, one of the most cited theories is the expectations theory. Many studies examine this theory empirically, including Campbell and Shiller (1991), Johnson (1997), Bekaert and Hodrick (2001), Carriero et al. (2006), Kalev and Inder (2006), Beyaert and Pérez-Castejón (2007), etc. However, this article employs a different point of view to study how the loss averse behaviour affects the shape of the term structure of real interest rates based on a consumption-based asset pricing model. Recently, different versions of consumption-based asset pricing models are used to investigate the historical average of the term premium of risk-less assets, or even further, matching the whole spectrum of the observed term structure of interest rates.

Abel (1999) separates the equity premium into the term and the risk premium based on a consumption-based asset pricing model with habit formation and ‘catching up with the Joneses’. The habit formation means that an individual’s habit level depends on his past consumption, but the ‘catching up with the Joneses’ formulation specifies an individual’s habit level depending on the history of aggregate consumption. The implied term premium between the long- and short-term risk-less assets in his model is around 226 basis points. However, the entire spectrum of the term structure of interest rates is not investigated in his study.

Brandt and Wang (2003) introduced a stochastic formulation of the relative risk aversion coefficient into the consumption-based asset pricing model for deriving the entire spectrum of the term structure. The stochastic part of the relative risk aversion coefficient consists of the unexpected news about the consumption growth and inflation. Based on the monthly or quarterly data on aggregate consumption and consumer prices from January 1959 to June 1998, the implied term premium generated from their model is too small to successfully match the entire term structure for the same period.

Piazzesi and Schneider (2006) considered the role of inflation as a bad news for future consumption growth. Relying on the negative correlation between consumption growth and lagged inflation, the Epstein–Zin recursive utility is adopted in their model to produce an upward sloping yield curve. For the period of 1952:I to 2005:IV, they are able to generate an average nominal yield curve with reasonable magnitude, but the curvature of their results cannot fit that in the empirical data. In addition, the average and the volatility of the term structure of real interest rates in their model are inconsistent with those implied from the data of US Treasury Inflation-Protection Securities (TIPS).

Wachter (2004, 2006) extends the framework of the consumption-based asset pricing model with habit formation in Campbell and Cochrane (1999) to study the spectrum of the term structure. With one more variable to balance the effects of consumption smoothing and precautionary saving, her model is able to generate the bond yields with different time to maturities. For the quarterly data on inflation and consumption from 1952:I to 2004:II, the magnitude of the means and the SDs of the yield curves is close to the empirical data. However, her model implies a nearly straight-line yield curve, which is not consistent with the ones in the empirical data.

In this article, we study the term structure of real interest rates under the consideration of loss aversion of prospect theory. In our consumption-based asset pricing model, loss aversion is incorporated to capture the phenomenon of the pro-cyclical conditional expected marginal rate of substitution, and the concept of habit formation is adopted to determine the reference point for this behavioural

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5The other solution of $\lambda_1$ is $-1.014$, which contradicts the assumption that the marginal utility with respect to the consumption must be positive.
finance utility. The entire spectrum of the term structure is derived, and the results show that the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion is the key factor of determining the curvature of the term structure of real interest rates.

The remainder of this article is organized as follows. In Section II, a consumption-based asset pricing model incorporating the prospect theory utility is proposed. The details of the simulation algorithm to derive the term structure are shown as well. Section III is dedicated to the results of our model, and the empirical studies for the moments of real yield curves in the US are performed in Section IV. Section V concludes this article.

II. The Loss Aversion Asset Pricing Framework

The economy

The utility of the representative agent. We assume that there exists a representative agent and one perishable consumption good in the economy, and at each time point $t$, he maximizes $E_t[U_t]$, where $E_t[\cdot]$ is the conditional expectation operator at time $t$, and $U_t$ is defined as

$$U_t = \sum_{i=0}^{\infty} \delta^i u(c_{t+i}, v_{t+i})$$

where $c_t$ and $v_t$ are the consumption level and the consumption reference point at time $t$, and $\delta_t$ is the subjective discount factor based on the information set at time $t$. The utility function $u(c_t, v_t)$ is defined as follows:

$$u(c_t, v_t) = \begin{cases} 1 - e^{-\beta(c_t - v_t)} & \text{if } c_t - v_t \geq 0 \\ \lambda_1 \left[ \frac{1}{1 - e^{-\beta_c(c_t - v_t)}} \right] & \text{if } c_t - v_t < 0 \end{cases}$$

where $\beta$ is the risk aversion coefficient. In this article, we follow Campbell and Cochrane (1999) and use the difference between the current consumption level and the consumption reference point as the proxy of the business cycle. Furthermore, when $c_t - v_t \geq 0$, we say the representative agent is in a good state (or in a boom). In this case, he is with the exponential risk aversion utility function $1 - e^{-\beta_c(c_t - v_t)}$. Otherwise, when $c_t - v_t < 0$, the representative agent is in a bad state (or in a recession). Under this condition, the representative agent is assumed to exhibit the loss aversive attitude, and his utility is $\lambda_1[1 - e^{\frac{\beta}{2}c_t - v_t}]$.

Different combinations of the values of $\lambda_1$ and $\lambda_2$ are able to represent different utility functions. Once the value of $\lambda_1$ is smaller than $-1$ and the value of $\lambda_2$ is smaller than 0, Equation 2 is the prospect theory utility. In this case, $\lambda_1$ is the loss averse coefficient, and when $c_t < v_t$, the representative agent, being in the bad state, becomes a risk lover with a negative risk aversion coefficient $\beta/\lambda_2$. In Equation 2, if the values of $\lambda_1$ and $\lambda_2$ happen to be 1 simultaneously, the utility function $u(c_t, v_t)$ does not depend on whether $c_t < v_t$. Equation 2 becomes an exponential utility function with the habit reference point $v_t$. Moreover, since the consumption level of the representative agent is always larger than zero, once setting $v_t = 0$, Equation 2 becomes the classic exponential utility.

Deciding the reference point. In this article, habit formation and ‘catching up with the Joneses’ are adopted to determine and update the reference point $v_t$ in the following form:

$$v_t = wC_{t-1} + (1-w)v_{t-1}, \quad 0 \leq w \leq 1$$

In Equation 3, an additional parameter $w$ is used to balance the two determining factors of $v_t$, the individual consumption level $c_{t-1}$ and the aggregate consumption level per capita $C_{t-1}$ at time $t-1$. When $w = 1$, the utility function displays habit formation, because the representative agent’s consumption reference point $v_t$, depends only on his last-period consumption. If $w = 0$, the utility function displays the phenomenon of ‘catching up with the Joneses,’ which indicates that the consumption reference level of the representative agent is the aggregate consumption level per capita in the last period.

In the previous works, if the consumption level were to fall unfortunately below the habit reference point, the investor’s marginal utility would not always remain finite and positive. Campbell and Cochrane (1999) adopt a highly persistent, nonlinear historic-consumption habit reference level such that the consumption level is guaranteed to be higher than this reference level. On the other hand, Abel (1999) replaces the subtract-form utility with a ratio-form utility, $(c_t/v_t)^{1-\beta}/(1-\beta)$, to avoid this problem. However, by treating the boom and recession separately in the prospect theory utility, the situation $c_t - v_t < 0$ can be handled properly in our model.

The return of risk-less asset

In this section, the risk-less asset returns of different maturities are derived via the Euler equation, which states that consumers will sacrifice today’s consumption level in exchange for increasing the possession of some assets, and holding the asset will bring them...
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returns that can be transformed into consumption goods in the future, i.e.,
\[ E\left[ -\frac{\partial U_t}{\partial c_t} + \delta_t R^i_{t+n} \left( \frac{\partial U_{t+n}}{\partial c_{t+n}} \right) \right] = 0 \]
where \( R^i_{t+n} \) is the real return of asset \( j \) between current time \( t \) and \( n \) year after. Rearranging the above equation, we obtain
\[ E\left[ \frac{\partial U_t}{\partial c_t} \right] = E\left[ \delta_t R^i_{t+n} \left( \frac{\partial U_{t+n}}{\partial c_{t+n}} \right) \right] \tag{4} \]

Following Equations 1 and 3, \( \frac{\partial U_t}{\partial c_t} \) is represented as follows:
\[ \frac{\partial U_t}{\partial c_t} = u_t(c_t, v_t) + \delta_t u_{c_{t+1}}(c_{t+1}, v_{t+1}) \frac{\partial v_{t+1}}{\partial c_t} \tag{5} \]

Similarly,
\[ \frac{\partial U_{t+n}}{\partial c_{t+n}} = u_{c_{t+n}}(c_{t+n}, v_{t+n}) + \delta_t u_{c_{t+n+1}}(c_{t+n+1}, v_{t+n+1}) \frac{\partial v_{t+n+1}}{\partial c_{t+n}} \tag{6} \]

Because \( \frac{\partial U_t}{\partial c_t} \) depends not only on the derivative of \( u(c_t, v_t) \) with respect to \( c_t \), but also on the derivative of \( u(c_{t+1}, v_{t+1}) \) with respect to \( c_{t+1} \), \( \frac{\partial U_t}{\partial c_t} \) does not belong to the information set of time \( t \). Therefore, different from previous studies, it is necessary to maintain the expectation of \( \frac{\partial U_t}{\partial c_t} \) at \( t \) in Equation 4. After dividing both sides of Equation 4 by \( E\left[ \frac{\partial U_t}{\partial c_t} \right] \), we have
\[ E\left[ \delta_t R^i_{t+n} \left( \frac{\partial U_{t+n}}{\partial c_{t+n}} \right) \right] = 1 \]

According to the general definition, \( M_t^{i+n} = \frac{\partial U_t}{\partial c_t} R^i_{t+n} \) is the intertemporal marginal rate of substitution, and the above equation can be rewritten as \( E\left[ \delta_t R^i_{t+n} M_t^{i+n} \right] = 1. \) Suppose \( R^i_{t+n} \) denotes the return of a risk-less zero coupon bond which is purchased at \( t \) and with the payment of one unit of the consumption good at \( t + n \). The corresponding Euler equation for \( R^b_{t+n} \) is as follows:
\[ R^b_{t+n} = \frac{1}{\delta_t E\left[ M_t^{i+n} \right]} \tag{7} \]

In order to derive \( R^b_{t+n} \), we must formulate \( M_t^{i+n} \) first. From Equations 5 and 6, one may have to take \( u(c_t, v_t) \), \( u(c_{t+1}, v_{t+1}) \), \( u(c_{t+n}, v_{t+n}) \), and \( u(c_{t+n+1}, v_{t+n+1}) \) into consideration while deriving \( M_t^{i+n} \).

Since the prospect theory utility is adopted, the utility function of each period may not be the same. For example, if \( c_t - v_t > 0 \), the utility function is \( 1 - e^{-\beta(c_t - v_t)} \), and if \( c_t - v_t < 0 \), the utility of the representative agent becomes \( \lambda_1[1-e^{-\frac{\beta}{\gamma}(c_t - v_t)}] \) due to the characteristic of loss aversion. Calculating the intertemporal marginal rate of substitution, \( M_t^{i+n} \) requires the comparisons between pairs of \( c_t \) and \( v_t \), \( c_{t+1} \) and \( v_{t+1} \), and \( c_{t+n} \) and \( v_{t+n} \). To simplify the equation of \( M_t^{i+n} \), we define the following indicator variables:
\[ p = I(c_t - v_t \geq 0), \quad \phi_t = 1 \cdot p + \frac{\lambda_1}{\gamma}, (1 - p), \]
\[ q = I(c_{t+1} - v_{t+1} \geq 0), \quad \phi_{t+1} = 1 \cdot q + \frac{\lambda_1}{\gamma}, (1 - q), \]
\[ r = I(c_{t+n} - v_{t+n} \geq 0), \quad \phi_{t+n} = 1 \cdot r + \frac{\lambda_1}{\gamma}, (1 - r), \]
\[ s = I(c_{t+n+1} - v_{t+n+1} \geq 0), \quad \phi_{t+n+1} = 1 \cdot s + \frac{\lambda_1}{\gamma}, (1 - s), \]
\[ \theta_{t+n+1} = 1 \cdot s + \lambda_2 \cdot (1 - s) \]

Following the definition of \( M_t^{i+n} \) and some derivative calculations, the marginal rate of substitution \( M_t^{i+n} \) conditional on \( p, q, r, s \) is
\[ M_t^{i+n}(p, q, r, s) = \frac{\phi_t \phi_{t+1} \phi_{t+n} \phi_{t+n+1} e^{-\beta(c_t - v_t)}}{\phi_{t+n+1} e^{-\beta(c_{t+n} - v_{t+n})}} \]

Similar to many consumption-based asset pricing models, we assume that the annual growth rates of both individual consumption and aggregate consumption per capita follow the same logarithmic normal distribution:
\[ \ln \frac{c_{t+1}}{c_t} = \ln \frac{c_{t+n+1}}{c_{t+n}} = \mu_g + z_g, \quad \text{where } z_g \sim N(0, \sigma_g) \]

In addition, Mehra and Prescott (1985) consider the first-order serial correlation of the consumption growth, denoted by \( \text{corr}(g_{t+1}', g_{t+1}';) = \rho_{gg} \), which is also taken into account in this article.

Simulation algorithm

In this article, similar to many previous studies, a simulation method is adopted to derive the rate of return with different time to maturities. In theory, at each point in time \( t \), knowing the rates of return of assets with different time to maturities, people decide how much \( c_t \) to consume today and \( c_{t+n} \) to consume in the future by maximizing their expected utility \( E_t[U_t] \) based on the reference point \( v_t \). Meanwhile, the
relation between the equilibrium rates of the return of assets and the marginal rate of substitution of consumption results from this maximization.

The simulation algorithm is formulated to mimic the above situation. However, we do not focus on how to derive the optimal consumption level \( c_t \). Instead, we would like to derive the term structure of real interest rates when taking the historical consumption data into consideration. Rather than applying the actual historical consumption data to our model directly, the distributions of the consumption level and consumption growth are derived from the historical consumption data, and simulated samples from these distributions are used to derive the expected term structure of real interest rates.

There are two main steps in our algorithm. First, based on the given reference point \( v_t \) and the chosen consumption level \( c_t \) at time \( t \), the conditional expected returns \( R_{t+n}^B(v_t, c_t) \) for \( n = 1, 2, \ldots, 30 \) are derived by simulation. Second, since the information \( v_t \) and the decision \( c_t \) are different for each time point, calculating the average return over different time points is equivalent to calculating the unconditional expectations of \( R_{t+n}^B(v_t, c_t) \) over \( v_t \) and \( c_t \). Based on the distributions of \( v_t \) and \( c_t \) from the historical data, the unconditional returns \( R_{t+n}^B \) are obtained through integration over possible combinations of the values of \( v_t \) and \( c_t \). The methods to derive the conditional and unconditional expected returns of the risk-less assets are further described as follows.

**Conditional expected returns.** Based on the given values of the reference point \( v_t \) and the consumption level \( c_t \), the detail steps to derive the conditional expected returns \( R_{t+n}^B(v_t, c_t) \) are the following.

First, following the assumptions in Equation 9 and the corr\((g_{t+1}^f, g_{t-1}^f) = \rho_{g_{t+1}^f, g_{t-1}^f} \). 60 000 sets of random samples of \( (g_{t+1}^f, g_{t+2}^f, \ldots, g_{t+31}^f) \) are generated, and 45 500 000 sets of \( (c_{t+1}, c_{t+2}, \ldots, c_{t+31}, v_{t+1}, v_{t+2}, \ldots, v_{t+31}) \) can be derived. In fact, it can be observed from Equation 8, for each \( n \), we need only \((c_{t+1}, c_{t+2}, c_{t+31}, v_{t+1}, v_{t+2}, v_{t+31})\).

Second, \( E_g[\Phi_{t+1}e^{-\Phi_{t+1}/(\beta_1)(v_{t+1}-r_t)}] \) is a quantity that needs to be computed before determining the value of \( M_t^{i+n}(p, q, r, s) \). In our model, across the 60 000 sampled sets, the arithmetic average of \( \Phi_{t+1}e^{-\Phi_{t+1}/(\beta_1)(v_{t+1}-r_t)} \) is used as an approximation of this expectation. Once we have the value of \( E_g[\Phi_{t+1}e^{-\Phi_{t+1}/(\beta_1)(v_{t+1}-r_t)}] \), for each set of \((c_{t+1}, c_{t+2}, c_{t+31}, v_{t+1}, v_{t+2}, v_{t+31})\), we can settle on the right form of \( M_t^{i+n}(p, q, r, s) \) for each set depending on whether \( c_{t+i} - v_{t+i} \) (\( i = 0, 1, n, n+1 \)) is larger than zero.

Finally, for risk-less assets, the arithmetic average of \( M_t^{i+n}(p, q, r, s) \) over these 60 000 sets gives us the unconditional distribution of \( M_t^{i+n}(p, q, r, s) \) for \( n = 1, 2, \ldots, 30 \) can be derived by Equation 7.

**Unconditional expected returns.** In literature, the reported average term structure is calculated by taking the arithmetic average across different time points in a period of time. In our model, considering different time points is equivalent to considering different information sets the representative agent may have. Thus, we calculate the unconditional expected return by the numerical integration over all possible values of \( v_t \) and \( c_t \). However, knowing \( v_t \) is equivalent to knowing last period’s consumption level \( c_{t-1} \). Therefore, there must be some relation between \( v_t \) and today’s consumption level \( c_t \). To avoid introducing more parameters to describe the relation between \( c_t \) and \( v_t \), we further assume that \( u_t \sim \mathcal{N}(\mu_t, \sigma_t) \) and \( c_t = e_{t-1}^0 - v_t \) as the initial distribution of the process \( (v_{t+1}, c_{t+1}) \), where \( \mu_t \) and \( \sigma_t \) are the mean and the SD of the annual consumption level estimated from the historical data of per capita consumption in the US, and \( g_{t-1}^f \) is the consumption growth rate from \( t-1 \) to \( t \) and its distribution is from Equation 9. With this assumption, the unconditional expected returns of return \( R_{t+n}^B \) can be calculated through the following equation:

\[
R_{t+n}^B = \int_{v_{t+n}} \int_{c_{t+n}} R_{t+n}^B(v_t, c_t, e_{t-1}^0, v_t) h(v_t, g_{t-1}^f) dg_{t-1}^f dv_t
\]

for \( n = 1, 2, \ldots, 30 \).

Based on the assumption of the independence characteristic between the last-period consumption information \( v_t \) and the consumption growth \( g_{t-1}^f \) between \( t-1 \) and \( t \), the probability density function \( h(v_t, g_{t-1}^f) \) is an independently bivariate normal distribution. Through the numerical integration, we are able to obtain the unconditional expected rates of return \( R_{t+n}^B \). Finally, the annualized real risk-less yield between \( t \) and \( t+n \) is computed by the following formula:

\[
r_{t+n}^B = \frac{\ln R_{t+n}^B}{n}
\]
Loss aversion and the term structure of interest rates

Table 1. Parameters and their values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of the consumption growth, $\mu_g$</td>
<td>0.0189</td>
</tr>
<tr>
<td>SD of the consumption growth, $\sigma_g$</td>
<td>0.015</td>
</tr>
<tr>
<td>Serial correlation between $g_{t+1}^r$ and $g_{t-1}^r$, $\rho_{gt}$</td>
<td>-0.14</td>
</tr>
<tr>
<td>Mean of the annualized consumption level, $\mu_c$ (US$1000)$</td>
<td>4.57336</td>
</tr>
<tr>
<td>SD of the annualized consumption level, $\sigma_c$ (US$1000)$</td>
<td>1.02046</td>
</tr>
<tr>
<td>Risk aversion coefficient, $\beta$</td>
<td>1</td>
</tr>
<tr>
<td>Subject discount factor, $\delta_i$</td>
<td>0.9725</td>
</tr>
<tr>
<td>In the case of prospect theory utility</td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$ as the loss averse coefficient</td>
<td>-1.3</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-1</td>
</tr>
<tr>
<td>$w$ (the benchmark level $v_t = wC_{t-1} + (1-w)C_{t-1}$)</td>
<td>0.5</td>
</tr>
<tr>
<td>In the case of exponential utility with habit reference point</td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1</td>
</tr>
<tr>
<td>$w$ (the benchmark level $v_t = wC_{t-1} + (1-w)C_{t-1}$)</td>
<td>0.5</td>
</tr>
<tr>
<td>In the case of exponential utility</td>
<td></td>
</tr>
<tr>
<td>$v_t$</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes: There are many parameters in our model and the table lists the value of each parameter. In order to demonstrate that our model's ability to solve the entire term structure is due to combining habit formation with loss aversion of prospect theory into the consumption-based asset pricing model, the values of parameters used in our model are mostly collected from existing consumption-based asset pricing models.

III. Numerical Results

In this section, the values of parameters used in our model are discussed first, followed by the results of the term structure of real interest rates for different cases of utilities. In addition, the comparisons between the generated yield curves of our model and those of the consumption-based asset pricing model in Wachter (2004, 2006) are presented. After that, the statistic analysis will be performed for each parameter of different utility cases in our model.

The parameters

Table 1 lists the parameters and the corresponding values used in our model. In order to demonstrate the superior performance of our model for solving the entire term structure, the data set and values of parameters in our model are mostly collected from Campbell and Cochrane (1999) and Mehra and Prescott (1985). First, following Campbell and Cochrane (1999), we assume the mean and the SD of the logarithmic consumption growth $g_{t+1}^r$ to be 0.0189 and 0.015, respectively. In addition, following Mehra and Prescott (1985), the value of the parameter $\rho_{gt}$, which is the first-order serial correlation between $g_{t+1}^r$ and $g_{t-1}^r$, is assumed to be -0.14.

We also need the distribution of consumption level per capita to derive the unconditional expected rates of return. Based on the same data set used in Campbell and Cochrane (1999), the annualized consumption level per capita is assumed to follow a normal distribution, of which the mean and SD are calculated from the quarterly data of US consumption level per capita from 1959:IV to 1996:I.

Second, in our base case, the risk averse coefficient $\beta$ is assumed to be 1, which represents a consumer possessing either mild degree of risk aversion or mild degree of risk loving. The discount factor $\delta_i$ is assumed to be a constant of 0.9725. Actually, the value of $\delta_i$ affects only the absolute magnitude of rates of return, but not the term premiums with different time to maturities.

Third, the primary feature of our model is to employ loss aversion of prospect theory. Hence, there are two more parameters in our model, $\lambda_1$ and $\lambda_2$, which are assumed to be -1.3 and -1 in our base case. Moreover, if we set $\lambda_1 = \lambda_2 = 1$, it represents the situation of taking only exponential utility with the habit reference point $v_t$ into consideration, and the utility in this case is $u(c_t, v_t) = 1 - e^{-\beta(C_{t-1})}$, and the corresponding marginal rate of substitution is

$$M_{t+1} = \frac{e^{-\beta(C_{t+1}-v_{t+1})} + \delta_t w E_t[\alpha_t]}{e^{-\beta(C_{t-1})} + \delta_t w E_t[\alpha_t]}$$

As to the parameter $w$ in the reference point $v_t$, we assume it to be 0.5, for which the utility of the representative agent features equally-weighted ‘catching up with the Joneses’ and habit formation. This is because different values of $w$ do not affect the shape of the term structure very much, and by this...
simplified assumption of \( w = 0.5 \), we can concentrate on studying the effect of loss aversion on the shape of the term structure of real interest rates.

Finally, since the consumption level is always larger than zero, once we set \( v_t = 0 \), it is always true that \( c_t \geq v_t \). The utility in Equation 2 will degenerate to the classic exponential utility, \( u(c_t, v_t) = 1 - e^{-\beta c_t} \), and the marginal rate of substitution becomes

\[
M^t_{t+1} = \frac{e^{-\beta c_{t+1}}}{e^{-\beta c_t}} = e^{-\beta(c_{t+1} - c_t)} \quad (12)
\]

530 **Term structures in different cases**

The term structures of real interest rates for different cases in Fig. 1 are our main results. If the representative agent is with the exponential utility, the term premium between \( n = 1 \) and \( n = 30 \) is about 3.5%. This term premium seems large enough, but the magnitudes of the interest rates are too large.\(^7\) On the other hand, in the case of the exponential utility with the habit reference point, the term premium between \( n = 1 \) and \( n = 30 \) is about 0.05%.\(^8\)

The underlying reason for these two cases is as follows. When \( n \) increases, the value of \( c_{t+n} - c_t \) on average rises, and according to Equation 12 the expected marginal rate of substitution decreases and thus \( R^t_{t+n} \) increases based on Equation 7. As a result, \( R^t_{t+n} \) increases with \( n \) to generate an upward sloping yield curve. However, when taking habit formation into consideration, because the habit benchmark \( v_{t+j} \) keeps a close trace behind \( c_{t+j} \), the values of \( c_{t+n} - v_{t+n} \) and \( c_{t+n+1} - v_{t+n+1} \) in Equation 11 increases little as \( n \) increases. Therefore, the expected marginal rate of substitution in this case decreases slightly when \( n \) increases, which results in little increase of \( R^t_{t+n} \) when \( n \) increases.\(^9\)

However, for the exponential utility, no matter whether habit formation is considered or not, these two yield curves look like straight lines, which do not fit the empirical data. Our results suggest that in the consumption-based asset pricing model, risk aversion does generate term premium, but its marginal term premium with respect to the time to maturity is nearly constant. This finding is consistent with the results in Brandt and Wang (2003), Piazzesi and Schneider (2006) and Wachter (2004, 2006). On the other hand, the term premium between \( n = 1 \) and \( n = 30 \) in the prospect theory utility case is 1.58% (Fig. 1). Meanwhile, the highly curved term structure of this behavioural finance utility is similar to the one in the empirical data. Therefore, we can infer that in order to match the yield curve in the empirical data, which is with decreasing marginal term premium with respect to the time to maturity, it is necessary to take loss aversion into consideration.

Let us remind readers that the analysis in Introduction has demonstrated the existence of the pro-cyclical conditional expected marginal rate of substitution in the US consumption data, and this phenomenon results from the loss aversion attitude of the representative agent. But how do loss aversion and the pro-cyclical conditional expected marginal rate of substitution affect the term structure?

In Equation 8, note that the way loss aversion affects the marginal rate of substitution is through \( \phi_{t+n} \), where \( i = 0, 1, n, n+1 \). The value of \( \phi_{t+n} \) is 1 if \( c_{t+n} \geq v_{t+n} \) and is \( \lambda_1/\lambda_2 \) otherwise, which means \( \phi_{t+n} \) equals 1 with \( \text{prob}(c_{t+n} \geq v_{t+n}) \) and equals \( \lambda_1/\lambda_2 \) with \( 1 - \text{prob}(c_{t+n} \geq v_{t+n}) \). Therefore, the conditional expected marginal rate of substitution in Equation 8 is higher for \( \phi_t = 1 \) than for \( \phi_t = \lambda_1/\lambda_2 = (-1.3)/(1-1) = 1.3(c_t < v_t) \), while \( \phi_{t+1}, \phi_{t+n}, \) and \( \phi_{t+n+1} \) have probabilities to be either 1.3 or 1. The above analysis shows that the phenomenon of the pro-cyclical conditional expected marginal rate of substitution is properly characterized in our model.

In addition, due to the consistent growing trend of the consumption process, the probability of \( c_t \geq v_t \) is significantly higher than the probability of \( c_t < v_t \),\(^9\) and as a result, the net effect of the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion is an increase in the expectation of the unconditional marginal rate of substitution.

Furthermore, our model implies that this sort of increment of the unconditional expected marginal rate of substitution is almost independent of the
time to maturity. Because the consumption process is consistently mildly growing and \( v_{t+1} \) contains the last-period consumption information, \( v_{t+1} \) keeps a close trace behind \( c_{t+1} \) in a way that the probabilities of \( c_{t+1} \geq v_{t+1} \) does not vary not much for different values of \( i \). In consequence, the possible realized values of \((\phi_t, \phi_{t+1}, \phi_{t+n}, \phi_{t+n+1})\) are distributed similarly given different values of \( n \), and the effect of loss aversion in Equation 8 is almost the same for different values of \( n \). Therefore, the net effect of loss aversion to increase the unconditional expected marginal rate of substitution is almost independent of the time to maturity.

Since the increase of the unconditional expected marginal rate of substitution driven by loss aversion is almost independent of \( n \), according to Equation 7, it is obvious that the decrease of the risk-less \( n \)-year return \( R^B_{t+n} \) caused by loss aversion is almost independent of \( n \) as well. Therefore, when we derive the annualized real risk-free zero rate \( R^B_{t+n} \) between today and \( n \)-year after. According to Equation 10, the effect of loss aversion to decrease \( R^B_{t+n} \) will be amortized among these \( n \) years. The decreasing level of \( R^B_{t+n} \) with relatively small \( n \) is more significant than that with relatively large \( n \), that results in the highly curved term structure for the case of the prospect theory utility in Fig. 1, which is similar to the ones in the empirical data.

Comparisons with Wachter’s results

In order to show that loss aversion is the dominating factor of the shape of the term structure, we compare the yield curve derived from our model with those derived from the model in Wachter (2004, 2006), in which both the consumption-based asset pricing model and habit formation are considered. Figure 2(a) illustrates the real yield curves derived from both Wachter’s model and ours, and Fig. 2(b)
illustrates the nominal yield curves derived from both models as well as the US historical data from 1952 to 1998 reported in Wachter (2004).\footnote{The reason why we do not use the results in Wachter (2006), the final publication of Wachter (2004), is that the longest maturity of the risk-less interest rates in Wachter (2006) is only 5 years, that is too short for us to compare the whole spectrum of the term structure.}

In Fig. 2, the real yield curve of our model is derived based on $\beta = 1$, $\lambda_1 = -1.1072$, $\lambda_2 = -1$ and $\delta = 0.9725$. The values of $\beta$, $\lambda_2$ and $\delta$ are from our basic setting in the case of prospect theory utility, the value of $\lambda_1$ is from the analysis in Introduction. Moreover, since our model only takes the real rates of return into account, in order to perform the comparison with the nominal yield curves in the historical data, the corresponding nominal yield curve of our model is based on our real yield curve plus the difference between the nominal and real yields in Wachter's model with the same maturity.

It is clear that although Wachter follows the framework of Campbell and Cochrane (1999) to take habit formation into consideration and further formulates the inflation in the consumption-based model, both the real and nominal yield curves of her model are still straight-line-like. It should be noted that in Fig. 1, no matter whether habit formation is considered or not, the traditional consumption-based asset pricing model is able to derive only straight-line-like term structures, which are very similar to the results of Wachter's model. However, once loss aversion is nested into the consumption-based model, we can obtain a highly curved term structure, which is apparently closer to the empirical data in the United States as illustrated in Fig. 2.

The comparative statics of parameters in different cases

The effects of $\beta$, $\lambda_1$ and $\lambda_2$ in the prospect theory utility case. Figures 3–5 show how $\beta$, $\lambda_1$ and $\lambda_2$ affect the term structure of real interest rates in the case of the loss averse utility. In Fig. 3, it can be seen that when the representative agent is more risk averse ($\beta$ increases), $r_{t+n}$ in general increases more when the time to maturity is relatively long and less when the time to maturity is relatively short. As a result, the term premium increases as risk aversion increases. This finding is in line with the theory based on the traditional asset pricing model that risk aversion is one of the major contributions of the term premium.

In Fig. 4, the effect of the loss averse coefficient is shown. According to the analysis in the previous section, the results again show that decreasing $\lambda_1$ (strengthening the degree of loss aversion) will decrease $r_{t+n}$ significantly when $n$ is small, decrease $r_{t+n}$ a little when $n$ is large, and therefore increase the term premium.

The above analysis has shown that the loss averse coefficient affects the curvature of the average term structure significantly. However, prospect theory says more than that the marginal utility will be different for gains and losses. Prospect theory also suggests that people will become risk loving when they are facing losses. In this article, we find this behaviour will affect the term structure in a novel way.

In Fig. 5, the cases of $\lambda_2$ equaling $-1$, $-1.3$ and $-3$ are examined, which represent the cases of $\lambda_2 > \lambda_1$, $\lambda_2 = \lambda_1$, and $\lambda_2 < \lambda_1$, respectively. The case of $\lambda_2 = -1$ is the base case in our model and the corresponding real yield curve is the same as that of the loss averse case in Fig. 1. For the case of $\lambda_2 = -3$, an inverted yield curve is derived, and the reason is as follows.

By observing that the conditional expected marginal rate of substitution in Equation 8 is smaller for $\phi_1 = 1(c_t \geq v_t)$ than for $\phi_1 = \lambda_1/\lambda_2 = (-1.3)/(-3) = 0.4333 (c_t < v_t)$, independently from the values of $\phi_{t+1}$, $\phi_{t+n}$ and $\phi_{t+n+1}$, the conditional expected marginal rate of substitution exhibits the counter-cyclical characteristic. Because the probability of $c_t \geq v_t$ is higher than the probability of $c_t < v_t$, the net effect of the counter-cyclical conditional expected marginal rate of substitution is to decrease the expectation of the unconditional marginal rate of substitution and therefore to increase $R_{t+n}^B$. When $R_{t+n}^B$ is amortized to derive $r_{t+n}$, because the increase in $R_{t+n}^B$ is almost independent of $n$, $r_{t+n}$ will increase significantly when $n$ is small and will increase a little when $n$ is large. This is why an inverted yield curve is derived in the case of $\lambda_2 = -3$.

When $\lambda_2 = -1.3$, since $\lambda_1 = \lambda_2$, $\phi_{t+n}$ is always equal to 1 no matter the representative agent is in the boom or in the recession. As a result, the effect of loss aversion is eliminated and only risk aversion affects the term premium, so only a straight-line-like yield curve is generated.

In addition to the viewpoint of studying the conditional expected marginal rate of substitution, we provide an alternative explanation of generating inverted yield curves by analysing the nature of the prospect theory utility function. In Fig. 6, it can be found that in the case of $\lambda_2 = -1$, it is a normal prospect theory utility function, and there is a concave kink at the reference point. When a smaller (more negative) value of $\lambda_2$ is considered, e.g. $\lambda_2 = -3$, not only the representative agent becomes...
Loss aversion and the term structure of interest rates

Figure 2. The comparisons between the results of Wachter's and our models
Notes: (a) Illustrates the real yield curves generated by Wachter's and our models, and (b) Shows not only the nominal yield curves derived from both models but also the historical averages on annual zero coupon yields reported in Wachter (2004). Since only real yields are considered in our model, in order to perform the comparison with the nominal yield curve in the historical data, the corresponding nominal yield curve of our model is based on our real yield curve plus the difference between the nominal and real yields in Wachter's model with the same maturity. Both Wachter's and our models are based on the consumption-based asset pricing model and take habit formation into consideration, but due to the lack of loss aversion, only straight-line-like yield curves are generated in Wachter (2004, 2006).
Figure 3. The effect of $\beta$ on the term structure in the prospect theory utility case

Notes: As $\beta$ increases, the term premium increases with the degree of risk aversion. This finding is consistent with the traditional asset pricing model in which risk aversion is the main contribution of the term premium.

Figure 4. The effect of $\lambda_1$ on the term structure in the prospect theory utility case

Notes: When the representative agent is more loss averse ($\lambda_1$ decreases), the yield with short time to maturity decreases significantly, while the yield with long time to maturity decreases relatively little. Therefore, considering loss aversion in the consumption-based asset pricing model can generate yield curves whose shape is similar to the ones in the empirical data.
Loss aversion and the term structure of interest rates

Figure 5. The effect of $\lambda_2$ on the term structure in the prospect theory utility case

Notes: For the case in which $\lambda_2 > \lambda_1$, a normal yield curve is derived based on the pro-cyclical conditional expected marginal rate of substitution. For the other case in which $\lambda_2 < \lambda_1$, an inverted yield curve is derived due to the counter-cyclical conditional expected marginal rate of substitution. When $\lambda_2 = \lambda_1$, the term premium in this case is very small because the effect of loss aversion $\lambda_1$ is offset by $\lambda_2$ and only the risk adverse coefficient $\beta$ is responsible for the term premium.

Figure 6. The prospect theory utility when given different values of $\lambda_2$

Notes: In the case of $\lambda_2 = -1$, a familiar shape of prospect theory utility is shown. When $\lambda_2$ becomes smaller, not only is the utility function less risk loving in the bad state, but also the utility function changes from concave (when $\lambda_2 = -1$) to convex (when $\lambda_2 = -3$) near the reference point. Due to introducing the consumption habit, for which $c_{t+1}$ keeps a close trace behind $c_t$, the net effect of decreasing $\lambda_2$ is dominated by the convexity near the reference point, which makes the investor behaves more risk loving overall.
less risk loving in the bad state, but also a convex kink is formed at the reference point. Since \( v_{t+1} \) always keeps a close trace behind \( c_{t+\beta} \), the effect of decreasing \( \lambda_2 \) is primarily dominated by the effect of the new-forming convex kink. So, the decrease of \( \lambda_2 \) effectively makes the representative agents more risk loving in our model and thus a negative term premium will be obtained for the case in which \( \lambda_2 = -3 \).

The effect of \( \beta \) in the exponential utility case. In Fig. 7, it can be seen that the term premium between the long-term and short-term risk-less assets increases as the risk averse coefficient \( \beta \) increases. In addition, we also find that once a sufficiently large enough term premium is generated, it is always accompanied by a very large magnitude of risk free interest rates. These results again demonstrate the existence of the risk free rate puzzle in the literature. Furthermore, the drawback of the traditional consumption-based asset pricing model for deriving the whole spectrum of the interest rate term structure still exists: the yield curves are all straight-line-like and not consistent with the shape of the term structure of real interest rates in the empirical data.

Figure 7. The effect of \( \beta \) on the term structure in the exponential utility case

Notes: The results show that the term premium between the long-term and the short-term risk-less assets increases as the risk averse coefficient \( \beta \) increases. However, the yield curves are all straight-line-like, which is not consistent with the shape of the term structure of real interest rates in the empirical data.

The effect of \( \beta \) in the exponential utility with the habit reference point case. In Fig. 8, the effect of \( \beta \) is shown when the habit reference point is introduced into the exponential utility asset pricing model. Because of the close-trace characteristic of habit formation, the term premium is smaller than that of the case of the exponential utility. When \( \beta \) is small, the derived yield curve looks like a straight line. As the risk averse coefficient increases, the shape of the yield curve seems to fit the highly curved term structure a little better, but the term premium is still too small compared to the empirical data.

IV. Empirical Studies

The purpose of this section is to study how well our model fits the moments of real yield curves of the empirical data. We adopt the real yield curves derived directly from the prices of US TIPS for comparison. The data sets of these real yield curves could be accessed on J. Huston McCulloch’s website.\(^\text{11}\) As for the corresponding consumption data, the per capita consumption of nondurables and services in the US

\(^{11}\)These data sets are within a page titled ‘The US Real Term Structure of Interest Rates with Implicit Inflation Premium’ on his website.
Loss aversion and the term structure of interest rates

The case of exponential with habit reference point

Figure 8. The effect of $\beta$ on the term structure in the case of the exponential utility with the habit reference point

Notes: When taking habit formation into consideration, it can be seen that when the risk averse coefficient $\beta$ increases, the shape of the yield curve becomes more similar to the ones in the empirical data, although the magnitude of the term premium is still much smaller than that in the empirical data.

Estimation of parameters

The values of parameters used for the empirical studies are listed in Table 2. The quarterly consumption data on the website of BEA is from 1947:I to 2006:III. The value of $\mu_g$, $\sigma_g$ and $\rho_{gg}$ regarding the consumption growth process are estimated based on this data set. In the empirical studies, only the prospect theory utility is considered, and we concentrate on the effect of the risk and loss aversion attitudes that are the most important factors to determine the term structure of interest rates in our model. Thus, the values of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$, we derived through optimizing the fit for the means and SDs of the real yields in the historical data, whereas the values of $\lambda_2$ and $w$ are fixed the same as those in the base case in Section III. In addition, the subject discount factor for each quarter are derived through minimizing the differences between the real yield curves from our model and from the historical data for that quarter.

Calibration process and simulation algorithm

For each pair of examined values of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$, we apply a similar simulation algorithm as suggested in Section ‘Simulation Algorithm’ to derive the quarterly real yield curves form 1997:II to 2006:III. However, in the empirical studies, the values of $c_t$ and $v_t$ for each time $t$ are exactly the historical per capital consumption this quarter and previous quarter, respectively.

Based on each pair of $v_t$ and $c_t$ for each quarter, 60,000 sets of random samples of $(g_t^{i+1}, g_t^{i+2}, \ldots, g_t^{i+40})$ are generated, and 60,000 sets of $(c_t^{i+1}, c_t^{i+2}, \ldots, c_t^{i+41}, v_t^{i+1}, v_t^{i+2}, \ldots, v_t^{i+41})$ can then be derived. Finally, determine $M^t_{i+n}(p, q, r, s)$ and take the arithmetic average of $M^t_{i+n}(p, q, r, s)$ over these 60000 sets that gives us the expectation of $M^t_{i+n}$ and

\[12\] The first issuance of the US TIPS is from 1997, so our studying period begins from that year.
thus the conditional $R^B_{t+1}(v_t, c_t)$, given $v_t$ and $c_t$. In addition, for each quarter, we repeat this step until the optimized subjective discount factor $\delta_t$ for that quarter is found by minimizing the sum of the squared errors between the $R^B_{t+1}(v_t, c_t)$ for $n = 4, 8, \ldots, 40$ and the historical real yields with the same maturity of that quarter.

As for the unconditional expected returns, it is not necessary to perform the numerical integration described in Section ‘Unconditional expected returns’. We simply compute the arithmetic average of all yield curves for 38 quarters from 1997:II to 2006:III to obtain the results. Meanwhile, the SDs of the real interest rates with different maturities are also computed.

The whole process is conducted recursively until the optimized values of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$ are found to minimize the sum of the Root Mean Squared Errors (RMSEs) of the means and SDs of real yields from our model and from the historical data. The optimized values of $\beta$ and $\lambda_1$ is 0.9922 and $-1.3516$, respectively.

\section*{V. Conclusion}

In this article, it is shown that the underlying mechanism of the pro-cyclical conditional expected marginal rate of substitution, implied from the US

\begin{table}[h]
\centering
\caption{Parameters for deriving quarterly real yield curves in the empirical studies}
\begin{tabular}{|l|c|}
\hline
Parameters & Value \\
\hline
Parameters of the quarterly per-capita consumption growth & \\
Mean of the quarterly consumption growth, $\mu_g$ & 0.0141 \\
SD of the quarterly consumption growth, $\sigma_g$ & 0.0077 \\
Serial correlation between $g^*_t$ and $g^*_t$, $\rho_{gg}$ & 0.4988 \\
\hline
Parameters of the utility of the representative agent in our model & \\
$\beta$ & 0.9922 \\
$\lambda_1$ & $-1.3516$ \\
$\lambda_2$ & -1 \\
w & 0.5 \\
\hline
\end{tabular}
\end{table}

Notes: The parameters of the consumption growth is based on the quarterly per capita consumption data in the US from 1997:II to 2006:III. As to the parameters of the utility of the representative agent, only the setting of the prospect theory utility is considered. Here we focus on the effects of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$, so their values are calibrated to minimize the RMSEs of the means and SDs of real yields from our model and historical data. As to $\lambda_2$ and $w$, their values inherit from the base case in Section III.

\small
\begin{itemize}
\item[840] We take the arithmetic average of every three monthly real yield curves on the website of McCulloch to derive the quarterly real yield curves for comparisons.
\end{itemize}

\small
\begin{itemize}
\item[845] The parameters of the utility of the representative agent, only the setting of the prospect theory utility is considered. Here we focus on the effects of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$, so their values are calibrated to minimize the RMSEs of the means and SDs of real yields from our model and historical data. As to $\lambda_2$ and $w$, their values inherit from the base case in Section III.
\end{itemize}

\small
\begin{itemize}
\item[845] As for the unconditional expected returns, it is not necessary to perform the numerical integration described in Section ‘Unconditional expected returns’. We simply compute the arithmetic average of all yield curves for 38 quarters from 1997:II to 2006:III to obtain the results. Meanwhile, the SDs of the real interest rates with different maturities are also computed.
\end{itemize}

\small
\begin{itemize}
\item[855] The whole process is conducted recursively until the optimized values of the risk averse coefficient $\beta$ and the loss averse coefficient $\lambda_1$ are found to minimize the sum of the Root Mean Squared Errors (RMSEs) of the means and SDs of real yields from our model and from the historical data.
\end{itemize}

\small
\begin{itemize}
\item[860] The optimized values of $\beta$ and $\lambda_1$ is 0.9922 and $-1.3516$, respectively.
\end{itemize}

\section*{V. Conclusion}

In this article, it is shown that the underlying mechanism of the pro-cyclical conditional expected marginal rate of substitution, implied from the US
Loss aversion and the term structure of interest rates

Table 3. The average and volatility of the quarterly real yield curve in the US from 1997:II to 2006:III

<table>
<thead>
<tr>
<th>Maturity (quarters)</th>
<th>Real yield curve from our model</th>
<th>Real yield curve from TIPS</th>
<th>Real yield curve from our model</th>
<th>Real yield curve from TIPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.67</td>
<td>1.94</td>
<td>3.52</td>
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Notes: This table consists of the means and the SDs of real interest rates for different maturities derived from our prospect utility model and those from the empirical data provided by McCulloch (2006). The results of RMSEs indicate that both the means and the SDs of real yield curves generated from our model are with a reasonable magnitude. In addition, benefiting from introducing the loss averse behaviour and the habit reference point, our model fits both characteristics of the highly curved average and the downward sloping volatility of the term structure of real interest rates successfully.

References


Campbell, J. Y. and Cochrane, J. H. (1999) By force of habit: a consumption-based explanation of aggregate consumption data, corresponds to the loss averse behaviour in prospect theory. Accordingly, we are inspired to incorporate prospect theory into the consumption-based asset pricing model. Not only the loss averse behaviour in prospect theory is applied to the traditional consumption-based asset pricing model, but also the concepts of habit formation and ‘catching up with the Joneses’ are used to decide endogenously the reference point of this behavioural finance utility.

The results of our model show that the term premium consists of both the effects of risk and loss aversions. In addition, the curvature of the term structure of real interest rates is determined primarily from the effect of the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion. Our model is also capable of generating an inverted yield curve if the combination of loss aversion and the degree of risk aversion in the bad state results in the counter-cyclical conditional expected marginal rate of substitution. The analysis of the utility function in this combination shows that there is a convex kink near the reference point such that the representative agent effectively becomes risk loving and requires a negative term premium to hold the risk-less assets.

Finally, benefiting from introducing the loss averse behaviour and the habit reference point, our model uses relatively fewer number of parameters to fit simultaneously the highly curved average and the downward sloping volatility of the quarterly real yield curves for TIPS from 1997:II to 2006:III. Our results demonstrate that combining habit formation with the loss averse attitude of prospect theory into the consumption-based asset pricing model is the key factor to improve the performance of this sort of model in terms of explaining the characteristics of the term structure of real interest rates.


