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Loss aversion and the term structure of interest rates

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This article studies how the loss averse behaviour affects the term structure of real interest rates. Since the pro-cyclical conditional expected marginal rate of substitution, implied from the US consumption data, is consistent with the proposition of loss aversion, we incorporate the loss averse behaviour of prospect theory into the consumption-based asset pricing model. Motivated by the similarity between habit formation and the prospect theory utility, habit formation is exploited to determine endogenously the reference point of this behavioural finance utility. The highly curved characteristic of the term structure of real interest rates can thus be captured by the additional consideration of loss aversion. This model also fits the downward sloping volatility of the real yield curve in the data of US Treasury Inflation-Protection Securities (TIPS). Moreover, depending on the effective risk attitude of the representative agent with the loss averse behaviour of prospect theory, our model is capable of generating a normal or an inverted yield curve.

I. Introduction

Due to the inability of the expected utility framework to explain the behaviour of asset returns, introducing findings in behavioural finance has been recognized as a possible alternative to improve the performance of asset pricing models.[1] One of the most famous findings in behavioural finance is prospect theory, proposed first by Kahneman and Tversky (1979).[2] Some psychological experiments have been conducted to determine how people make decisions when facing different types of gambles, the results of which show that the major factor affecting people's decisions is not their wealth level after the gamble but the amount of gains or losses from the gamble. They also discovered that people are more sensitive about the losses than the gains and are more willing to take risks to avoid losses. This phenomenon is often termed loss aversion.

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[1] Other alternatives include: (1) nonexpected utilities in Weil (1989) and Epstein and Zin (1990); (2) habit formation in Abel (1999), Constantinides (1990) and Campbell and Cochrane (1999); (3) some types of market incompleteness, such as Rietz (1988), the asymmetric underlying process in Hung (1994), the transaction cost in Aiyagari and Gertler (1991) and Heaton and Lucas (1996), heterogeneous agents in Mankiw (1986), Mankiw and Zeldes (1991), Weil (1992), Lucas (1994), Constantinides and Duffie (1996), etc.

[2] Tversky and Kahneman (1992) further extended the original prospect theory to the cumulative prospect theory to solve a variety of experiment evidence inconsistent with standard expected utility theory. Afterwards many studies tried to analyse the characteristic of this theory, including Schmidt (2003), Law and Peel (2007), Schmidt and Zank (2008), Cain *et al*. (2008), etc.

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Various asset pricing models combining the feature of loss aversion have been applied to many financial and economic studies. However, as pointed out in Campbell (2000), there are some unsettled issues when loss aversion is incorporated into asset pricing models. For instance, one issue is the argument of the objective function, and another is the determination and updating of the reference point. For the argument of the objective function, since individuals derive their utilities from the consumption rather than the increase of wealth, we use the consumption-based prospect theory utility instead of the wealth-based prospect theory utility, the latter of which is commonly adopted in the previous studies.[3] In addition, it is generally believed that what people really care about is not the absolute value of their consumption level but the changes in their level of consumption. Namely, people exhibit the phenomenon of habit formation. Motivated by the similarity between habit formation and the prospect theory utility, habit formation is employed to determine and update endogenously the reference point in our model. Finally, the exponential utility-based prospect theory is adopted in our model to avoid the undesired characteristic that the first-order derivative of the power utility approaches infinity at the reference point.

The importance of combining loss aversion with the consumption-based asset pricing model is illustrated as follows. Mehra and Prescott (1985) demonstrate that the traditional consumption-based asset pricing model with a proper degree of risk aversion cannot generate a large enough equity premium. In their research, a two-state Markov process is proposed for the consumption growth rate, $g_t^{t+1} = c_{t+1}/c_t$, where c_t and g_t^{t+1} denote the consumption level at t and the consumption growth rate from t to $t+1$ respectively. Matching the data of annual consumption growth in the US from 1889 to 1978, $E[g_t^{t+1}] = 1.018$, $\mathrm{var}[g_t^{t+1}] = 0.036^2$, and the first-order serial correlation of the consumption growth, $\mathrm{corr}(g_t^{t+1}, g_{t+1}^t)$, equal to -0.14, the consumption growth for each state is $g_L = 0.982$ and $g_H = 1.054$, and the transition probabilities are

$$\prod = \begin{bmatrix} \pi_{LL} & \pi_{LH} \\ \pi_{HL} & \pi_{HH} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix}$$

Along the line, Melino and Yang (2003)[4] study the transition of the conditional marginal rate of substitution between the states g_L and g_H. Calibrated with the historical means and variances of the real annual returns of the stock index and the risk-less assets from 1889 to 1978: $E[R^S] = 1.07$, $\mathrm{var}[R^S] = 0.165^2$, $E[R^B] = 1.008$, $\mathrm{var}[R^B] = 0.056^2$, and the Euler equation $E_t[M_t^{t+1} R_{t+1}^j] = 1$ for both these assets, where M_t^{t+1} is the marginal rate of substitution, the values of the conditional marginal rates of substitution are

$$\begin{bmatrix} M_{LL} & M_{LH} \\ M_{HL} & M_{HH} \end{bmatrix} = \begin{bmatrix} 1.862 & 0.244 \\ 1.127 & 0.949 \end{bmatrix}$$

Our work is motivated by a further observation that the conditional expected marginal rate of substitution is pro-cyclical. Based on the aforementioned matrices of conditional marginal rates of substitution and transition probabilities, one can show that conditional on the recession, the expected marginal rate of substitution is $E_t[M_t^{t+1} | g_{t-1}^t = g_L] = 0.43 \times 1.862 + 0.57 \times 0.244 = 0.939$, and conditional on the boom, the expected marginal rate of substitution is $E_t[M_t^{t+1} | g_{t-1}^t = g_H] = 0.57 \times 1.127 + 0.43 \times 0.949 = 1.0546$.

The pro-cyclical conditional expected marginal rate of substitution may be attributed to loss aversion, which is elaborated by the following example. Consider a representative agent economy, and suppose the representative agent exhibits the loss averse attitude during the period of recession. To focus on the effect of loss aversion, we also assume that the representative agent is with a piecewise linear (risk-neutral) loss averse utility on consumption, of which the first-order derivative with respect to c_t is as follows:

$$u'(c_t) = \begin{cases} 1 & \text{if } g_{t-1}^t = g_H \\ \lambda_1 & \text{if } g_{t-1}^t = g_L \end{cases}$$

Based on the above utility and the two-state Markov process of the consumption growth rate in Mehra and Prescott (1985), the expectations of the marginal rates of substitution conditional on $g_{t-1}^t = g_L$ and $g_{t-1}^t = g_H$ are shown as follows:

$$E_t\left[M_t^{t+1} | g_{t-1}^t = g_L\right] = E_t\left[\frac{\delta u'(c_t+1)}{u'(c_t)} | g_{t-1}^t = g_L\right] = \frac{\delta_t(\lambda_1 \cdot 0.43 + 1 \cdot 0.57)}{\lambda_1}$$

$$E_t\left[M_t^{t+1} | g_{t-1}^t = g_H\right] = E_t\left[\frac{\delta u'(c_{t+1})}{u'(c_t)} | g_{t-1}^t = g_H\right] = \frac{\delta_t(\lambda_1 \cdot 057 + 1 \cdot 0.43)}{1}$$

[3] For example, one of the pioneer articles to apply the loss aversion of the prospect theory to solving the equity premium puzzle is Benartzi and Thaler (1995), in which they consider a wealth-based loss averse utility for investors. In addition, in a more recent research, Lien (2001) study the effect of the loss aversion of the prospect theory on the optimal futures hedge ratio, in which the investor is assumed to maximize the expected utility on his period-end wealth.
[4] Routledge and Zin (2003) report a similar exercise to that of Melino and Yang (2003).

where δ_t is the subject discount factor at t. Based on the aforementioned results, the ratio of the above two equations equals 0.939/1.0546, i.e.

$$\frac{E_t\left[M_t^{t+1}|g_{t-1}^t = g_L\right]}{E_t\left[M_t^{t+1}|g_{t-1}^t = g_H\right]} = \frac{0.939}{1.0546} = \frac{\lambda_1 \cdot 0.43 + 1 \cdot 0.57}{\lambda_1 \cdot (\lambda_1 \cdot 0.57 + 1 \cdot 0.43)}$$

Solving the above equation, the value of λ_1 is 1.1072.[5] The value of λ_1 derived from the historical data is indeed larger than 1, which is consistent with the proposition that the representative agent is with the loss averse attitude.

Note that in the recession, the conditional expected marginal rate of substitution, $E_t[M_t^{t+1}|\, g_{t-1}^t = g_L] = \delta_t(0.43 + 0.57/\lambda_1)$, declines with the increase of λ_1. On the other hand, in the boom, the conditional expected marginal rate of substitution, $E_t(M_t^{t+1}|\, g_{t-1}^t = g_H] = \delta_t(\lambda_1 \cdot 0.57 + 0.43)$, increases with the increase of λ_1. Based on the above reasoning, it is believed that if the representative agent is with a higher degree of loss aversion, i.e. if λ_1 increases, the conditional expected marginal rate of substitution is more inclined to exhibit the pro-cyclical characteristic. This finding that US consumption data exhibits the implications of the loss averse behaviour motivates us to combine the prospect theory utility and the consumption-based asset pricing model.

To explain the determinants of the shape of the term structure of interest rates, one of the most cited theories is the expectations theory. Many studies examine this theory empirically, including Campbell and Shiller (1991), Johnson (1997), Bekaert and Hodrick (2001), Carriero *et al.* (2006), Kalev and Inder (2006), Beyaert and Pérez-Castejón (2007), etc. However, this article employs a different point of view to study how the loss averse behaviour affects the shape of the term structure of real interest rates based on a consumption-based asst pricing model. Recently, different versions of consumption-based asseet pricing models are used to investigate the historical average of the term premium of risk-less assets, or even further, matching the whole spectrum of the observed term structure of interest rates.

Abel (1999) separates the equity premium into the term and the risk premium based on a consumption-based asset pricing model with habit formation and 'catching up with the Joneses'. The habit formation means that an individual's habit level depends on his past consumption, but the 'catching up with the Joneses' formulation specifies an individual's habit level depending on the history of aggregate consumption. The implied term premium between the long- and short-term risk-less assets in his model is around 226 basis points. However, the entire spectrum of the term structure of interest rates is not investigated in his study.

Brandt and Wang (2003) introduced a stochastic formulation of the relative risk averse coefficient into the consumption-based asset pricing model for deriving the entire spectrum of the term structure. The stochastic part of the relative risk aversion coefficient consists of the unexpected news about the consumption growth and inflation. Based on the monthly or quarterly data on aggregate consumption and consumer prices from January 1959 to June 1998, the implied term premium generated from their model is too small to successfully match the entire term structure for the same period.

Piazzesi and Schneider (2006) considered the role of inflation as a bad news for future consumption growth. Relying on the negative correlation between consumption growth and lagged inflation, the Epstein–Zin recursive utility is adopted in their model to produce an upward sloping yield curve. For the period of 1952:II to 2005:IV, they are able to generate an average nominal yield curve with reasonable magnitude, but the curvature of their results cannot fit that in the empirical data. In addition, the average and the volatility of the term structure of real interest rates in their model are inconsistent with those implied from the data of US Treasury Inflation-Protection Securities (TIPS).

Wachter (2004, 2006) extends the framework of the consumption-based asset pricing model with habit formation in Campbell and Cochrane (1999) to study the spectrum of the term structure. With one more variable to balance the effects of consumption smoothing and precautionary saving, her model is able to generate the bond yields with different time to maturities. For the quarterly data on inflation and consumption from 1952:I to 2004:II, the magnitude of the means and the SDs of the yield curves is close to the empirical data. However, her model implies a nearly straight-line yield curve, which is not consistent with the ones in the empirical data.

In this article, we study the term structure of real interest rates under the consideration of loss aversion of prospect theory. In our consumption-based asset pricing model, loss aversion is incorporated to capture the phenomenon of the pro-cyclical conditional expected marginal rate of substitution, and the concept of habit formation is adopted to determine the reference point for this behavioural

[5] The other solution of λ_1 is -1.014, which contradicts the assumption that the marginal utility with respect to the consumption must be positive.

finance utility. The entire spectrum of the term structure is derived, and the results show that the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion is the key factor of determining the curvature of the term structure of real interest rates.

The remainder of this article is organized as follows. In Section II, a consumption-based asset pricing model incorporating the prospect theory utility is proposed. The details of the simulation algorithm to derive the term structure are shown as well. Section III is dedicated to the results of our model, and the empirical studies for the moments of real yield curves in the US are performed in Section IV. Section V concludes this article.

II. The Loss Aversion Asset Pricing Framework

The economy

The utility of the representative agent. We assume that there exists a representative agent and one perishable consumption good in the economy, and at each time point t, he maximizes $E_t[U_t]$, where $E_t[\cdot]$ is the conditional expectation operator at time t, and U_t is defined as

$$U_t = \sum_{i=0}^{\infty} \delta_t^i u(c_{t+i}, v_{t+i}) \tag{1}$$

where c_t and v_t are the consumption level and the consumption reference point at time t, and δ_t is the subjective discount factor based on the information set at time t. The utility function $u(c_t, v_t)$ is defined as follows:

$$u(c_t, v_t) = \begin{cases} 1 - e^{-\beta(c_t - v_t)} & \text{if } c_t - v_t \geq 0 \\ \lambda_1\left[1 - e^{-\frac{\beta}{\lambda_2}(c_t - v_t)}\right] & \text{if } c_t - v_t < 0 \end{cases} \tag{2}$$

where β is the risk aversion coefficient. In this article, we follow Campbell and Cochrane (1999) and use the difference between the current consumption level and the consumption reference point as the proxy of the business cycle. Furthermore, when $c_t - v_t \geq 0$, we say the representative agent is in a good state (or in a boom). In this case, he is with the exponential risk averse utility function $1 - e^{-\beta(c_t - v_t)}$. Otherwise, when $c_t - v_t < 0$, the representative agent is in a bad state (or in a recession). Under this condition, the representative agent is assumed to exhibit the loss averse attitude, and his utility is $\lambda_1[1 - e^{-\frac{\beta}{\lambda_2}(c_t - v_t)}]$.

Different combinations of the values of λ_1 and λ_2 are able to represent different utility functions. Once the value of λ_1 is smaller than -1 and the value of λ_2 is smaller than 0, Equation 2 is the prospect theory utility. In this case, λ_1 is the loss averse coefficient, and when $c_t < v_t$, the representative agent, being in the bad state, becomes a risk lover with a negative risk averse coefficient β/λ_2. In Equation 2, if the values of λ_1 and λ_2 happen to be 1 simultaneously, the utility function $u(c_t, v_t)$ does not depend on whether $c_t < v_t$. Equation 2 becomes an exponential utility function with the habit reference point v_t. Moreover, since the consumption level of the representative agent is always larger than zero, once setting $v_t = 0$, Equation 2 becomes the classic exponential utility.

Deciding the reference point. In this article, habit formation and 'catching up with the Joneses' are adopted to determine and update the reference point v_t in the following form:

$$v_t = wc_{t-1} + (1 - w)\overline{C_{t-1}}, \quad 0 \leq w \leq 1 \tag{3}$$

In Equation 3, an additional parameter w is used to balance the two determining factors of v_t, the individual consumption level c_{t-1} and the aggregate consumption level per capita $\overline{C_{t-1}}$ at time $t-1$. When $w = 1$, the utility function displays habit formation, because the representative agent's consumption reference point, v_t, depends only on his last-period consumption. If $w = 0$, the utility function displays the phenomenon of 'catching up with the Joneses,' which indicates that the consumption reference level of the representative agent is the aggregate consumption level per capita in the last period.

In the previous works, if the consumption level were to fall unfortunately below the habit reference point, the investor's marginal utility would not always remain finite and positive. Campbell and Cochrane (1999) adopt a highly persistent, nonlinear historic-consumption habit reference level such that the consumption level is guaranteed to be higher than this reference level. On the other hand, Abel (1999) replaces the subtract-form utility with a ratio-form utility, $(c_t/v_t)^{1-\beta}/(1-\beta)$, to avoid this problem. However, by treating the boom and recession separately in the prospect theory utility, the situation $c_t - v_t < 0$ can be handled properly in our model.

The return of risk-less asset

In this section, the risk-less asset returns of different maturities are derived via the Euler equation, which states that consumers will sacrifice today's consumption level in exchange for increasing the possession of some assets, and holding the asset will bring them returns that can be transformed into consumption goods in the future, i.e.

$$E_t\left[-\frac{\partial U_t}{\partial c_t}+\delta_t^n R_{t+n}^j\left(\frac{\partial U_{t+n}}{\partial c_{t+n}}\right)\right]=0$$

where R_{t+n}^j is the real return of asset j between current time t and n year after. Rearranging the above equation, we obtain

$$E_t\left[\frac{\partial U_t}{\partial c_t}\right]=E_t\left[\delta_t^n R_{t+n}^j\left(\frac{\partial U_{t+n}}{\partial c_{t+n}}\right)\right] \tag{4}$$

Following Equations 1 and 3, $\partial U_t/\partial c_t$ is represented as follows:

$$\frac{\partial U_t}{\partial c_t}=u_{c_t}(c_t,v_t)+\delta_t u_{v_{t+1}}(c_{t+1},v_{t+1})\frac{\partial v_{t+1}}{\partial c_t} \tag{5}$$

Similarly,

$$\frac{\partial U_{t+n}}{\partial c_{t+n}}=u_{c_{t+n}}(c_{t+n},v_{t+n})+\delta_t u_{v_{t+n+1}}(c_{t+n+1},v_{t+n+1})\frac{\partial v_{t+n+1}}{\partial c_{t+n}} \tag{6}$$

Because $\partial U_t/\partial c_t$ depends not only on the derivative of $u(c_t,v_t)$ with respect to c_t but also on the derivative of $u(c_{t+1},v_{t+1})$ with respect to c_t, $\partial U_t/\partial c_t$ does not belong to the information set of time t. Therefore, different from previous studies, it is necessary to maintain the expectation of $\partial U_t/\partial c_t$ at t in Equation 4. After dividing both sides of Equation 4 by $E_t[\frac{\partial U_t}{\partial c_t}]$, we have

$$E_t\left[\delta_t^n R_{t+n}^j\left(\frac{\frac{\partial U_{t+n}}{\partial c_{t+n}}}{E_t\left[\frac{\partial U_t}{\partial c_t}\right]}\right)\right]=1$$

According to the general definition, $M_t^{t+n}=\frac{\partial U_{t+n}/\partial c_{t+n}}{E_t[\partial U_t/\partial c_t]}$ is the intertemporal marginal rate of substitution, and the above equation can be rewritten as $E_t[\delta_t^n R_{t+n}^j M_t^{t+n}]=1$. Suppose R_{t+n}^B denotes the return of a risk-less zero coupon bond which is purchased at t and with the payment of one unit of the consumption good at $t+n$. The corresponding Euler equation for R_{t+n}^B is as follows:

$$R_{t+n}^B=\frac{1}{\delta_t^n E_t\left[M_t^{t+n}\right]} \tag{7}$$

In order to derive R_{t+n}^B, we must formulate M_t^{t+n} first. From Equations 5 and 6, one may have to take $u(c_t,v_t)$, $u(c_t+1,v_t+1)$, $u(c_{t+n},v_{t+n})$, and $u(c_{t+n+1},v_{t+n+1})$ into consideration while deriving M_t^{t+n}.

Since the prospect theory utility is adopted, the utility function of each period may not be the same. For example, if $c_t-v_t>0$, the utility function is $1-e^{-\beta(c_t-v_t)}$, and if $c_t-v_t<0$, the utility of the representative agent becomes $\lambda_1[1-e^{-\frac{\beta}{\lambda_2}(c_t-v_t)}]$ due to the characteristic of loss aversion. Calculating the intertemporal marginal rate of substitution, M_t^{t+n} requires the comparisons between pairs of c_t and v_t, c_{t+1} and v_{t+1}, c_{t+n} and v_{t+n}, and c_{t+n+1} and v_{t+n+1}. To simplify the equation of M_t^{t+n}, we define the following indicator variables:

$$\begin{aligned}
p&=I_{\{c_t-v_t\geq 0\}},\quad \phi_t=1\cdot p+\frac{\lambda_1}{\lambda_2}\cdot(1-p),\\
\theta_t&=1\cdot p+\lambda_2\cdot(1-p)\\
q&=I_{\{c_{t+1}-v_{t+1}\geq 0\}},\quad \phi_{t+1}=1\cdot q+\frac{\lambda_1}{\lambda_2}\cdot(1-q),\\
\theta_{t+1}&=1\cdot q+\lambda_2\cdot(1-q)\\
r&=I_{\{c_t+n-v_{t+n}\geq 0\}},\quad \phi_{t+n}=1\cdot r+\frac{\lambda_1}{\lambda_2}\cdot(1-r),\\
\theta_{t+n}&=1\cdot r+\lambda_2\cdot(1-r)\\
s&=I_{\{c_{t+n+1}-v_{t+n+1}\geq 0\}},\quad \phi_{t+n+1}=1\cdot s+\frac{\lambda_1}{\lambda_2}\cdot(1-s),\\
\theta_{t+n+1}&=1\cdot s+\lambda_2\cdot(1-s)
\end{aligned}$$

Following the definition of M_t^{t+n} and some derivative calculations, the marginal rate of substitution M_t^{t+n} conditional on p, q, r and s is

$$M_t^{t+n}(p,q,r,s)=\frac{\phi_{t+n}e^{-(\beta/\theta_{t+n})(c_{t+n}-v_{t+n})}-\delta_t w\phi_{t+n+1}e^{-(\beta/\theta_{t+n+1})(c_{t+n+1}-v_{t+n+1})}}{\phi t e^{-(\beta/\theta_t)(c_t-v_t)}-\delta_t w E_t\left[\phi_{t+1}e^{-(\beta/\theta_{t+1})(c_{t+1}-v_{t+1})}\right]} \tag{8}$$

Similar to many consumption-based asset pricing models, we assume that the annual growth rates of both individual consumption and aggregate consumption per capita follow the same logarithmic normal distribution:

$$\ln\frac{c_{t+1}}{c_t}=\ln\frac{\overline{C_{t+1}}}{\overline{C_t}}=g_t^{t+1}=\mu_g+z_g,\quad \text{where } z_g\sim\mathcal{N}(0,\sigma_g) \tag{9}$$

In addition, Mehra and Prescott (1985) consider the first-order serial correlation of the consumption growth, denoted by $\operatorname{corr}(g_t^{t+1},g_{t-1}^t)=\rho_{gg}$, which is also taken into account in this article.

Simulation algorithm

In this article, similar to many previous studies, a simulation method is adopted to derive the rate of return with different time to maturities. In theory, at each point in time t, knowing the rates of return of assets with different time to maturities, people decide how much c_t to consume today and c_{t+n} to consume in the future by maximizing their expected utility $E_t[U_t]$ based on the reference point v_t. Meanwhile, the relation between the equilibrium rates of the return of assets and the marginal rate of substitution of consumption results from this maximization.

The simulation algorithm is formulated to mimic the above situation. However, we do not focus on how to derive the optimal consumption level c_t. Instead, we would like to derive the term structure of real interest rates when taking the historical consumption data into consideration. Rather than applying the actual historical consumption data to our model directly, the distributions of the consumption level and consumption growth are derived from the historical consumption data, and simulated samples from these distributions are used to derive the expected term structure of real interest rates.

There are two main steps in our algorithm. First, based on the given reference point v_t and the chosen consumption level c_t at time t, the conditional expected returns $R^B_{t+n}(v_t, c_t)$ for $n = 1, 2, \ldots, 30$ are derived by simulation. Second, since the information v_t and the decision c_t are different for each time point, calculating the average return over different time points is equivalent to calculating the unconditional expectations of $R^B_{t+n}(v_t, c_t)$ over v_t and c_t. Based on the distributions of v_t and c_t from the historical data, the unconditional returns R^B_{t+n} are obtained through integration over possible combinations of the values of v_t and c_t. The methods to derive the conditional and unconditional expected returns of the risk-less assets are further described as follows.

Conditional expected returns. Based on the given values of the reference point v_t and the consumption level c_t, the detail steps to derive the conditional expected returns $R^B_{t+n}(v_t, c_t)$ are the following.

First, following the assumptions in Equation 9 and the $\mathrm{corr}(g_t^{t+1}, g_{t-1}^t) = \rho_{gg}$, 60 000 sets of random samples of $(g_t^{t+1}, g_{t+1}^{t+2}, \ldots, g_{t+30}^{t+31})$ are generated,[6] and 60 000 sets of $(c_{t+1}, c_{t+2}, \ldots, c_{t+31}, v_{t+1}, v_{t+2}, \ldots, v_{t+31})$ can be derived. In fact, it can be observed from Equation 8, for each n, we need only $(c_{t+1}, c_{t+n}, c_{t+n+1}, v_{t+1}, v_{t+n}, v_{t+n+1})$.

Second, $E_t[\phi_{t+1} e^{-(\beta/\theta_{t+1})(c_{t+1}-v_{t+1})}]$ is a quantity that needs to be computed before determining the value of $M_t^{t+n}(p, q, r, s)$. In our model, across the 60 000 sampled sets, the arithmetic average of $\phi_{t+1} e^{-(\beta/\theta_{t+1})(c_{t+1}-v_{t+1})}$ is used as an approximation of this expectation. Once we have the value of $E_t[\phi_{t+1} e^{-(\beta/\theta_{t+1})(c_{t+1}-v_{t+1})}]$, for each set of $(c_{t+1}, c_{t+n}, c_{t+n+1}, v_{t+1}, v_{t+n}, v_{t+n+1})$, we can settle on the right form of $M_t^{t+n}(p, q, r, s)$ for each set depending on whether $c_{t+i} - v_{t+i}$ $(i = 0, 1, n, n+1)$ is larger than zero.

Finally, for risk-less assets, the arithmetic average of $M_t^{t+n}(p, q, r, s)$ over these 60 000 sets gives us the expectation of M_t^{t+n} conditional on v_t and c_t, and thus the corresponding values of the conditional $R^B_{t+n}(v_t, c_t)$ for $n = 1, 2, \ldots, 30$ can be derived by Equation 7.

Unconditional expected returns. In literature, the reported average term structure is calculated by taking the arithmetic average across different time points in a period of time. In our model, considering different time points is equivalent to considering different information sets the representative agent may have. Thus, we calculate the unconditional expected return by the numerical integration over all possible values of v_t and c_t. However, knowing v_t is equivalent to knowing last period's consumption data, c_{t-1} and $\overline{C_{t-1}}$. Therefore, there must be some relation between v_t and today's consumption level c_t. To avoid introducing more parameters to describe the relation between c_t and v_t, we further assume that $u_t \sim \mathcal{N}(\mu_c, \sigma_c)$ and $c_t = e^{g_{t-1}^t} v_t$ as the initial distribution of the process (v_{t+i}, c_{t+i}), where μ_c and σ_c are the mean and the SD of the annual consumption level estimated from the historical data of per capita consumption in the US, and g_{t-1}^t is the consumption growth rate from $t-1$ to t and its distribution is from Equation 9. With this assumption, the unconditional expected rates of return R^B_{t+n} can be calculated through the following equation:

$$R^B_{t+n} = \int_{v_t} \int_{g_{t-1}^t} R^B_{t+n}(v_t, e^{g_{t-1}^t} v_t) h(v_t, g_{t-1}^t) \mathrm{d}g_{t-1}^t \mathrm{d}v_t \quad \text{for } n = 1, 2, \ldots, 30$$

Based on the assumption of the independence characteristic between the last-period consumption information v_t and the consumption growth g_{t-1}^t between $t-1$ and t, the probability density function $h(v_t, g_{t-1}^t)$ is an independently bivariate normal distribution. Through the numerical integration, we are able to obtain the unconditional expected rates of return R^B_{t+n}. Finally, the annualized real risk-less yield between t and $t+n$ is computed by the following formula:

$$r^B_{t+n} = \frac{\ln R^B_{t+n}}{n} \quad (10)$$

[6] These 60 000 sets of random samples are employed to describe the possible states of nature of the world in our model. They are sampled once and used to calculate all the results under different values of the parameters. Using common random samples helps to isolate the effects of applying different values of the parameters from the effects of different realizations of the simulated samples on the interest rate term structure.

Table 1. Parameters and their values

Parameters	Value
Mean of the consumption growth, μ_g	0.0189
SD of the consumption growth, σ_g	0.015
Serial correlation between g_t^{t+1} and g_{t-1}^t, ρ_{gg}	−0.14
Mean of the annualized consumption level, μ_c (US$ 1000)	4.57336
SD of the annualized consumption level, σ_c (US$ 1000)	1.02046
Risk aversion coefficient, β	1
Subject discount factor, δ_t	0.9725
In the case of prospect theory utility	
λ_1 as the loss averse coefficient	−1.3
λ_2	−1
w (the benchmark level $v_t = wc_{t-1} + (1-w)\overline{C_{t-1}}$)	0.5
In the case of exponential utility with habit reference point	
λ_1	1
λ_2	1
w (the benchmark level $v_t = wc_{t-1} + (1-w)\overline{C_{t-1}}$)	0.5
In the case of exponential utility	
v_t	0

Notes: There are many parameters in our model and the table lists the value of each parameter. In order to demonstrate that our model's ability to solve the entire term structure is due to combining habit formation with loss aversion of prospect theory into the consumption-based asset pricing model, the values of parameters used in our model are mostly collected from existing consumption-based asset pricing models.

III. Numerical Results

In this section, the values of parameters used in our model are discussed first, followed by the results of the term structure of real interest rates for different cases of utilities. In addition, the comparisons between the generated yield curves of our model and those of the consumption-based asset pricing model in Wachter (2004, 2006) are presented. After that, the statistic analysis will be performed for each parameter of different utility cases in our model.

The parameters

Table 1 lists the parameters and the corresponding values used in our model. In order to demonstrate the superior performance of our model for solving the entire term structure, the data set and values of parameters in our model are mostly collected from Campbell and Cochrane (1999) and Mehra and Prescott (1985). First, following Campbell and Cochrane (1999), we assume the mean and the SD of the logarithmic consumption growth g_t^{t+1} to be 0.0189 and 0.015, respectively. In addition, following Mehra and Prescott (1985), the value of the parameter ρ_{gg}, which is the first-order serial correlation between g_t^{t+1} and g_{t-1}^t, is assumed to be −0.14. We also need the distribution of consumption level per capita to derive the unconditional expected rates of return. Based on the same data set used in Campbell and Cochrane (1999), the annualized consumption level per capita is assumed to follow a normal distribution, of which the mean and SD are calculated from the quarterly data of US consumption level per capita from 1959:IV to 1996:I.

Second, in our base case, the risk averse coefficient β is assumed to be 1, which represents a consumer possessing either mild degree of risk aversion or mild degree of risk loving. The discount factor δ_t is assumed to be a constant of 0.9725. Actually, the value of δ_t affects only the absolute magnitude of rates of return, but not the term premiums with different time to maturities.

Third, the primary feature of our model is to employ loss aversion of prospect theory. Hence, there are two more parameters in our model, λ_1 and λ_2, which are assumed to be −1.3 and −1 in our base case. Moreover, if we set $\lambda_1 = \lambda_2 = 1$, it represents the situation of taking only exponential utility with the habit reference point v_t into consideration, and the utility in this case is $u(c_t, v_t) = 1 - e^{-\beta(c_t - v_t)}$, and the corresponding marginal rate of substitution is

$$M_t^{t+1} = \frac{e^{-\beta(c_{t+n} - v_{t+n})} - \delta_t w e^{-\beta(c_{t+n+1} - v_{t+n+1})}}{e^{-\beta(c_t - v_t)} - \delta_t w E_t[e^{-\beta(c_{t+1} - v_{t+1})}]} \qquad (11)$$

As to the parameter w in the reference point v_t, we assume it to be 0.5, for which the utility of the representative agent features equally-weighted 'catching up with the Joneses' and habit formation. This is because different values of w do not affect the shape of the term structure very much, and by this simplified assumption of $w = 0.5$, we can concentrate on studying the effect of loss aversion on the shape of the term structure of real interest rates.

Finally, since the consumption level is always larger than zero, once we set $v_t = 0$, it is always true that $c_t \geq v_t$. The utility in Equation 2 will degenerate to the classic exponential utility, $u(c_t, v_t) = 1 - e^{-\beta c_t}$, and the marginal rate of substitution becomes

$$M_t^{t+1} = \frac{e^{-\beta c_{t+n}}}{e^{-\beta c_t}} = e^{-\beta(c_{t+n} - c_t)} \tag{12}$$

Term structures in different cases

The term structures of real interest rates for different cases in Fig. 1 are our main results. If the representative agent is with the exponential utility, the term premium between $n = 1$ and $n = 30$ is about 3.5%. This term premium seems large enough, but the magnitudes of the interest rates are too large.[7] On the other hand, in the case of the exponential utility with the habit reference point, the term premium between $n = 1$ and $n = 30$ is about 0.05%.

The underlying reason for these two cases is as follows. When n increases, the value of $c_{t+n} - c_t$ on average rises, and according to Equation 12 the expected marginal rate of substitution decreases and thus R_{t+n}^B increases based on Equation 7. As a result, r_{t+n}^B increases with n to generate an upward sloping yield curve. However, when taking habit formation into consideration, because the habit benchmark v_{t+i} keeps a close trace behind c_{t+i}, the values of $c_{t+n} - v_{t+n}$ and $c_{t+n+1} - v_{t+n+1}$ in Equation 11 increases little as n increases. Therefore, the expected marginal rate of substitution in this case decreases slightly when n increases, which results in little increase of r_{t+n}^B when n increases.[8]

However, for the exponential utility, no matter whether habit formation is considered or not, these two yield curves look like straight lines, which do not fit the empirical data. Our results suggest that in the consumption-based asset pricing model, risk aversion does generate term premium, but its marginal term premium with respect to the time to maturity is nearly constant. This finding is consistent with the results in Brandt and Wang (2003), Piazzesi and Schneider (2006) and Wachter (2004, 2006). On the other hand, the term premium between $n = 1$ and $n = 30$ in the prospect theory utility case is 1.58% (Fig. 1). Meanwhile, the highly curved term structure of this behavioural finance utility is similar to the one in the empirical data. Therefore, we can infer that in order to match the yield curve in the empirical data, which is with decreasing marginal term premium with respect to the time to maturity, it is necessary to take loss aversion into consideration.

Let us remind readers that the analysis in Introduction has demonstrated the existence of the pro-cyclical conditional expected marginal rate of substitution in the US consumption data, and this phenomenon results from the loss averse attitude of the representative agent. But how do loss aversion and the pro-cyclical conditional expected marginal rate of substitution affect the term structure?

In Equation 8, note that the way loss aversion affects the marginal rate of substitution is through ϕ_{t+i}, where $i = 0, 1, n, n+1$. The value of ϕ_{t+i} is 1 if $c_{t+i} \geq v_{t+i}$ and is λ_1/λ_2 otherwise, which means ϕ_{t+i} equals 1 with $\text{prob}(c_{t+i} \geq v_{t+i})$ and equals λ_1/λ_2 with $1 - \text{prob}(c_{t+i} \geq v_{t+i})$. Therefore, the conditional expected marginal rate of substitution in Equation 8 is higher for $\phi_t = 1(c_t \geq v_t)$ than for $\phi_t = \lambda_1/\lambda_2 = (-1.3)/(-1) = 1.3(c_t < v_t)$, while ϕ_{t+1}, ϕ_{t+n}, and ϕ_{t+n+1} have probabilities to be either 1.3 or 1. The above analysis shows that the phenomenon of the pro-cyclical conditional expected marginal rate of substitution is properly characterized in our model.

In addition, due to the consistent growing trend of the consumption process, the probability of $c_t \geq v_t$ is significantly higher than the probability of $c_t < v_t$,[9] and as a result, the net effect of the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion is an increase in the expectation of the unconditional marginal rate of substitution.

Furthermore, our model implies that this sort of increment of the unconditional expected marginal rate of substitution is almost independent of the

[7] The numerical in Section 'The effect of β in the exponential utility case' will show that if the values of the parameters are adjusted to let the interest rates back to the normal magnitude, the term premium will become much smaller.

[8] This result seems contradictory to previous studies, in which habit formation is thought to be a valid way to increase the degree of risk aversion and therefore enlarge the degree of the risk or term premium. This could be attributed to the fact that our model employs the exponential utility, whereas the previous models employ the power utility. In the power utility, when the consumption level c_{t+i} is close to the reference point v_{t+i}, the degree of risk aversion is enlarged effectively. As for the exponential utility, except the case in which the value of $c_{t+i} - v_{t+i}$ is negative and very small, the degree of risk aversion will not be enlarged effectively. In our model, however, since v_{t+i} keeps a close trace behind c_{t+i}, the value of $c_{t+i} - v_{t+i}$ is not small enough to enlarge the degree of risk aversion effectively. Therefore, the exponential utility with the habit reference point cannot generate a high enough term premium.

[9] In fact, the probability of $c_t \geq v_t$ is about 90% in our data set.

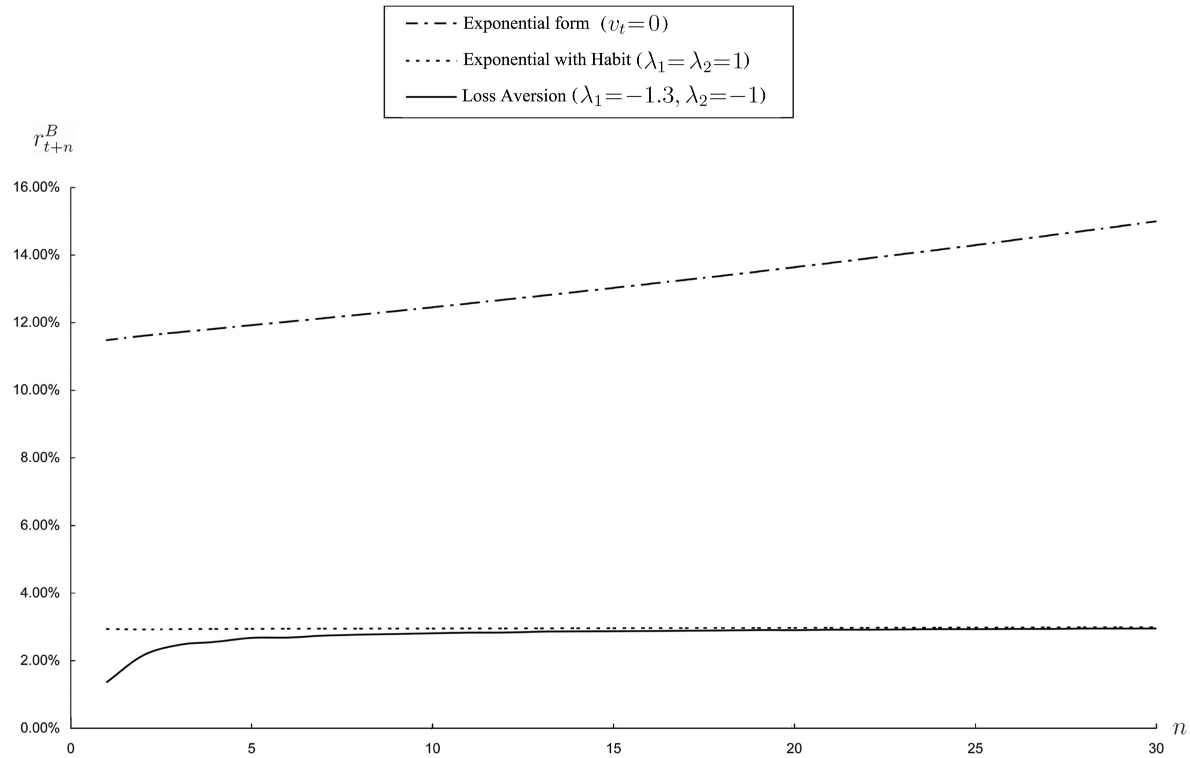

Figure 1. The term structures in different cases when $\beta = 1$

Note: This figure shows the results of term structures of different utilities. In the case of the exponential utility, regardless whether habit formation is considered, only straight-line-like yield curves can be derived. In the case of considering the prospect theory utility, because loss aversion properly formulates the pro-cyclical conditional expected marginal rate of substitution, the resulted yield curve is similar to the ones in the empirical data.

time to maturity. Because the consumption process is consistently mildly growing and v_{t+i} contains the last-period consumption information, v_{t+i} keeps a close trace behind c_{t+i} in a way that the probabilities of $c_{t+i} \geq v_{t+i}$ does not vary not much for different values of i. In consequence, the possible realized values of $(\phi_t, \phi_{t+1}, \phi_{t+n}, \phi_{t+n+1})$ are distributed similarly given different values of n, and the effect of loss aversion in Equation 8 is almost the same for different values of n. Therefore, the net effect of loss aversion to increase the unconditional expected marginal rate of substitution is almost independent of the time to maturity.

Since the increase of the unconditional expected marginal rate of substitution driven by loss aversion is almost independent of n, according to Equation 7, it is obvious that the decrease of the risk-less n-year return R^B_{t+n} caused by loss aversion is almost independent of n as well. Therefore, when we derive the annualized real risk-free zero rate r^B_{t+n} between today and n-year after. According to Equation 10, the effect of loss aversion to decrease R^B_{t+n} will be amortized among these n years. The decreasing level of r^B_{t+n} with relatively small n is more significant than that with relatively large n, that results in the highly curved term structure for the case of the prospect theory utility in Fig. 1, which is similar to the ones in the empirical data.

According to the results in Fig. 1 and the above analysis, it is believed that the term premium consists of both the effects of risk aversion and loss aversion, but the shape of the term structure of real interest rates is primarily determined by the new introduction of loss aversion. If only the risk averse coefficient is introduced into the asset pricing model, no matter whether habit formation is considered or not, we obtain nothing more than a straight-line-like term structure of real interest rates.

Comparisons with Wachter's results

In order to show that loss aversion is the dominating factor of the shape of the term structure, we compare the yield curve derived from our model with those derived from the model in Wachter (2004, 2006), in which both the consumption-based asset pricing model and habit formation are considered. Figure 2(a) illustrates the real yield curves derived from both Wachter's model and ours, and Fig. 2(b) illustrates the nominal yield curves derived from both models as well as the US historical data from 1952 to 1998 reported in Wachter (2004).[10]

In Fig. 2, the real yield curve of our model is derived based on $\beta = 1$, $\lambda_1 = -1.1072$, $\lambda_2 = -1$ and $\delta_t = 0.9725$. The values of β, λ_2 and δ_t are from our basic setting in the case of prospect theory utility, the value of λ_1 is from the analysis in Introduction. Moreover, since our model only takes the real rates of return into account, in order to perform the comparison with the nominal yield curves in the historical data, the corresponding nominal yield curve of our model is based on our real yield curve plus the difference between the nominal and real yields in Wachter's model with the same maturity.

It is clear that although Wachter follows the framework of Campbell and Cochrane (1999) to take habit formation into consideration and further formulates the inflation in the consumption-based model, both the real and nominal yield curves of her model are still straight-line-like. It should be noted that in Fig. 1, no matter whether habit formation is considered or not, the traditional consumption-based asset pricing model is able to derive only straight-line-like term structures, which are very similar to the results of Wachter's model. However, once loss aversion is nested into the consumption-based model, we can obtain a highly curved term structure, which is apparently closer to the empirical data in the United States as illustrated in Fig. 2.

The comparative statics of parameters in different cases

The effects of β, λ_1 and λ_2 in the prospect theory utility case. Figures 3–5 show how β, λ_1 and λ_2 affect the term structure of real interest rates in the case of the loss averse utility. In Fig. 3, it can be seen that when the representative agent is more risk averse (β increases), r^B_{t+n} in general increases more when the time to maturity is relatively long and less when the time to maturity is relatively short. As a result, the term premium increases as risk aversion increases. This finding is in line with the theory based on the traditional asset pricing model that risk aversion is one of the major contributions of the term premium.

In Fig. 4, the effect of the loss averse coefficient is shown. According to the analysis in the previous section, the results again show that decreasing λ_1 (strengthening the degree of loss aversion) will decrease r^B_{t+n} significantly when n is small, decrease r^B_{t+n} a little when n is large, and therefore increase the term premium.

The above analysis has shown that the loss averse coefficient affects the curvature of the average term structure significantly. However, prospect theory says more than that the marginal utility will be different for gains and losses. Prospect theory also suggests that people will become risk loving when they are facing losses. In this article, we find this behaviour will affect the term structure in a novel way.

In Fig. 5, the cases of λ_2 equalling -1, -1.3 and -3 are examined, which represent the cases of $\lambda_2 > \lambda_1$, $\lambda_2 = \lambda_1$, and $\lambda_2 < \lambda_1$, respectively. The case of $\lambda_2 = -1$ is the base case in our model and the corresponding real yield curve is the same as that of the loss averse case in Fig. 1. For the case of $\lambda_2 = -3$, an inverted yield curve is derived, and the reason is as follows. By observing that the conditional expected marginal rate of substitution in Equation 8 is smaller for $\phi_t = 1 (c_t \geq v_t)$ than for $\phi_t = \lambda_1/\lambda_2 = (-1.3)/(-3) = 0.4333\ (c_t < v_t)$, independently from the values of ϕ_{t+1}, ϕ_{t+n} and ϕ_{t+n+1}, the conditional expected marginal rate of substitution exhibits the counter-cyclical characteristic. Because the probability of $c_t \geq v_t$ is higher than the probability of $c_t < v_t$, the net effect of the counter-cyclical conditional expected marginal rate of substitution is to decrease the expectation of the unconditional marginal rate of substitution and therefore to increase R^B_{t+n}. When R^B_{t+n} is amortized to derive r^B_{t+n}, because the increase in R^B_{t+n} is almost independent of n, r^B_{t+n} will increase significantly when n is small and will increase a little when n is large. This is why an inverted yield curve is derived in the case of $\lambda_2 = -3$.

When $\lambda_2 = -1.3$, since $\lambda_1 = \lambda_2$, ϕ_{t+i} is always equals to 1 no matter the representative agent is in the boom or in the recession. As a result, the effect of loss aversion is eliminated and only risk aversion affects the term premium, so only a straight-line-like yield curve is generated.

In addition to the viewpoint of studying the conditional expected marginal rate of substitution, we provide an alternative explanation of generating inverted yield curves by analysing the nature of the prospect theory utility function. In Fig. 6, it can be found that in the case of $\lambda_2 = -1$, it is a normal prospect theory utility function, and there is a concave kink at the reference point. When a smaller (more negative) value of λ_2 is considered, e.g. $\lambda_2 = -3$, not only the representative agent becomes

[10] The reason why we do not use the results in Wachter (2006), the final publication of Wachter (2004), is that the longest maturity of the risk-less interest rates in Wachter (2006) is only 5 years, that is too short for us to compare the whole spectrum of the term structure.

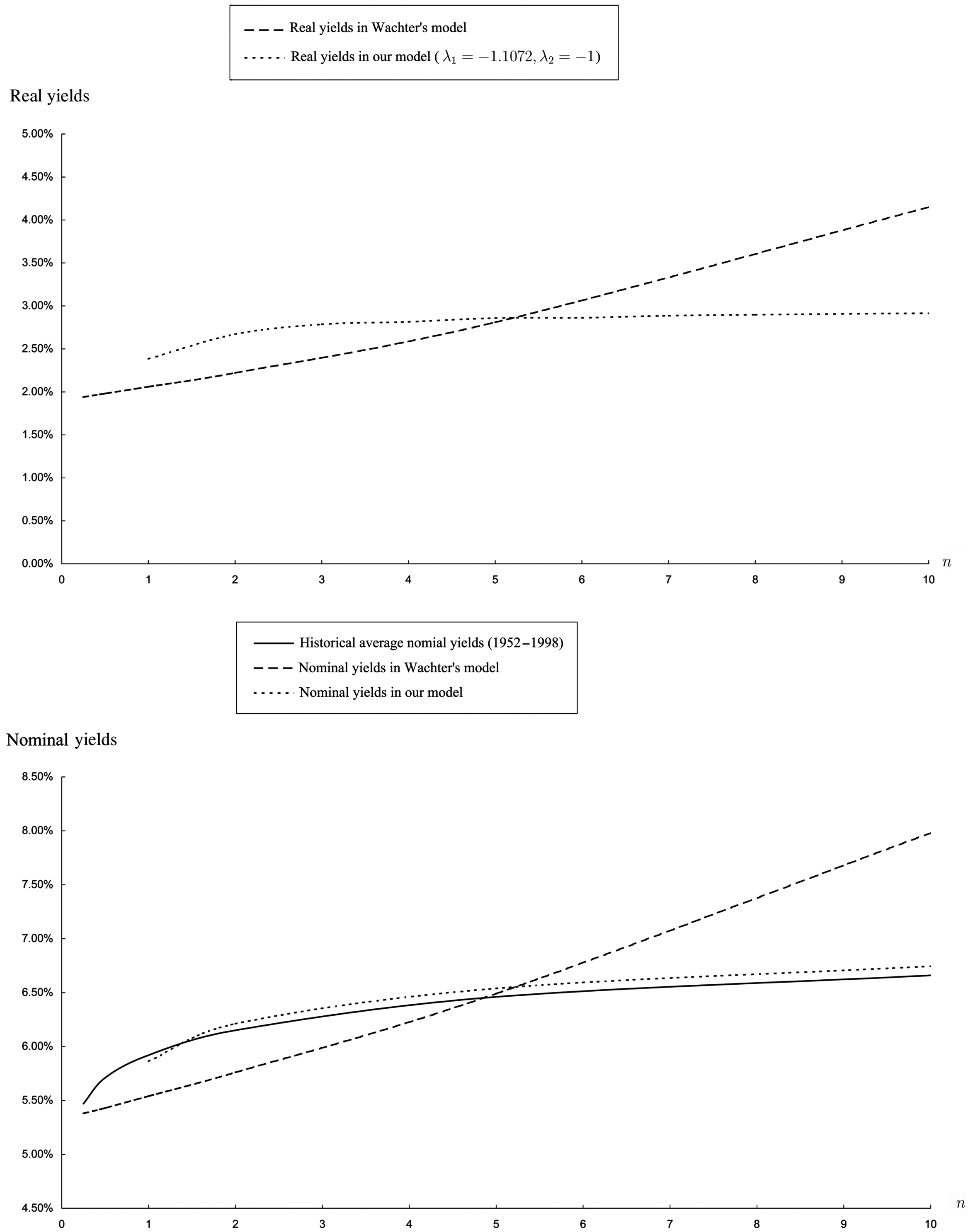

Figure 2. The comparisons between the results of Wachter's and our models
Notes: (a) Illustrates the real yield curves generated by Wachter's and our models, and (b) Shows not only the nominal yield curves derived from both models but also the historical averages on annual zero coupon yields reported in Wachter (2004). Since only real yields are considered in our model, in order to perform the comparison with the nominal yield curve in the historical data, the corresponding nominal yield curve of our model is based on our real yield curve plus the difference between the nominal and real yields in Wachter's model with the same maturity. Both Wachter's and our models are based on the consumption-based asset pricing model and take habit formation into consideration, but due to the lack of loss aversion, only straight-line-like yield curves are generated in Wachter (2004, 2006).

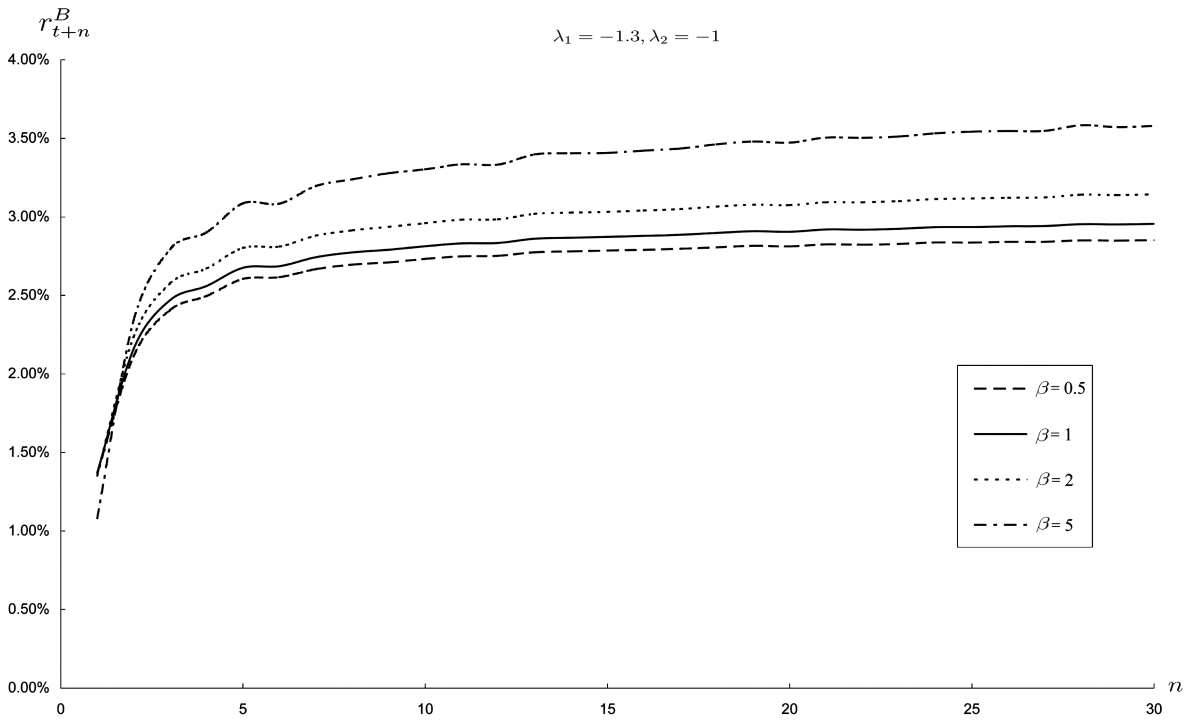

Figure 3. The effect of β on the term structure in the prospect theory utility case
Notes: As β increases, the term premium increases with the degree of risk aversion. This finding is consistent with the traditional asset pricing model in which risk aversion is the main contribution of the term premium.

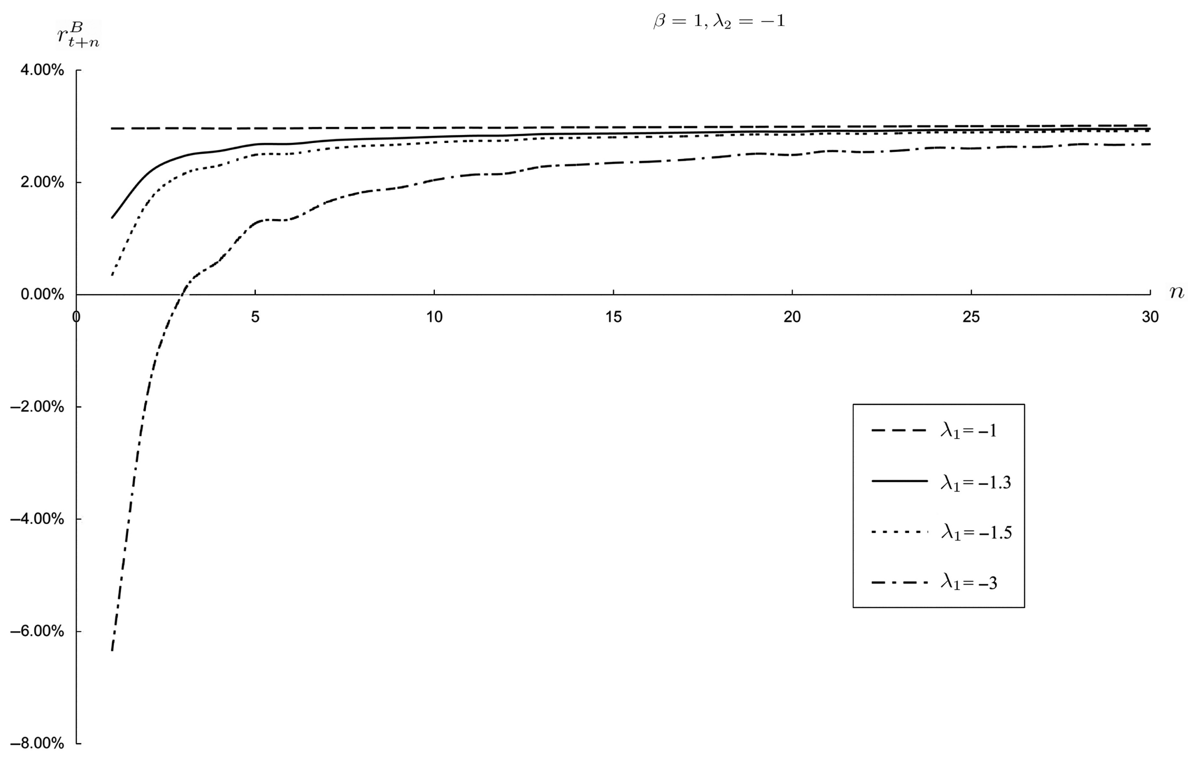

Figure 4. The effect of λ_1 on the term structure in the prospect theory utility case
Notes: When the representative agent is more loss averse (λ_1 decreases), the yield with short time to maturity decreases significantly, while the yield with long time to maturity decreases relatively little. Therefore, considering loss aversion in the consumption-based asset pricing model can generate yield curves whose shape is similar to the ones in the empirical data.

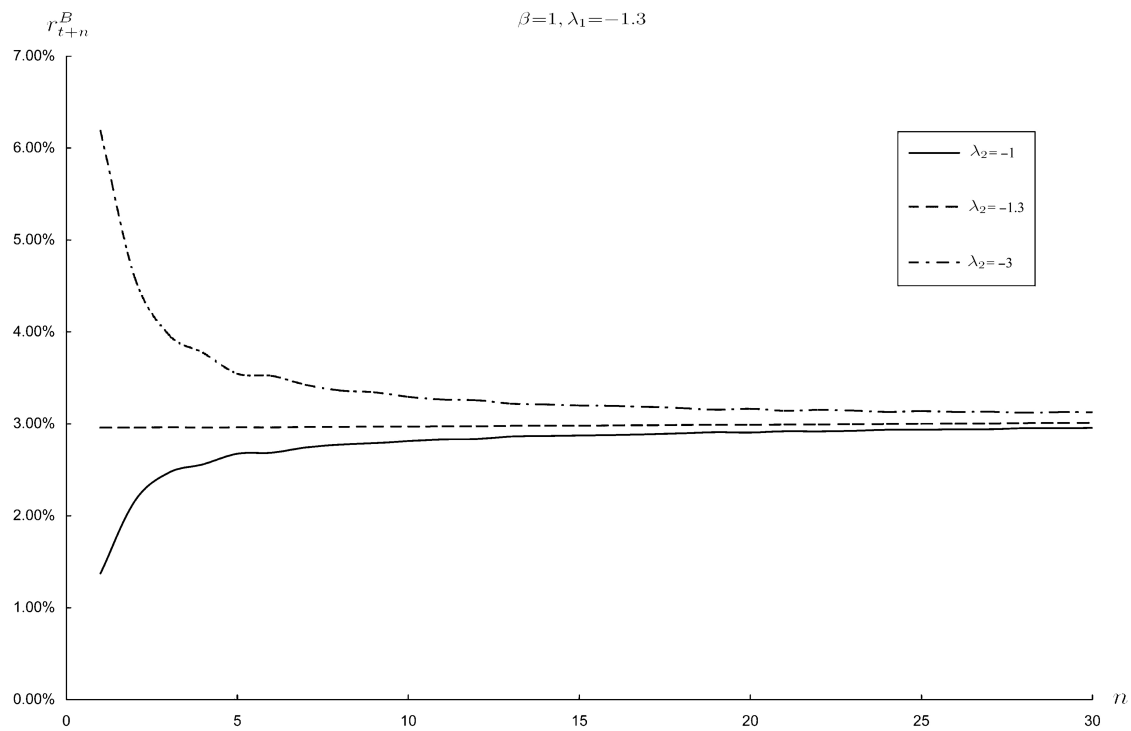

Figure 5. The effect of λ_2 on the term structure in the prospect theory utility case
Notes:For the case in which $\lambda_2 > \lambda_1$, a normal yield curve is derived based on the pro-cyclical conditional expected marginal rate of substitution. For the other case in which $\lambda_2 < \lambda_1$, an inverted yield curve is derived due to the counter-cyclical conditional expected marginal rate of substitution. When $\lambda_2 = \lambda_1$, the term premium in this case is very small because the effect of loss aversion λ_1 is offset by λ_2 and only the risk averse coefficient β is responsible for the term premium.

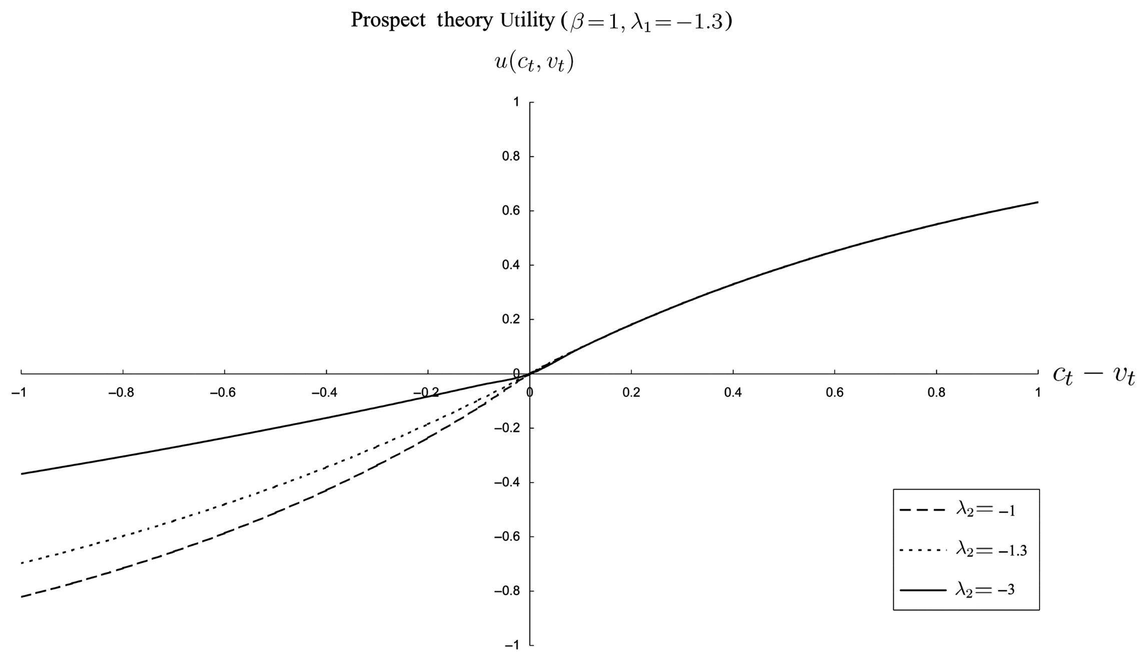

Figure 6. The prospect theory utility when given different values of λ_2
Notes: In the case of $\lambda_2 = -1$, a familiar shape of prospect theory utility is shown. When λ_2 becomes smaller, not only is the utility function less risk loving in the bad state, but also the utility function changes from concave (when $\lambda_2 = -1$) to convex (when $\lambda_2 = -3$) near the reference point. Due to introducing the consumption habit, for which v_{t+i} keeps a close trace behind c_{t+i}, the net effect of decreasing λ_2 is dominated by the convexity near the reference point, which makes the investor behaves more risk loving overall.

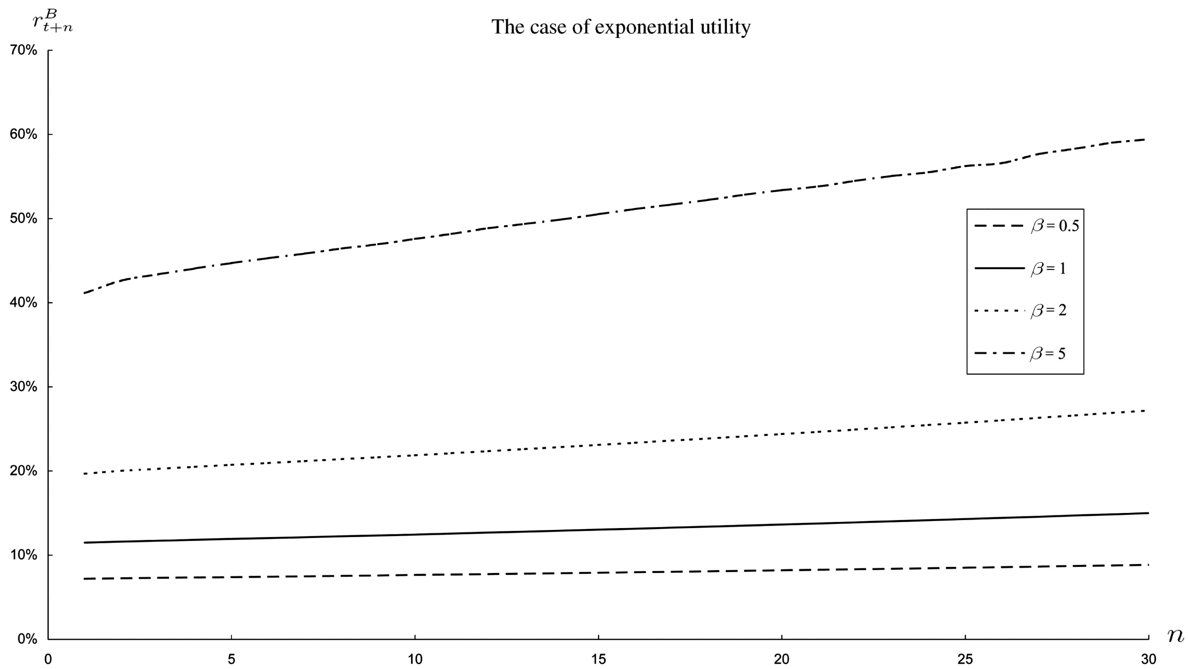

Figure 7. The effect of β on the term structure in the exponential utility case
Notes: The results show that the term premium between the long-term and the short-term risk-less assets increases as the risk averse coefficient β increases. However, the yield curves are all straight-line-like, which is not consistent with the shape of the term structure of real interest rates in the empirical data.

less risk loving in the bad state, but also a convex kink is formed at the reference point. Since v_{t+i} always keeps a close trace behind c_{t+i}, the effect of decreasing λ_2 is primarily dominated by the effect of the new-forming convex kink. So, the decrease of λ_2 effectively makes the representative agents more risk loving in our model and thus a negative term premium will be obtained for the case in which $\lambda_2 = -3$.

The effect of β in the exponential utility case. In Fig. 7, it can be seen that the term premium between the long-term and short-term risk-less assets increases as the risk averse coefficient β increases. In addition, we also find that once a sufficiently large enough term premium is generated, it is always accompanied by a very large magnitude of risk free interest rates. These results again demonstrate the existence of the risk free rate puzzle in the literature. Furthermore, the drawback of the traditional consumption-based asset pricing model for deriving the whole spectrum of the interest rate term structure still exists: the yield curves are all straight-line-like and not consistent with the shape of the term structure of real interest rates in the empirical data.

The effect of β in the exponential utility with the habit reference point case. In Fig. 8, the effect of β is shown when the habit reference point is introduced into the exponential utility asset pricing model. Because of the close-trace characteristic of habit formation, the term premium is smaller than that of the case of the exponential utility. When β is small, the derived yield curve looks like a straight line. As the risk averse coefficient increases, the shape of the yield curve seems to fit the highly curved term structure a little better, but the term premium is still too small compared to the empirical data.

IV. Empirical Studies

The purpose of this section is to study how well our model fits the moments of real yield curves of the empirical data. We adopt the real yield curves derived directly from the prices of US TIPS for comparison. The data sets of these real yield curves could be accessed on J. Huston McCulloch's website.[11] As for the corresponding consumption data, the per capita consumption of nondurables and services in the US

[11] These data sets are within a page titled 'The US Real Term Structure of Interest Rates with Implicit Inflation Premium' on his website.

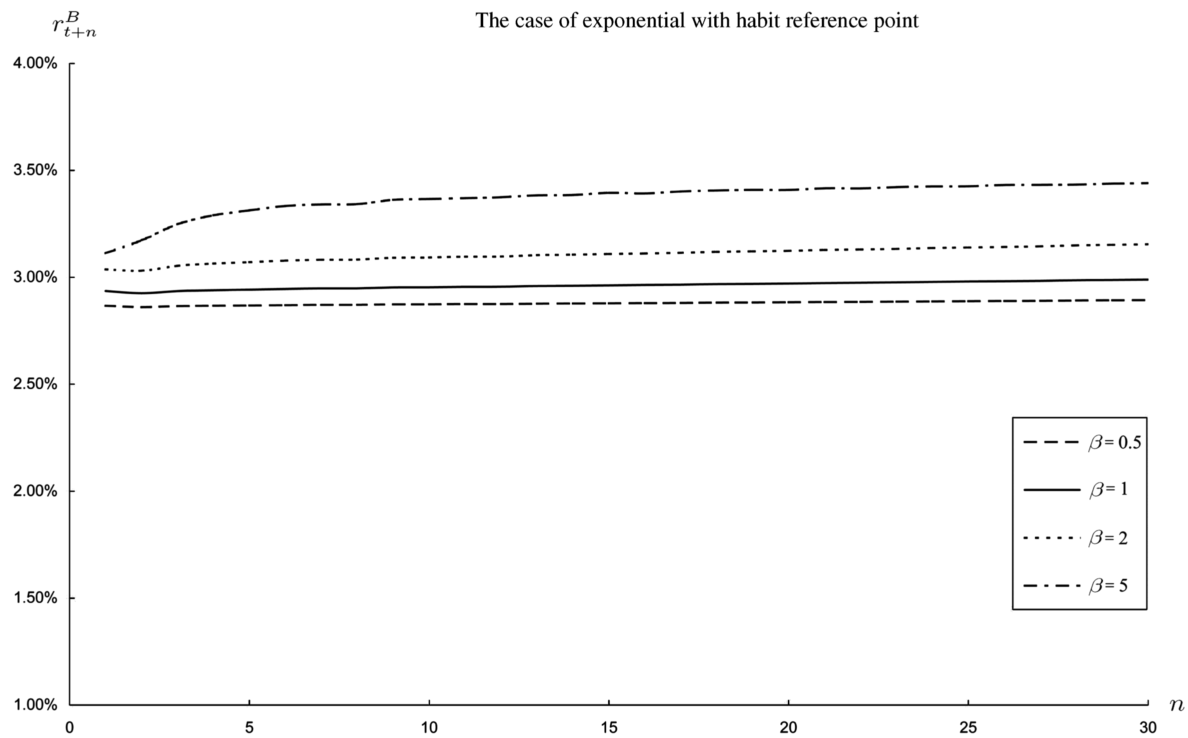

Figure 8. The effect of β on the term structure in the case of the exponential utility with the habit reference point
Notes: When taking habit formation into consideration, it can be seen that when the risk averse coefficient β increases, the shape of the yield curve becomes more similar to the ones in the empirical data, although the magnitude of the term premium is still much smaller than that in the empirical data.

on the website of the Bureau of Economic Analysis (BEA) is used. The studying period of real yield curves is from 1997:II[12] to 2006:III. In contrast with solving the annual term structure of real interest rates in Sections II and III, we will study the quarterly term structure of real interest rates here for the sake of having enough observations for c_t and v_t.

Estimation of parameters

The values of parameters used for the empirical studies are listed in Table 2. The quarterly consumption data on the website of BEA is from 1947:I to 2006:III. The value of μ_g, σ_g and ρ_{gg} regarding the consumption growth process are estimated based on this data set. In the empirical studies, only the prospect theory utility is considered, and we concentrate on the effect of the risk and loss averse attitudes that are the most important factors to determine the term structure of interest rates in our model. Thus, the values of the risk averse coefficient β and the loss averse coefficient λ_1 are derived through optimizing the fit for the means and SDs of the real yields in the historical data, whereas the values of λ_2 and w are fixed the same as those in the base case in Section III. In addition, the subject discount factor for each quarter are derived through minimizing the differences between the real yield curves from our model and from the historical data for that quarter.

Calibration process and simulation algorithm

For each pair of examined values of the risk averse coefficient β and the loss averse coefficient λ_1, we apply a similar simulation algorithm as suggested in Section 'Simulation Algorithm' to derive the quarterly real yield curves form 1997:II to 2006:III. However, in the empirical studies, the values of c_t and v_t for each time t are exactly the historical per capital consumption this quarter and previous quarter, respectively.

Based on each pair of v_t and c_t for each quarter, 60 000 sets of random samples of $(g_t^{t+1}, g_{t+1}^{t+2}, \ldots, g_{t+40}^{t+41})$ are generated, and 60 000 sets of $(c_{t+1}, c_{t+2}, \ldots, c_{t+41}, \quad v_{t+1}, v_{t+2}, \ldots, v_{t+41})$ can then be derived. Finally, determine $M_t^{t+n}(p,q,r,s)$ and take the arithmetic average of $M_t^{t+n}(p,q,r,s)$ over these 60 000 sets that gives us the expectation of M_t^{t+n} and

[12]The first issuance of the US TIPS is from 1997, so our studying period begins from that year.

Table 2. Parameters for deriving quarterly real yield curves in the empirical studies

Parameters	Value
Parameters of the quarterly per-capita consumption growth	
Mean of the quarterly consumption growth, μ_g	0.0141
SD of the quarterly consumption growth, σ_g	0.0077
Serial correlation between g_t^{t+1} and g_{t-1}^t, ρ_{gg}	0.4988
Parameters of the utility of the representative agent in our model	
β	0.9922
λ_1	−1.3516
λ_2	−1
w	0.5

Notes: The parameters of the consumption growth is based on the quarterly per capita consumption data in the US from 1997:II to 2006:III. As to the parameters of the utility of the representative agent, only the setting of the prospect theory utility is considered. Here we focus on the effects of the risk averse coefficient β and the loss averse coefficient λ_1, so their values are calibrated to minimize the RMSEs of the means and SDs of real yields from our model and historical data. As to λ_2, and w, their values inherit from the base case in Section III.

thus the conditional $R_{t+n}^B(v_t, c_t)$, given v_t and c_t. In addition, for each quarter, we repeat this step until the optimized subjective discount factor δ_t for that quarter is found by minimizing the sum of the squared errors between the $R_{t+n}^B(v_t, c_t)$ for $n = 4, 8, \ldots, 40$ and the historical real yields with the same maturity of that quarter.

As for the unconditional expected returns, it is not necessary to perform the numerical integration described in Section 'Unconditional expected returns'. We simply compute the arithmetic average of all yield curves for 38 quarters from 1997:II to 2006:III to obtain the results. Meanwhile, the SDs of the real interest rates with different maturities are also computed.

The whole process is conducted recursively until the optimized values of the risk averse coefficient β and the loss averse coefficient λ_1 are found to minimize the sum of the Root Mean Squared Errors (RMSEs) of the means and SDs of real yields from our model and from the historical data. The optimized values of β and λ_1 is 0.9922 and −1.3516, respectively.

Moments of quarterly real yield curves from 1997:II to 2006:III

Table 3 compares the first two moments of real yield curves from our model and from the quarterly[13] real yield curves on the website of McCulloch. The RMSE between average real yield curves from our model and from TIPS is only 0.20%, indicating that the magnitude of the real yield curves generated from our model is similar to that from the empirical data. In addition, the highly curved characteristic of the term structure of real interest rates is also captured by our model. As for the SDs of the real yield curves, the 0.41% of RMSE between real yield curves from our model and from TIPS indicates that our model is able to produce reasonable volatility of the real yield curve. Meanwhile the phenomenon of the downward sloping volatility of the term structure of real interest rates is also exhibited in our model.

In summary, Table 3 shows that our model can simultaneously fit both the average and the volatility of the real yield curve very well. Comparing to existing literatures about the consumption-based asset pricing model, benefiting from introducing the loss averse behaviour and the habit reference point, our model uses relatively fewer number of parameters while fitting the first two moments of the real yield curve simultaneously. In addition, to the best of our knowledge, our model is the first consumption-based asset pricing model that can capture both characteristics of the highly curved average and the downward sloping volatility of the term structure of real interest rates.

V. Conclusion

In this article, it is shown that the underlying mechanism of the pro-cyclical conditional expected marginal rate of substitution, implied from the US

[13] We take the arithmetic average of every three monthly real yield curves on the website of McCulloch to derive the quarterly real yield curves for comparisons.

Table 3. The average and volatility of the quarterly real yield curve in the US from 1997:II to 2006:III

	Mean (%)		SD (%)	
Maturity (quarters)	Real yield curve from our model	Real yield curve from TIPS	Real yield curve from our model	Real yield curve from TIPS
4	1.67	1.94	3.52	3.48
8	2.40	2.16	2.30	3.06
12	2.66	2.35	2.04	2.75
16	2.76	2.51	1.95	2.51
20	2.85	2.65	1.91	2.33
24	2.90	2.75	1.90	2.17
28	2.95	2.82	1.89	2.05
32	2.97	2.88	1.88	1.95
36	3.01	2.92	1.88	1.86
40	3.03	2.96	1.88	1.79
RMSE	0.20		0.41	

Notes: This table consists of the means and the SDs of real interest rates for different maturities derived from our prospect utility model and those from the empirical data provided by McCulloch (2006). The results of RMSEs indicate that both the means and the SDs of real yield curves generated from our model are with a reasonable magnitude. In addition, benefiting from introducing the loss averse behaviour and the habit reference point, our model fits both characteristics of the highly curved average and the downward sloping volatility of the term structure of real interest rates successfully.

consumption data, corresponds to the loss averse behaviour in prospect theory. Accordingly, we are inspired to incorporate prospect theory into the consumption-based asset pricing model. Not only the loss averse behaviour in prospect theory is applied to the traditional consumption-based asset pricing model, but also the concepts of habit formation and ‘catching up with the Joneses’ are used to decide endogenously the reference point of this behavioural finance utility.

The results of our model show that the term premium consists of both the effects of risk and loss aversions. In addition, the curvature of the term structure of real interest rates is determined primarily from the effect of the pro-cyclical conditional expected marginal rate of substitution driven by loss aversion. Our model is also capable of generating an inverted yield curve if the combination of loss aversion and the degree of risk aversion in the bad state results in the counter-cyclical conditional expected marginal rate of substitution. The analysis of the utility function in this combination shows that there is a convex kink near the reference point such that the representative agent effectively becomes risk loving and requires a negative term premium to hold the risk-less assets.

Finally, benefiting from introducing the loss averse behaviour and the habit reference point, our model uses relatively fewer number of parameters to fit simultaneously the highly curved average and the downward sloping volatility of the quarterly real yield curves for TIPS from 1997:II to 2006:III. Our results demonstrate that combining habit formation with the loss averse attitude of prospect theory into the consumption-based asset pricing model is the key factor to improve the performance of this sort of model in terms of explaining the characteristics of the term structure of real interest rates.

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