## Chapter 9

Section 9.1 (page 447)

1. f
2. a
3. b

4. 


11.

13.

15.

17.

19.

23.

21.

25.

27.

29. (a) No
(b) No
(c) Yes
(d) Yes
31. (a) No
(b) Yes
(c) Yes
(d) Yes
33.

35.

37.

39.

41.

45.

49. $y \leq 4-x$

$$
\begin{aligned}
& x \geq 0 \\
& y \geq 0
\end{aligned}
$$

53. $2 \leq x \leq 5$
$1 \leq y \leq 7$
54. $10 x-7 y \geq-46$
$3 x+5 y \leq 43$
$10 x-7 y \leq 25$
$3 x+5 y \geq-28$
55. 


47.

51. $y<2 x+1$
$x \geq 0$
$y \geq 0$
$x \leq 4$
57. $y \leq \frac{3}{2} x$
$y \leq-x+5$
$y \geq 0$
59. $x=$ amount in one account
$y=$ amount in other account
$x+y \leq 20,000$

$$
\begin{aligned}
x & \geq 5000 \\
y & \geq 5000 \\
2 x & \leq y
\end{aligned}
$$


61. $x+\frac{3}{2} y \leq 12$
$\frac{4}{3} x+\frac{3}{2} y \leq 15$
$x \geq 0$
$y \geq 0$

63. $x=$ \# of ounces of food $X$ $y=\#$ of ounces of food Y
$20 x+10 y \geq 300$
$15 x+10 y \geq 150$
$10 x+20 y \geq 200$

$$
\begin{aligned}
& x \geq 0 \\
& y \geq 0
\end{aligned}
$$


65. The solution points lie above the boundary line. Explanations will vary.
67.


## Section 9.2 (page 454)

1. (a) Minimum is 0 at $(0,0)$.

Minimum is 41 at $(3,4)$.
(a) The graph includes the boundary.
(b) The graph is shaded above the boundary.
(b) Minimum is 0 at $(0,0)$. Maximum is 20 at $(4,0)$.
3.

(a) Minimum is 0 at $(0,0)$.

Maximum is 740 at $(60,20)$.
(b) Minimum is 0 at $(0,0)$.

Maximum is 2100 at any point on the line segment between $(30,45)$ and $(60,20)$.
5.

7.

(a) Minimum is 0 at $(0,0)$. Maximum is 21 at $(4,1)$.
(b) Minimum is -3 at $(0,3)$. Maximum is 10 at $(5,0)$.
(c) Minimum is -25 at $(5,0)$. Maximum is 3 at $(0,3)$.
(a) Minimum is 0 at $(0,0)$. Maximum is 12 at $(12,0)$.
(b) Minimum is 0 at $(0,0)$. Maximum is 20 at $(0,20)$.
(c) Minimum is 0 at $(0,0)$. Maximum is 22 at $(6,16)$.
9. Minimum is 0 at $(0,0)$.

Maximum is 12 at $(3,6)$.
13. Minimum is 0 at $(0,0)$.

Maximum is 50 at $(0,10)$.
15.

17.

19.

21. 100 units of $\$ 80$ model 100 units of $\$ 100$ model Maximum profit: $\$ 5500$
25. 12 audits, 0 tax returns
11. Minimum is 0 at $(0,0)$.

Maximum is 10 at $(0,10)$.
$z$ is maximum at any point on the line segment between points $(2,0)$ and $\left(\frac{20}{19}, \frac{45}{19}\right)$.

The constraint $x \leq 10$ is extraneous.

The feasible set is empty, no maximum.
23. 3 bags of brand $X$ 6 bags of brand $Y$ Minimum cost: $\$ 240$
27. (a) $t \geq 6$
(b) $2.4 \leq t \leq 6$
(c) $t \leq 2.4$
(d) Not possible
29. Answers will vary. Sample answer: $z=x+5 y$
31. Answers will vary. Sample answer: $z=4 x+y$

## Section 9.3 (page 468)

1. Objective function should be maximized, not minimized.
2. All constraints must be $\leq$.
3. 

| $x_{1}$ | $x_{2}$ | $s_{1}$ | $s_{2}$ | $b$ | Basic <br> Variables |
| ---: | ---: | ---: | ---: | ---: | :---: |
| 2 | 1 | 1 | 0 | 8 | $s_{1}$ |
| 1 | 1 | 0 | 1 | 5 | $s_{2}$ |
| -1 | -2 | 0 | 0 | 0 |  |

7. 

| $x_{1}$ | $x_{2}$ | $x_{3}$ | $s_{1}$ | $s_{2}$ | $b$ | Basic <br> Variables |
| ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 2 | 0 | 1 | 0 | 12 | $s_{1}$ |
| 1 | 0 | 1 | 0 | 1 | 8 | $s_{2}$ |
| -2 | -3 | -4 | 0 | 0 | 0 |  |

9. $(8,0,112,0,4)$
10. Maximum is 8 at $(8,0)$.
11. Maximum is 17 at $(3,4)$.
12. Maximum is 740 at $(60,20)$.
13. Maximum is 43 at $(7,3)$.
14. Maximum is 210 at $(0,21,21)$.
15. Maximum is 25 at $(23,0,2)$ or $\left(\frac{43}{3}, 0, \frac{32}{3}\right)$.
16. Maximum is 24 at $(0,12,0,0)$.
17. Maximum is 2640 at $(105,150,70)$.
18. 8 audits, 8 tax returns
19. $\frac{5000}{3}$ liters of the the first drink $\frac{2500}{3}$ liters of the second drink Maximum profit: about $\$ 1416.67$
20. 322 of model A 764 of model B 484 of model C Maximum profit: $\$ 79,310$
21. 50 acres of crop $X$ 0 acres of crop Y and crop Z Maximum profit: $\$ 3000$
22. $t \geq 5 / 2$
23. After one iteration, the simplex tableau is as follows.

Basic

| $x_{1}$ | $x_{2}$ | $s_{1}$ | $s_{2}$ | $b$ | Variables |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $-\frac{1}{2}$ | 0 | 1 | $\frac{3}{2}$ | 7 | $s_{1}$ |
| $-\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ | 2 | $x_{2}$ |
| -2 | 0 | 0 | 1 | 4 |  |

39. After one iteration, $x_{1}=2, x_{2}=0$, and $z=5$. Bringing $x_{2}$ into the basis after another iteration, $x_{1}=\frac{20}{19}, x_{2}=\frac{45}{19}$, and $z$ still equals 5 .
40. Maximum is about 480.8 at $(0,5.16,53.20,31.37)$.
41. Maximum is about 346.88 at $(14.78,0,60.51,0)$.
42. False. The entering variable corresponds to the most negative entry.

## Section 9.4 (page 478)

1. (Maximize)

Objective function:
$z=6 y_{1}+6 y_{2}$
Constraints:

$$
\begin{array}{r}
2 y_{1}+y_{2} \leq 2 \\
y_{1}+2 y_{2} \leq 2 \\
y_{1}, y_{2} \geq 0
\end{array}
$$

3. (Maximize)

Objective function:
$z=5 y_{1}+8 y_{2}+6 y_{3}$
Constraints:
$y_{1}+2 y_{2}+2 y_{3} \leq 9$
$2 y_{1}+2 y_{2}+y_{3} \leq 6$
$y_{1}, y_{2}, y_{3} \geq 0$
5. (Maximize)

Objective function: $z=7 y_{1}+4 y_{2}$
Constraints:
$y_{1}+y_{2} \leq 14$
$y_{1}+2 y_{2} \leq 20$
$2 y_{1}+y_{2} \leq 24$ $y_{1}, y_{2} \geq 0$
7. (a) Minimum is 6 at $(1,1)$
(b) (Maximize) Objective function: $z=3 y_{1}+5 y_{2}$
Constraints:

$$
\begin{array}{r}
y_{1}+3 y_{2} \leq 3 \\
2 y_{1}+2 y_{2} \leq 3 \\
y_{1}, y_{2} \geq 0
\end{array}
$$

(c) Maximum is 6 at $\left(\frac{3}{4}, \frac{3}{4}\right)$.
9. (a) Minimum is 13 at $(1,1)$.
(b) (Maximize) Objective function:

$$
z=3 y_{1}+5 y_{2}
$$

Constraints:

$$
\begin{array}{r}
y_{1}+3 y_{2} \leq 5 \\
2 y_{1}+2 y_{2} \leq 8 \\
y_{1}, y_{2} \geq 0
\end{array}
$$

(c) Maximum is 13 at $\left(\frac{7}{2}, \frac{1}{2}\right)$.
11. (a) Minimum is 8 at $\left(\frac{4}{3}, \frac{5}{3}\right)$.
(b) (Maximize)

Objective function: $z=3 y_{1}+2 y_{2}$
Constraints:

$$
\begin{aligned}
y_{1}-y_{2} & \leq 1 \\
y_{1}+2 y_{2} & \leq 4 \\
y_{1}, y_{2} & \geq 0
\end{aligned}
$$

(c) Maximum is 8 at $(2,1)$.
13. (a) Minimum is 9 at $\left(\frac{1}{2}, 2\right)$.
(b) (Maximum) Objective function: $z=4 y_{1}+2 y_{2}$ Constraints:

$$
\begin{aligned}
4 y_{1} & \leq 6 \\
y_{1}+y_{2} & \leq 3 \\
y_{1,} y_{2} & \geq 0
\end{aligned}
$$

(c) Maximum is 9 at $\left(\frac{3}{2}, \frac{3}{2}\right)$.
15. Minimum is $\frac{9}{5}$ at $\left(1, \frac{9}{5}\right)$. 17. Minimum is 8 at $(0,8)$.
19. Minimum is 5 at $(5,0)$.
21. Minimum is 18 at $\left(\frac{1}{5}, 2, \frac{7}{5}\right)$.
23. Minimum is 64 at $\left(\frac{4}{3}, 4,16\right)$.
25. 22.5 days for plant 1
10.5 days for plant 2
4.75 days for plant 3

Minimum operating cost: $\$ 2,152,500$
27. Answers will vary.
29. 1 liter of drink $A$

1 liter of drink B
Minimum cost: \$5
31. 3 liters of drink $A$

0 liters of drink B
Minimum cost: \$3
33. Minimum is 87.14 at $(21.43,2.86,25.71,0)$.

## Section 9.5 (page 488)

1. 

| $x_{1}$ | $x_{2}$ | $s_{1}$ | $s_{2}$ | $b$ | Variables |
| ---: | ---: | ---: | ---: | ---: | :---: |
| 2 | 1 | -1 | 0 | 4 | $s_{1}$ |
| 1 | 1 | 0 | 1 | 8 | $s_{2}$ |
| -10 | -4 | 0 | 0 | 0 |  |

3. 

| $x_{1}$ | $x_{2}$ | $s_{1}$ | $s_{2}$ | $b$ | Basic <br> Variables |
| ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| 2 | 1 | 1 | 0 | 4 | $s_{1}$ |
| 1 | 3 | 0 | -1 | 2 | $s_{2}$ |
| 1 | 1 | 0 | 0 | 0 |  |

5. 


7.

| $x_{1}$ | $x_{2}$ | $x_{3}$ | $s_{1}$ | $s_{2}$ | $b$ | Basic <br> Variables |
| ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| 5 | 4 | 5 | -1 | 0 | 12 | $s_{1}$ |
| 1 | 6 | 0 | 0 | 1 | 5 | $s_{2}$ |
| 4 | 2 | 1 | 0 | 0 | 0 |  |

9. Maximum is 12 at $(0,6)$.
10. Minimum is 16 at any point on the line segment between $(0,8)$ and $(2,7)$.
11. Maximum is 25 at $(5,20,0)$.
12. Maximum is 12 at $(0,6)$. The solution is the same.
13. Maximum is 16 at $(2,7)$. The value of $w$ is the same, but the point $\left(x_{1}, x_{2}\right)$ is different. (Note: Any point on the line segment between $(0,6)$ and $(2,7)$ is an optimal solution.)
14. Maximum is 25 at $(5,20,0)$. The solution is the same.
15. Maximum is 40 at $(0,8)$.
16. Maximum is 108 at $(0,36)$.
17. Minimum is 15 at $(5,10)$.
18. Maximum is 30 at $(5,20,0)$.
19. Minimum is -20 at $\left(11, \frac{1}{2}, 0\right)$. (Note: Any point on the line segment between $\left(11, \frac{1}{2}, 0\right)$ and $(10,0,0)$ is an optimal solution.)
20. Maximum is 9 at $(4,1)$.
21. Maximum is 4 at $(1,4)$.
22. Maximum is 6 at $(0,3)$.
23. Maximum is 24 at $(1,4)$.
24. Maximum is 0 at $(2,0)$.
25. Maximum is 9 at $(4,5)$.
26. Maximum is 3 at $(0,3)$.
27. Maximum is -4 at $(2,0)$.
28. 300 tires from $S_{1}$ to $C_{1}$ 600 tires from $S_{1}$ to $C_{2}$ 200 tires from $S_{2}$ to $C_{1}$
29. 600 tires from $S_{1}$ to $C_{2}$ 500 tires from $S_{2}$ to $C_{1}$ Minimum cost: $\$ 1100$
Minimum cost $\$ 1100$
30. (a)

|  | Outlet I | Outlet II |
| :---: | ---: | :---: |
| Plant $A$ | $a$ | $5000-a$ |
| Plant $B$ | $3000-a$ | $a$ |

(b) Minimum cost: $\$ 40,000$
53. (a)

|  | Customer 1 | Customer 2 |
| :--- | :--- | :--- |
| Factory 1 | 0 | 200 |
| Factory 2 | 200 | 100 |

(b) Minimum cost: $\$ 14,500$
55. 9 television ads

4 newspaper ads
Maximum audience: 147,000,000
57. Feasible solution; $s_{1}$ and $s_{2}$ are positive.
59. Feasible solution; $s_{1}$ and $s_{2}$ are positive.
61. Not a feasible solution; $s_{3}$ is negative.
63. True. See paragraph before Example 2.

Review Exercises (page 491)
1.


5.

7. Minimum is 0 at $(0,0)$.

Maximum is 47 at $(5,8)$.
9. Minimum is 0 at $(0,0)$.

Maximum is 20 at $(5,0)$.
11. Minimum is 0 at $(0,0)$.

Maximum is 2100 at $y=-\frac{5}{6} x+70$ where $30 \leq x \leq 60$.
13. Minimum is 0 at $(0,0)$.
15. Minimum is 3 at $(3,0)$.

Maximum is 125 at $(25,0)$.
Maximum is 11 at $(5,2)$.
17. Minimum is -6 at $(0,6)$.
19. Maximum is 26 at $(12,7)$.

Maximum is $\frac{64}{3}$ at $\left(8, \frac{8}{3}\right)$.
21. Maximum is 20 at $\left(0, \frac{48}{5}, \frac{4}{5}\right)$. (Note: Any point on the line segment between $\left(0, \frac{48}{5}, \frac{4}{5}\right)$ and $(0,0,20)$ is an optimal solution.)
23. Maximum is 232 at $(100,132)$.
25. Maximum is 3599 at ( $110,537,146$ ).
27. (Maximize)

Objective function: $z=30 y_{1}+75 y_{2}$
Constraints:

$$
\begin{aligned}
y_{1}+3 y_{2} & \leq 7 \\
y_{1}+6 y_{2} & \leq 3 \\
2 y_{1}+4 y_{2} & \leq 1 \\
y_{1}, y_{2} & \geq 0
\end{aligned}
$$

29. Minimum is 75 at $(5,2)$.
30. Minimum is 6006 at $\left(\frac{81}{2}, 138,111\right)$.
31. Minimum is $\frac{118}{3}$ at $\left(\frac{4}{3}, \frac{1}{3}, 0\right)$. 35. Maximum is 31 at $(1,5)$.
32. Maximum is 67 at $(7,27,26)$.
33. Minimum is 90 at $(10,0,0)$.
34. $x+y \leq 1500, x \geq 400, y \geq 600$

35. (a) 2 vests, 5 purses (b) $\$ 500$
36. $\frac{5}{3}$ liters of dietary drink I
37. 5 liters of dietary drink I $\frac{4}{3}$ liters of dietary drink II 0 liters of dietary drink II Minimum cost: \$19

Minimum cost: \$5
49. 3 bags of Brand $X$ 6 bags of Brand Y Minimum cost: \$345
51. Minimum cost is $\$ 4800$ when mines $A, B$, and $C$ are operated for 7, 5, and 24 days, respectively. (Note: Any point in the triangular region bounded by $(7,5,24),(12,0,24)$, and $(7,0,34)$, is an optimal solution.)

