Chapter 5

Market-Risk Measurement

- Five common approaches to measure the market risk
  - Sensitivity analysis
  - Stress testing
  - Scenario testing
  - Capital Asset Pricing Model (CAPM)
  - Value at Risk

Sensitivity Analysis

$$\text{Sensitivity} = \frac{V(r+\varepsilon)-V(r)}{\varepsilon}$$

$$\text{When} \quad \varepsilon \to 0 \Rightarrow \frac{\partial V}{\partial r}$$

* The main risk factors (*r*) include interest rates, credit spreads, equity prices, exchange rates, implied volatility, commodity prices, forward prices, etc.

* Sensitivity analysis is a description of how much the portfolio's value (*V*) is expected to change if there is a small change in one of the market-risk factors

* Sensitivity also called the relative change, the first derivative, or the best linear approximation

Sensitivity Analysis

- For bonds

$$V = \sum \frac{C_t}{(1+r)^t}$$

$$\frac{\partial V}{\partial r} = \sum \frac{-tC_t}{(1+r)^{t+1}} = -\text{Duration\$}$$

$$\delta V = \frac{\partial V}{\partial r} \delta r$$

* The (-Duration$ $\times$ 0.0001) is also called PVBP or DV01. PVBP is the present value of a basis-point change in interest rates, and DV01 is the delta value for a 1-basis point change.

$$\delta r = 1 \text{ bp} = 0.01\% = 0.0001 \Rightarrow \text{PVBP} = \text{DV01} = -\text{Duration\$} \times 0.0001$$

$$= \frac{\partial V}{\partial r} \times 0.0001$$

Sensitivity Analysis

- The credit spread is taken into consideration

if $\quad r = r_f + S$

$$\frac{\partial V}{\partial r_f} = \sum \frac{-tC_t}{\left(1 + r_f + S\right)^{t+1}} = -\text{Duration\$}$$

$$\frac{\partial V}{\partial S} = \sum \frac{-tC_t}{\left(1 + r_f + S\right)^{t+1}} = -\text{Duration\$}$$

$$\delta V = \frac{\partial V}{\partial r_f}\delta r_f + \frac{\partial V}{\partial S}\delta S$$

Sensitivity Analysis

- The composition of the credit spread:

$$e^{-R_t} = (1-h_t)\cdot e^{-r_t} + h_t e^{-r_t}(1-L_t)$$

$$1-R_t = (1-h_t)\cdot(1-r_t) + h_t(1-r_t)(1-L_t)$$

$$1-R_t = 1-h_t - r_t + h_t r_t + h_t - h_t r_t - h_t L_t + h_t r_t L_t$$

$$1-R_t = 1-r_t - h_t L_t + h_t r_t L_t$$

$$R_t \approx r_t + h_t L_t$$

h_t : hazard rate

L_t : loss rate

Sensitivity Analysis

- For equities

$$V = N \times S$$

$$\frac{\partial V}{\partial S} = N \;\Rightarrow\; \delta V = N\delta S$$

$$S = S_0\left[1 + \beta m + \varepsilon\right] \;, \quad \text{where} \;\; m = \frac{M - M_0}{M_0} \;(\text{指數的變動率})$$

$$\frac{\partial S}{\partial m} = S_0 \beta$$

- For an equity portfolio

$$\frac{\partial V_P}{\partial m} = \sum_{k=1}^{P} \frac{\partial V_P}{\partial S_k}\frac{\partial S_k}{\partial m} = \sum_{k=1}^{P} N_k S_{k,0}\beta_k$$

$$\delta V = \frac{\partial V_P}{\partial m} \times \delta m$$

Sensitivity Analysis

- For foreign exchange (the same as equities)

$$V = X \cdot C$$

X : exchange rate

C : currency

Sensitivity Analysis

- For forward and futures

$$V = N\frac{D_C - D_0}{\left(1+r_f\right)^t}, \quad \frac{\partial V}{\partial D_C} = \frac{N}{\left(1+r_f\right)^t}, \quad \frac{\partial V}{\partial r_f} = \frac{-tN}{\left(1+r_f\right)^{t+1}}\left(D_C - D_0\right)$$

$$\delta V = \frac{\partial V}{\partial D_C}\delta D_C + \frac{\partial V}{\partial r_f}\delta r_f$$

D_C : current forward or futures price

D_0 : original delivery price

Sensitivity Analysis

- For options

$$\Delta = \frac{\partial C}{\partial S} \ (\text{Delta}), \ \Gamma = \frac{\partial^2 C}{\partial S^2} \ (\text{Gamma})$$

$$\nu = \frac{\partial C}{\partial \sigma} \ (\text{Vega}), \ \rho = \frac{\partial C}{\partial r} \ (\text{Rho})$$

$$\theta = \frac{\partial C}{\partial T} \ (\text{Theta})$$

$$\delta C = \Delta \cdot \delta S + \frac{1}{2}\Gamma \cdot \delta S^2 + \nu \cdot \delta\sigma + \rho \cdot \delta r + \theta \cdot \delta T$$

Stress Testing

- If the change in a risk factor is large (e.g., in a crisis), the linear sensitivity will not give a good estimate to the change in the value of a portfolio

- Steps of the stress testing
  1. 決定主要影響之risk factors
  2. 相關高的risk factors合起來看，一方面減少risk factor之數目，一方面確保risk factor間是independent movement
  3. 4σ or 6σ of daily movements for each risk factor
  4. 將要測試的各組risk factors的值，代入pricing models，重算一次被這些risk factors影響的商品之價值
  5. 回報投資組合的價值變化 (p. 94 Fig. 5-1 and 5-2)

Stress Testing

- 三個缺點
  1. 不知那個factor影響力最大
  2. Arbitrarily choose 4 σ or 6 σ 不好，因為此選擇跟機率沒關係
  3. 隱含了假設各factor之間的correlation不是0 即是 1，例：歐元與英鎊，都當作risk factor，則其間之相關係數假設為0，若想成一個risk factor，則其間之相關係數被假設為1

Scenario Testing

- Scenario testing and stress testing are similar in that both use specified changes in the market-risk factors and reprice the portfolio with full, nonlinear pricing models
- In scenario testing, the changes are tailored and subjectively chosen. For example, each scenario corresponds to a specific type of market crisis, such as U.K. equities market crashes, a default by China, or raising of oil prices by OPEC.
- How to choose scenario?
  - previous crises
  - the bank's current portfolio
  - the opinion of the bank's experts
    - head trader
    - bank economists
    - risk management group

Scenario Testing

- Steps of the scenario testing
  1. Choose 5 ~ 10 scenarios
  2. Estimate the changes in each risk factor based on the crisis scenarios you have identified
  3. Value portfolio under the given scenario
  4. Test the portfolio each day to see how much would be lost under each scenario
  5. Update the set of the scenarios quarterly or more often

Scenario Testing

- 四個缺點
  1. The process is time-consuming
  2. 只能考慮有限數量的 scenarios
  3. 在某些人為假設的scenarios中，risk factor 之值的選取非常主觀
  4. 通常提出最壞scenario的人通常會take risk and make the trade

  * 之前發生的情況，未來未必會再發生，但是若永遠考慮之前的crisis scenario，雖然可以確保若再一次發生這種情況時，銀行不會倒，但同時也限制的銀行可以承擔的風險，並減少了銀行的獲利

- Capital asset pricing model (CAPM):

  Assume in an efficient market, an investor can diversify the portfolio that removes all the risks except the systemic risk.

$$r_a = r_f + \beta\left(r_m - r_f\right) + \varepsilon$$

$$\beta = \frac{\operatorname{cov}(r_a, r_m)}{\sigma_m^2} = \frac{\rho_{a,m}\sigma_a}{\sigma_m}$$

$\varepsilon \sim N(0,\ \sigma^2)$ (σ measures the idiosyncratic risk)

- Sharpe Ratio

$$S = \frac{r_P - r_f}{\sigma_{(r_P - r_f)}}$$

- Treynor Ratio

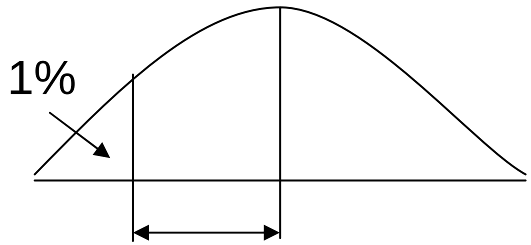
$$T = \frac{r_P - r_f}{\beta}$$

Value at Risk

- VaR is a measure of market risk that tries objectively to combine the sensitivity of the portfolio to market changes and the probability of a given market change
- VaR is the best single risk-measurement technique available now
- However, VaR has some limitations that always require the continued use of stress and scenario tests as a backup

Value at Risk

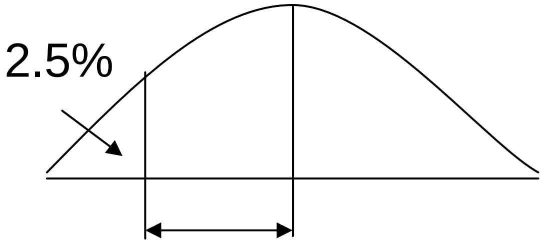
- The basic concept of VaR



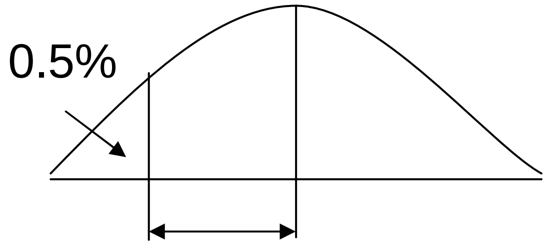
$\text{VaR}=2.32\cdot\sigma \Longrightarrow$ 只有1%之機率，最大損失會超過VaR

- VaR has been adopted by the Basel Committee to set the standard for the minimum amount of capital to be held against market risks

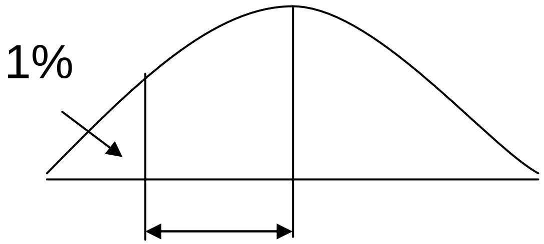
Value at Risk



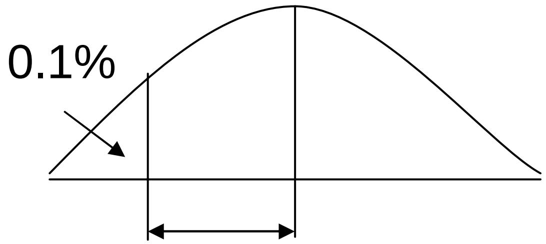
VaR=1.96$\cdot\sigma$



VaR=2.57$\cdot\sigma$



VaR=2.32$\cdot\sigma$



VaR=3.09$\cdot\sigma$

Value at Risk

- For bonds

$$\text{VaR} = -\frac{\partial V}{\partial r} \times \delta r_{\text{worst case}}$$

$$-\frac{\partial V}{\partial r} : \text{Duration Dollars (Duration \$)}$$

$$\delta r_{\text{worst case}} = 2.32\sigma_r$$

Value at Risk

- For equities

$$\text{VaR} = \underbrace{2.32\sigma_E}_{} \times N$$

只有1%，equity price loss會超過$2.32\sigma_E$

Value at Risk

- For call options

1. $\text{VaR} = \left| -2.32 \cdot \sigma_S \cdot \Delta \right|$

$$= \left| -2.32 \cdot \sigma_S \cdot \Delta + \frac{1}{2}\Gamma\left(-2.32 \times \sigma_S\right)^2 \right|$$

2. $\text{VaR} = C\left(S\right) - C\left(S - 2.32 \times \sigma_S\right)$ (the most reliable way)

VaR over multiple days

- If VaR is used without a specified time, it means one-day VaR (DEaR: daily earnings at risk)
- Under the following assumptions, the multiple-day VaR is

$$VaR_T = VaR_1 \sqrt{T}$$

1. Changes in market factors are normally distributed
2. One-day VaR is constant over the time period
3. There is no serial correlation (no one day is dependent of the results on previous days)

Limitation of VaR

- VaR only describe what happen on bad days (e.g., twice a year) rather than terrible days (e.g., once every 10 years)
- VaR is good for avoiding bad days, but to avoiding terrible days, stress and scenario tests are needed