The Greek Letters

Example

- A bank has sold for \$300,000 a European call option on 100,000 shares of a nondividend paying stock
- $S_0 = 49$, $K = 50$, $r = 5\%$, $\sigma = 20\%$, $T = 20$ weeks, $\mu = 13\%$
- The Black-Scholes value of the option is \$2.4 x 100,000 = \$240,000
- How does the bank hedge its risk to lock in a \$60,000 profit?

Naked & Covered Positions

Naked position

Take no action

Covered position

Buy 100,000 shares today

Both strategies leave the bank exposed to significant risk

Stop-Loss Strategy

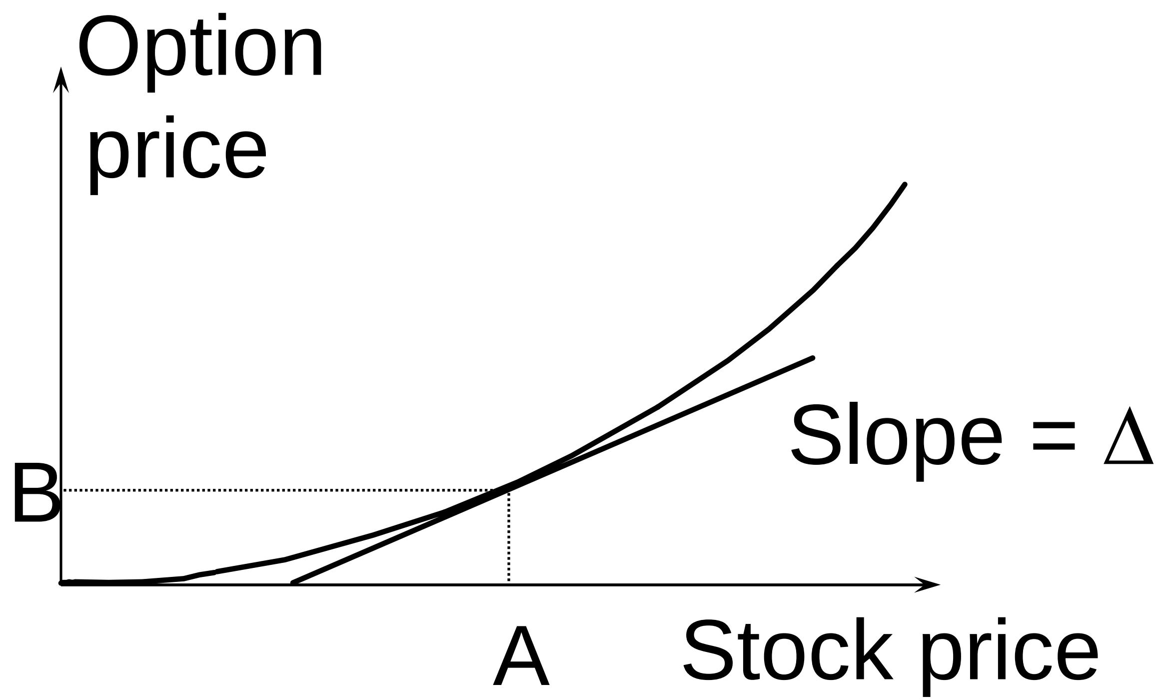
This involves:

- Buying 100,000 shares as soon as price reaches $50
- Selling 100,000 shares as soon as price falls below $50

This deceptively simple hedging strategy does not work well

Delta (See Figure 15.2, page 345)

- Delta (Δ) is the rate of change of the option price with respect to the underlying



Delta Hedging

- This involves maintaining a delta neutral portfolio
- The delta of a European call on a stock paying dividends at rate q is $N(d_1)e^{-qT}$
- The delta of a European put is

$$e^{-qT}[N(d_1) - 1]$$

Delta Hedging (continued)

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a "buy high, sell low" trading rule
- See Tables 15.2 (page 350) and 15.3 (page 351) for examples of delta hedging

Delta Hedging (continued)

Table 15.2 Simulation of delta hedging. Option closes in the money and cost of hedging is $263,300.

Week	*Stock price*	*Delta*	*Shares purchased*	*Cost of shares purchased ($000)*	*Cumulative cost including interest ($000)*	*Interest cost ($000)*
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	48.12	0.458	(6,400)	(308.0)	2,252.3	2.2
2	47.37	0.400	(5,800)	(274.7)	1,979.8	1.9
3	50.25	0.596	19,600	984.9	2,966.6	2.9
4	51.75	0.693	9,700	502.0	3,471.5	3.3
5	53.12	0.774	8,100	430.3	3,905.1	3.8
6	53.00	0.771	(300)	(15.9)	3,893.0	3.7
7	51.87	0.706	(6,500)	(337.2)	3,559.5	3.4
8	51.38	0.674	(3,200)	(164.4)	3,398.5	3.3
9	53.00	0.787	11,300	598.9	4,000.7	3.8
10	49.88	0.550	(23,700)	(1,182.2)	2,822.3	2.7
11	48.50	0.413	(13,700)	(664.4)	2,160.6	2.1
12	49.88	0.542	12,900	643.5	2,806.2	2.7
13	50.37	0.591	4,900	246.8	3,055.7	2.9
14	52.13	0.768	17,700	922.7	3,981.3	3.8
15	51.88	0.759	(900)	(46.7)	3,938.4	3.8
16	52.87	0.865	10,600	560.4	4,502.6	4.3
17	54.87	0.978	11,300	620.0	5,126.9	4.9
18	54.62	0.990	1,200	65.5	5,197.3	5.0
19	55.87	1.000	1,000	55.9	5,258.2	5.1
20	57.25	1.000	0	0.0	5,263.3	

Delta Hedging (continued)

Table 15.3 Simulation of delta hedging. Option closes out of the money and cost of hedging is $256,600.

Week	*Stock price*	*Delta*	*Shares purchased*	*Cost of shares purchased ($000)*	*Cumulative cost including interest ($000)*	*Interest cost ($000)*
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	49.75	0.568	4,600	228.9	2,789.2	2.7
2	52.00	0.705	13,700	712.4	3,504.3	3.4
3	50.00	0.579	(12,600)	(630.0)	2,877.7	2.8
4	48.38	0.459	(12,000)	(580.6)	2,299.9	2.2
5	48.25	0.443	(1,600)	(77.2)	2,224.9	2.1
6	48.75	0.475	3,200	156.0	2,383.0	2.3
7	49.63	0.540	6,500	322.6	2,707.9	2.6
8	48.25	0.420	(12,000)	(579.0)	2,131.5	2.1
9	48.25	0.410	(1,000)	(48.2)	2,085.4	2.0
10	51.12	0.658	24,800	1,267.8	3,355.2	3.2
11	51.50	0.692	3,400	175.1	3,533.5	3.4
12	49.88	0.542	(15,000)	(748.2)	2,788.7	2.7
13	49.88	0.538	(400)	(20.0)	2,771.4	2.7
14	48.75	0.400	(13,800)	(672.7)	2,101.4	2.0
15	47.50	0.236	(16,400)	(779.0)	1,324.4	1.3
16	48.00	0.261	2,500	120.0	1,445.7	1.4
17	46.25	0.062	(19,900)	(920.4)	526.7	0.5
18	48.13	0.183	12,100	582.4	1,109.6	1.1
19	46.63	0.007	(17,600)	(820.7)	290.0	0.3
20	48.12	0.000	(700)	(33.7)	256.6	

Using Futures for Delta Hedging

- The delta of a futures contract is $e^{(r-q)T}$ times the delta of a spot contract
- The position required in futures for delta hedging is therefore $e^{-(r-q)T}$ times the position required in the corresponding spot contract

Theta

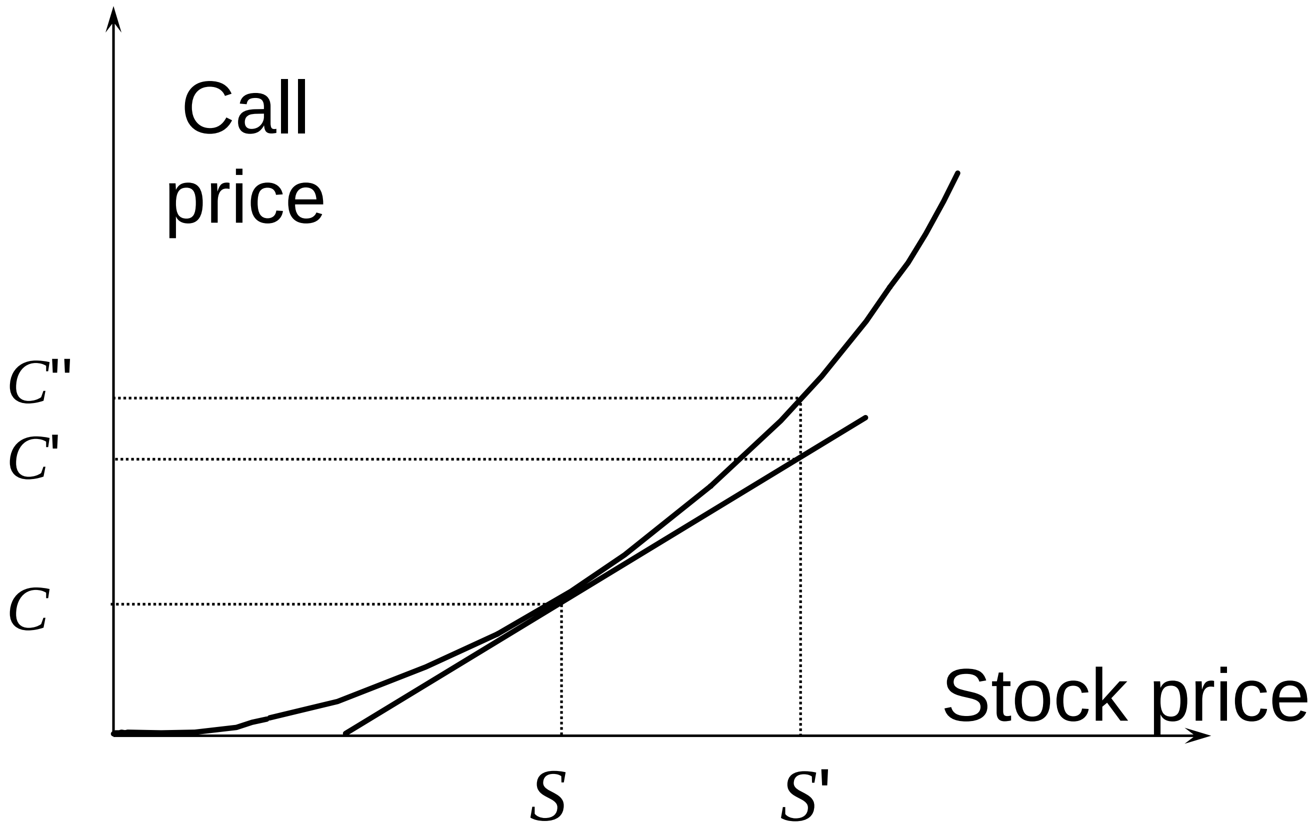
- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of the option declines
- $$\Theta = -\frac{S_0 N'(d_1)\sigma e^{-qT}}{2\sqrt{T}} + qS_0 N(d_1)e^{-qT} - rKe^{-rT}N(d_2) \text{ (for calls)}$$
  $$\Theta = -\frac{S_0 N'(d_1)\sigma e^{-qT}}{2\sqrt{T}} - qS_0 N(-d_1)e^{-qT} + rKe^{-rT}N(-d_2) \text{ (for puts)}$$

Gamma

- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- Gamma is greatest for options that are at the money (see Figure 15.9, page 358)
- $$\Gamma = \frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}} \text{ (for calls and puts)}$$

Gamma Addresses Delta Hedging Errors Caused By Curvature

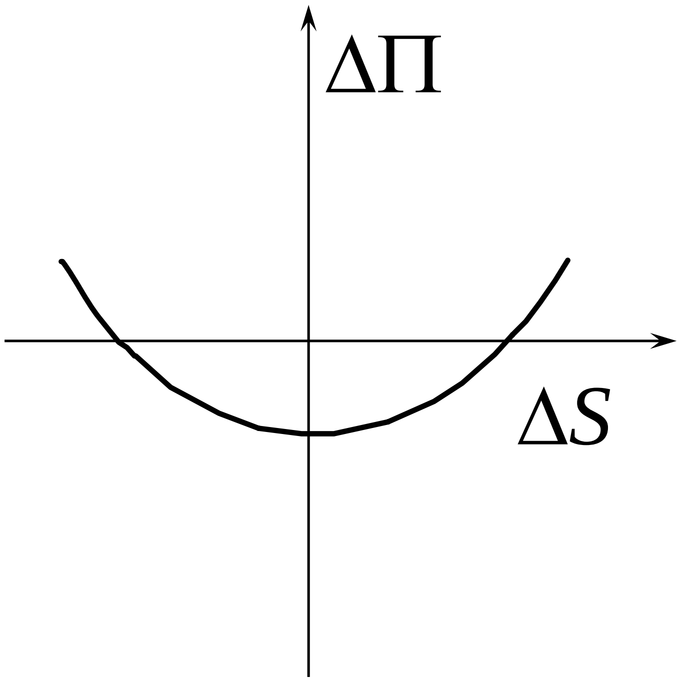
(Figure 15.7, page 355)



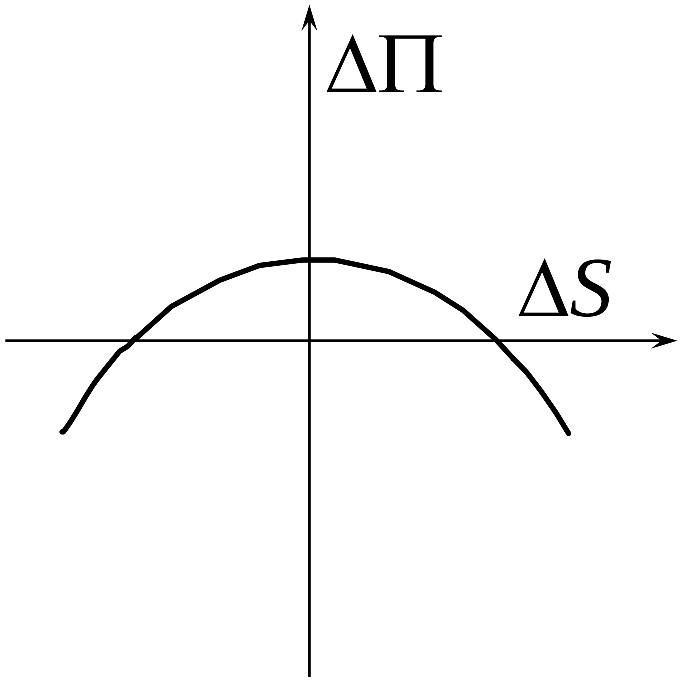
Interpretation of Gamma

- For a delta neutral portfolio,

$$\Delta\Pi \approx \Theta\,\Delta t + \tfrac{1}{2}\Gamma\Delta S^2$$



Positive Gamma



Negative Gamma

Relationship Between Delta, Gamma, and Theta

For a portfolio of derivatives on a stock paying a continuous dividend yield at rate q

$$\Theta + (r - q)S\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

which is the same as the partial differential equation mentioned before

Vega

- Vega (ν) is the rate of change of the value of a derivatives portfolio with respect to volatility
- Vega tends to be greatest for options that are at the money (See Figure 15.11, page 361)
- $\nu = S_0\sqrt{T}\,N'(d_1)e^{-qT}$ (for calls and puts)

Managing Delta, Gamma, & Vega

- Δ can be changed by taking a position in the underlying
- To adjust Γ & ν it is necessary to take a position in an option or other derivative

Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate
- $\rho = KTe^{-rT}N(d_2)$ (for calls)

  $\rho = -KTe^{-rT}N(-d_2)$ (for calls)
- For currency options there are 2 rhos

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger, hedging becomes less expensive

Scenario Analysis

A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities

Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset Δ, Γ, ν, etc.
- When we create an option synthetically we take positions that match Δ, Γ, & ν

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the Δ of the put option

Portfolio Insurance continued

- As the value of the portfolio increases, the Δ of the put becomes less negative and some of the original portfolio is repurchased
- As the value of the portfolio decreases, the Δ of the put becomes more negative and more of the portfolio must be sold

Portfolio Insurance continued

The strategy did not work well on October 19, 1987...

*Real puts work, but synthetic puts fail