Chapter 19

Estimating Parameter Values for Single Facilities

Introduction

- Three important parameters needed to be estimated:
  - The probability of default (PD)
  - The loss in the event of default (LIED)
  - The exposure at default (EAD)
- Example of calculating EL and UL for a loan
- Information (data) requirements

Estimating the Probability of Default

1. Expert Credit Grading

- Three Steps

  First: 定義buckets (或grades)

  Second: 將客戶分類到各個buckets中 (最困難的一步)

  - For large loans, banks often rely on the expert opinion, which may be from the credit-rating staff or the rating agency

    (Expert system is a database of rules and questions that tries to mirror the credit expert's decision process)

  - For large-volume, but small loans, the decision mainly depends on quantitative data

  Third: 由歷史資料算出每個buckets中所有客戶的平均破產機率

- Ratings used by Standard & Poor's, Fitch, and Moody's (p.270 Table19-1)
- 從前的rating，同時包括了PD與LIED的資訊，已考慮期望損失，但現在的rating，只考慮PD

2. Quantitative Scores Based on Customer Data

- The quantitative rating models are often called scorecards because they produce a score based on the given information (p.272 Table 19-2 and 19-3)
- 若客戶破產後的還款行為、債權轉讓時的賣出價格與催收成本也能清楚記錄，則可以估LIED
- 若能將客戶行為與是否破產做連結，則可能可以預測破產

- Two common approaches
  1. Discriminant Analysis (分辨會破產與不會破產的公司)
     - Altman's Z Score (working capital, retained earnings, EBIT, market value of equity, and sales) (Z<1.81，很可能破產；Z>2.99，應不會破產) (p.273~274)

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

     - 可根據年初各公司的Z-score來分組，再看各組年尾時到底有多少比例會破產，如此可得不同Z值的破產機率
  2. Logistic Regression (直接將score與破產機率連結) (p.274~275 maximum likelihood estimation MLE)

$$P_C = \frac{1}{1+e^{Y_C}}, \text{ where } Y_C = w_0 + \sum_i w_i X_{i,C}$$

- Testing Quantitative Scorecards
  - Power Curve
    - 客戶的歷史資料分為兩個部份: the model set (估計參數)與the test set (驗證模型的預測能力)
    - Sorting the customers according to their scores (假設由低到高)
    - Constructing a graph with the percentage of all the sorted customers on the *x*-axis and the percentage of all the defaults on the *y*-axis

$$x_k = \frac{k}{N}, \quad y_k = \frac{1}{N_D}\sum_{C=1}^{k} I_C, \ \text{for } k = 1, \ldots, N$$

    - 如果power curve很快就上升到100%，則表示模型的判別率高 (p.276 Figure 19-2)；反之，若幾乎要把所有的客戶都排完，power curve 才上升到100%，則表示模型的判別率低 (p.277 Figure 19-3)

3. Equity-Based Credit Scoring (simplified KMV)

- 考慮 $\tilde{A}=\tilde{E}+D$，且 $\tilde{E}\sim N(\bar{E},\sigma_E)$，則 $\tilde{A}\le D \Leftrightarrow \tilde{E}<0$

$$P=\int_{-\infty}^{0} p(\tilde{E};\bar{E},\sigma_E)\,d\tilde{E}$$

$$=\int_{-\infty}^{(0-\bar{E})/\sigma_E} \phi(z)\,dz=\Phi(-\bar{E}/\sigma_E)$$

- The value $\bar{E}/\sigma_E$ is called the critical value of the distance to default (p.279 Table 19-4) (要注意的是，股價其實是lognormal而非normal分配)
- 實務上可用 $\bar{E}/\sigma_E$ 將公司分類，之後再算出每個分組中的平均破產機率，之後只要將新公司的資訊代入，看是屬於那一組，即可求出其破產機率
- 與其他的模型不同，因考慮了股價，亦即也考慮了最新的市場資訊

4. Cash-Flow Simulation

- Project finance is used for large projects, where a project company is established and raises funds in the form of debt or equity.
- Because the operations of the project company are so well defined, it is possible to build a cash-flow model that predicts the company's profits under different scenarios
- The structure of the cash-flow model is illustrated in Figure 19-5. The simulation can be used to not only give the PD, but also the EAD, LIED, and the net present value of losses
- An oil-refinery example (p.280~281)

Estimating the Exposure at Default

- For loans
  - The exposure amount is set by the amortization rate
  - The exposure is assumed to be fixed for each year and equal to the average outstanding for the year
- For derivatives (by simulation in Ch17)
- For credit lines (p.283 Table 19-5)
  - $\text{EAD} = L(\overline{E} + (1 - \overline{E})e_d)$

  (e_d is the additional use of the normally unused line at the time of default)

Estimating the Loss in the Event of Default

- For illiquid securities, ex. loans

$$\text{LIED} = \frac{\text{EAD} - \text{Recovery\$} + \text{Admin\$}}{\text{EAD}} \approx 1 - \text{Recovery\%}$$

- For liquid securities, ex. bonds

$$\text{LIED} = \frac{\text{Value Before - Value After}}{\text{Value Before}}$$

Estimating the Loss in the Event of Default

- The standard deviation of LIED (與collateral, structure和industry有關) is required to estimate the UL

$$\sigma_{LIED} \approx A \times \sigma_{LIED,Worst} = A \times \sqrt{\overline{LIED} - \overline{LIED}^2} = A \times \sqrt{\overline{R} - \overline{R}^2}$$

★A is derived by comparing the actual standard deviation with the worst case

★所謂worst case，指的是當破產一發生，則LIED=100%，此時可由二項式分配，得出 $\sigma_{LIED,Worst}$

★銀行的loan之recovery rate的分配情況，p.285 Figure 19-6，用回收部份的NPV來看，Figure 19-7，用loan之可賣得價值所反推之回收率

★Recovery rate之分析 (p.286~287 Tables 19-6, 19-7, and 19-8)

Example of Calculating EL and UL

- Consider a 1-year line of credit of $100 million to a BBB-rated public utility, with a 40% utilization
  - P.279 Table 19-4, PD=0.22%
  - P.283 Table 19-5, addition exposure at default=65%
  - P.287 Table 19-8, $\overline{R} = 70\%, \sigma_R = 19\%$
- Calculate EL and UL if changes in exposure and severity are uncorrelated (p.288~289)
- Summary of calculation (p.289 Table 19-9)
- Calculate EL and UL for different credit rating (p.290 Table 19-10)

Information Requirements

- Three types of information must be collected:
  1. Information on the customer and facility at the time the loan was granted (Table 19-2, 19-3)
  2. Information on the results of the models used to approve the facility (ex. Credit rating, predicted exposure at default, predicted loss in the event of default) (for back testing)
  3. Information on later default behavior (p. 291 Table 19-11)