Chapter 13
Measurement of Interest-Rate Risk for ALM

Introduction

- Estimate structural IR risk，並建立EC來對抗此risk
- 除此之外，還可由買賣interest rate derivatives來降低structural IR risk
- VaR不適用，原因: (1) 須考慮customer behavior，(2) 長天期之利率的變動，不易用VaR之簡單假設來補捉
- 三種ALM desk估structural IR risk之方法
  - Gap reports
  - Rate-shift scenarios
  - Simulation methods

Gap Reports

- Gap reports: Gap = CF of Assets－CF of liabilities
  - 使用歷史悠久
  - 也可用來measure liquidity risk (因其考慮了CF之maturity or duration)
  - Easy to create
  - Intuitive view of the balance sheet of the bank
  - 沒法考慮 customers exercise their option (customer behavior)，原因: 此法中未來的CF與r之變化無關
  - Three types of Gap reports
    - Contractual maturity
    - Repricing frequency
    - Effective maturity

- Contractual-Maturity Gap reports
  - 將assets與liabilities 中多筆CF，依照**合約到期日**整理出來，p.192 Figure 13.1
    - 例：checking accounts 之contractual maturity= 0，maturity為3個月之certificate of deposit投資組合，假設總合共 4千萬元，則等分成四個CF，到期日各為0、1、2、3個月
  - 通常此法假設CF都發生在合約到期日那天 (亦即不考慮顧客的prepayment behavior)，如此會使得風險的估計被扭曲
  - Show the mismatch of cash flows and possible required cash out flow，此mismatch之部位才會受interest risk所影響，若所有都match，可想成net holding=0，亦即沒風險
  - This method is useful in showing **liquidity** characteristics, but gives little information on **structural IR risk**

- Repricing Gap reports
  - 不是看合約到期日，而是看在合約下面之risk factor (r) 是多久reset (repricing)一次，就把此合約之cash flow放到此reset期間之籃子
  - The report matches together all assets and liabilities that have the same interest rate basis, e.g., 3-month LIBOR, 1-year fixed rate
  - 此法考慮了不同的interest rate，但沒考慮customer behavior

	Contractual-maturity	Repricing
personal loan	2- 4 yrs	3-mon
floating mortgage	10- 20yrs	1-yr

- Effective-Maturity Gap reports (考慮了customer behavior 後，CF之到期日)
  - 例如房貸需考慮與T, r相關的prepayment behavior之effective maturity
  - The effective maturity for checking accounts is no more zero, and the total amount in the checking accounts will have a core component that will be not be withdraw in the near future
  - Assets：因顧客會prepay → effective maturity < contractual maturity
  - Liability：即使contractual maturity 到了，存戶也未必會將所有的錢領回 → effective maturity > contractual maturity

- 如何根據Gap Reports 來算EC against structural IR risk

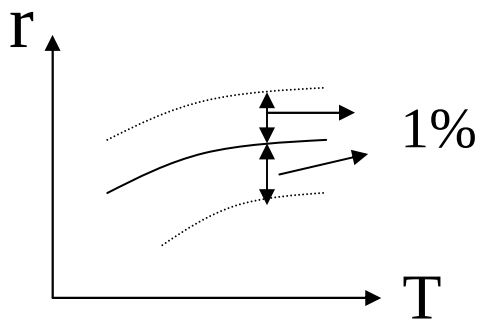
  - (i) 將每個bucket之net CF想成bond之C_t
  - (ii) 算 $\frac{\partial}{\partial r} = -\text{Duration \$}$
  - (iii) 估annualized σ_r (一般都用quarterly data $\times\sqrt{4}$ 來估計)
  - (iv) $\text{EC} = \text{Duration \$} \times \square \times \sigma_r$ (若要達成10 bps (0.1%) 之 default prob.，□約為3.1)

  - 需要之assumptions

    1. linearity 成立 (因duration dollars抓不到non-linearity之利率變化)
    2. duration is constant over the whole year
    3. portfolio 可持續一年不變
    4. r ~ N(,)

    → 但上述假設很常違背，使得此法估出之EC有20%~50%之誤差

Rate-Shift Scenarios



- After shifting the rates, the cash flows are changed according to the behavior expected in the new environment (測試對asset與liability portfolio有何影響)
- 如此可抓到portfolio 對 interest rate的nonlinear變化 (p.194之例子)
- 但此法補捉不到term structure複雜之變化
  - 例如twist, flex, or wiggle，或是yield curve先上升再下降
- 將r_t上升$3\sigma_r$ 或下降$3\sigma_r$，看最大損失為何，即為所需的EC

Simulation Methods

- 模擬整條interest rate path，此外，除了將 r 代入financial instruments model之外，還要將customer behavior與behavior of administered rate (prime rate)考慮進來
  1. 隨機產生下一期 (下一個月) 的interest rate
  2. 除了估計prime rate的變化，還要估計各個product之餘額變化 (考慮customer behavior 受利率的影響)
  3. 計算下一期的net income，包括net payment of interest and net payment of principal
  4. 繼續進行下一期的模擬，大約模擬3~5年，此時，假設各個product的餘額到一個穩定的狀況
  5. Calculate the NPV for this interest path (considering the generated cash flow and the remaining balance)
  6. 模擬多次後，除了可得到NPV的平均與分布之外，還可得到在每個時間點銀行的portfolio所帶來的net cash flow的分布

例子: p.196~201

1. Models to Create Interest Rate Scenarios Randomly

- General-equilibrium (GE) interest-rate models (對volatility之估計與描述較好，用在risk management，這邊採用此種設定) vs. arbitrage-free (AF) interest-rate models (對price之fit較好，適合trading用)
- 考慮mean reverting與 $r_t > 0$

$$r_{t+1} = r_t + \kappa(\theta - r_t)\Delta t + \sigma\sqrt{\Delta t} \times r_t^{\gamma} \times z_t,\ z_t \sim N(0,1)$$

  - If $\gamma = 0.5$, Cox-Ingersoll-Ross (CIR) Model
  - If $\gamma = 0$, Vasicek Model

★p.196~198 simulate random interest rate paths (p.199 Figure 13-3)

2. Stochastic Product Models
   - p.198~199 checking account balance is taken as an example
   - 想辦法將customer behavior與r連上關係，看對checking account balance 有何影響

$$B_{t+1} = B_t(1 + g + ar_{3m,t} + z_{b,t}\frac{\sigma_b}{\sqrt{\Delta t}})\Delta t,\ z_{b,t} \sim N(0,1)$$

3. 利用1.與2.，create stochastic CF of one given path
   - p.200, Figure 13-4, 13-5

4. 計算NPV之distribution (模擬多條interest rate path後)，可得VaR與所需的EC
   - 因為模擬都是很多年，但EC只需考慮一年，若A-rating bank需99.9%不倒閉for one year，它並不需要99.9%不倒閉for ten years之EC
     - 用99% for ten years當作EC (99% for ten years 可想為99.9% for one year and surviving 10 years)
     - 若不用maturity來考慮，只想一年一年來模擬，需模擬cash flow up to 1 year與一年後remaining balance，實際上，上述過程太複雜，且並沒增加accuracy