Chapter10
Overcoming VaR's Limitations

VaR的限制

- Assume variances and covariances between the risk factors are stable
- It does not give a good description of extreme losses beyond the 99% level
- It does not take the liquidity risk into account

* 除了搭配stress or scenario testing，本章是要探討如何直接改進VaR的方法

解决方法

- Allowing variance to change over time
- Approaches for assessing extreme events
- Approaches to quantifying liquidity risk

Allowing variance to change over time

- 一般之variance

$$\sigma_{T+1}^2 = \frac{1}{N-1} \sum_{t=T-N}^{T} \underbrace{X_t^2}_{change\ on\ day\ t}$$

其中假設 the mean of changes is relatively small and can be neglected

(此法是用歷史資料估計未來的variance，且假設未來的variance為constant，但是實際上將來的variance是會隨時間變動的)

- 改用GARCH (Generalized Autoregressive Conditional Heteroscedasticity)

$$\sigma_{T+1}^2 = w + \alpha \cdot X_T^2 + \beta\sigma_T^2$$

- 迴歸時，因並無法直接觀察到 volatility，而是用market price來估計出 w、α、β
  - GARCH也可用來估covariance between X and Y

$$\sigma_{xy,T+1}^2 = w + \alpha \cdot X_T \cdot Y_T + \beta\,\sigma_{xy,T}^2$$

$$\sigma_{T+1}^2 = (1-\lambda)X_T^2 + \lambda\sigma_T^2 \quad (\underline{\underline{EWMA}})$$

Exponentially weighted moving average

  - If $w = 0$，$\beta = \lambda$，$\alpha = 1 - \lambda$

Approaches for assessing extreme events

- 如何預估extreme events
  - Jump Diffusion
  - Historical Simulation
  - Adjustments to Monte Carlo Simulation
  - Extreme Value Theory

- Jump Diffusion

X: with probability P → $N(\mu_t, \sigma_t)$ 一般之分配

with probability $1-P$ → $N(\mu_c, \sigma_c)$ crisis之分配

※合起來之分配，比原來之normal來的厚尾

※很難估出 $\mu_t, \sigma_t, \mu_c, \sigma_c, P$，可能原因：

- crisis難界定
- crisis之資料少

- Historical Simulation:

  Historical Simulation並不需要對分配作假設，此外，原本利用historical simulation，就可有效的反應risk factor中複雜之interaction與extreme events，但是要注意的是，過去之crisis並不代表未來之crisis

- Adjustments to Monte Carlo Simulation

  原本Monte-Carlo是假設normal，現在可以改為jump，or leptokurtic distribution，但Monte-Carlo在risk factors多時不好用，需先用Eigenvalue decomposition to find primary risk factors

- Extreme Value Theory
  - Concentrate on estimating the shape of only the tail of a probability distribution
    - 原本只能估計99%之VaR，現在可估99.9%VaR
  - Generalized Pareto Distribution

$$\text{Prob}\left(\text{Result} \geq X\right) = \left(ax + b\right)^{-c}$$

    - 只能用在single risk factor
    - a, b, c難估，因為extreme case資料少

Quantifying liquidity risk

- Liquidity risk can increase a bank's losses; therefore, they should be included in the calculation of VaR and EC
  1. Liquidity in funding (run out of liquid cash to pay its debts, considered as a problem in ALM in later chapters)
  2. Liquidity in trading (unable to quickly sell a security at a fair price)
     - 可能發生在持有小公司股票或是在crisis發生時
     - 此時trader可能以低賣立即賣出資產(因很緊急)或是慢慢以公平價格賣資產，但可能遭受額外損失
- 為什麼要考慮liquidity risk？
  - 流動性不好，會增加market risk所造成的損失
  - 若不考慮流動性，則trader會選擇illiquid securities to earn high yield

- 如何quantifying the liquidity
  - 方法一：紀錄下在真實世界中，trader需花多久之時間來close out持有部位，並看賣出時之discount有多少 (但不可能每次都這樣來嘗試)
  - 方法二：預估close-out time

$$T = \frac{\text{Position Size}}{F \times \text{Daily Volume}}$$

    - T之意義：在沒有discount下，處理部位要幾天
    - F：在不影響成交價下能賣出之量佔當天交易量之比例
    - Daily Volume：某標的物之每日成交量
  - 方法三：看bid-ask spread
    - if bid-ask spread大 $\Rightarrow$ liquidity差
    - if bid-ask spread小 $\Rightarrow$ liquidity好

- Liquidity risk如何影響VaR

①用Close-Out Time

- The most common approach to assessing the liquidity risk is to use the "square-root-of-T" adjustment for VaR: A trader holding a position with a T-day closeout period is taking $\sqrt{T}$ times as much risk as a trader holding an equivalent liquid position for one day
- 若假設在這個T天當中之loss是i.i.d $\sim \mathrm{N}(0, \sigma_\ell^2)$

$$\Rightarrow L_T = \ell_1 + \ell_2 + \cdots + \ell_T$$

$$\Rightarrow \sigma_{L_T}^2 = \sigma_\ell^2 + \sigma_\ell^2 + \cdots + \sigma_\ell^2 = T \cdot \sigma_\ell^2$$

$$\Rightarrow \sigma_{L_T} = \sqrt{T}\sigma_\ell \Rightarrow VaR_T = 2.32 \cdot \sigma_{L_T} = 2.32 \cdot \sqrt{T}\sigma_\ell = \sqrt{T} \times VaR_1$$

- 稍微改進的方法是假設position is closed out linearly over T days

$$\sigma_{L_T}^2 = \left(\frac{T}{T}\sigma_\ell\right)^2 + \left(\frac{T-1}{T}\sigma_\ell\right)^2 + \cdots + \left(\frac{1}{T}\sigma_\ell\right)^2$$

$$VaR_T = 2.32 \cdot \sigma_{L_T}^2$$

$$= 2.32 \cdot \sigma_\ell \cdot \sqrt{\left(\frac{T}{T}\right)^2 + \left(\frac{T-1}{T}\right)^2 + \cdots + \left(\frac{1}{T}\right)^2}$$

- 例如：若T = 10，原始的 $VaR_{10} = \sqrt{10} \times VaR_1 = 3.16 \times VaR_1$

  改進的 $VaR_{10} = 1.96 \times VaR_1$

- It compares the amount that could be lost over 1 day for a liquid position with the amount that could be lost over T days for the illiquid position

②用simulation (同時模擬市場變動與交易者的反應)

比較同一資產 (i) be sold within a day

(ii) be sold for a year

*且若(i)決定不賣出，也持有一年，則兩邊之損失一樣，但(i)立即賣出，可能的損失會有效的降低

* Increased liquidity gives the trader greater options to buy and sell the instrument, but if these options are not exercised, they are worthless.

* Compare the loss suffered on a liquid position with the loss that would be suffered if the position was illiquid. This gives a measure of the relative risk

* 方法(i)中，假設銀行可以在crisis中出清其position，也可在好之時買回，但同時，銀行亦需維持所需之資本。但方法(ii)中，銀行不行做如此的決策

⇒方法(ii)需約2倍之capital of方法(i)

③用Bid-ask Spread

The trader may sell out his position immediately by giving a discount that brings the price down to the bid price

假設bid - ask spread 是r.v.，且change over time

$$\text{Additional Drop}_{99\%} = 0.5\bar{s} + 2.32\sigma_s$$

$$\text{Liquidity Adjusted VaR} = \text{VaR} + \text{Additional Drop}$$

where $\bar{s}$ is the average spread

σ_s is the standard deviation of the spread

- ⊙ 適用於單一之標的物或是 risk factor
- ⊙ 但也可以用Liquidity Adjusted VaR反推出相對應之新的σ，放入variance-covariance matrix中，此時σ除了market risk還包括liquidity risk