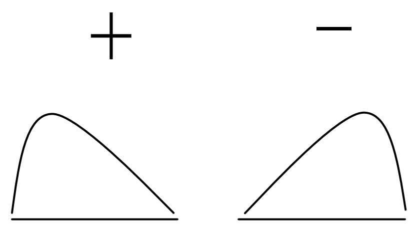
# Chapter 3
## Review of Statistics

- Histogram → probability density
  → cumulative probability

- Mean, Standard deviation, Skew, and Kurtosis
  (p.27~p.37)

- Normal Distribution

  Mean
  s.d.
  skew = 0 (偏態)
  kurtosis = 3 (峰態)

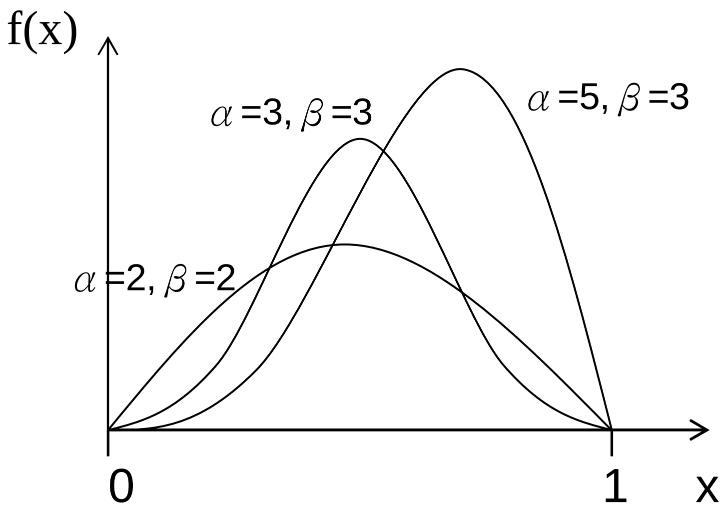



- Log Normal Distribution ( X > 0 才能取log)
  - If X～LogNormal, then Log(X)～Normal
  - If Z～Normal, them $e^{z}$～LogNormal

- Beta Distribution (常用來描述credit risk)

$$f(x)=\begin{cases}\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} & ,0<x<1\\ 0 & ,0/w\end{cases}$$

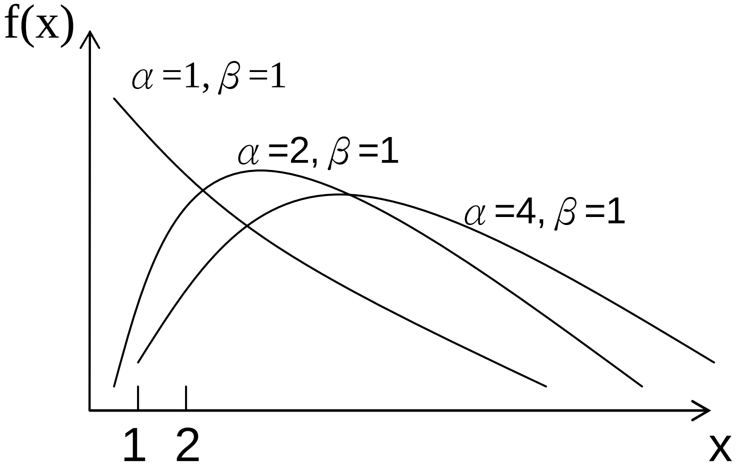
其中 $\alpha>0$, $\beta>0$, 且 $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 f(x)dx = 1$

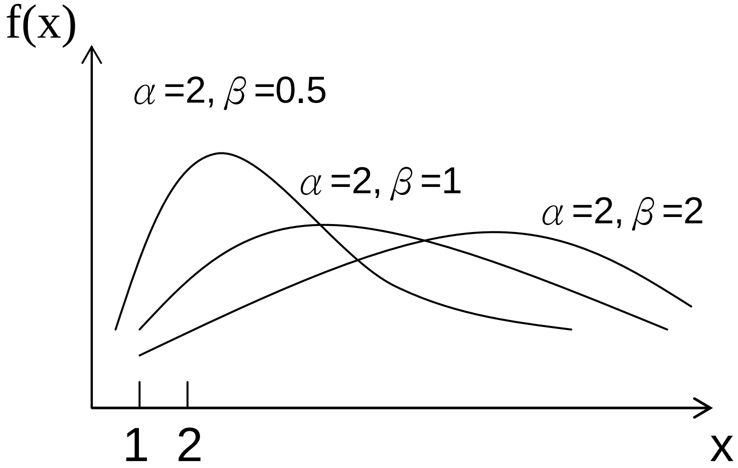


- Gamma distribution

$$f(x)=\begin{cases}\dfrac{1}{\Gamma(\alpha)\cdot\beta^{\alpha}}x^{\alpha-1}\cdot e^{-\frac{x}{\beta}} & x>0\\ 0 & x\le 0\end{cases}$$

其中 $\alpha>0$, $\beta>0$, 且 $\Gamma(\alpha)=\int_0^{\infty}y^{\alpha-1}e^{-y}dy$ , which is known as the Gamma function

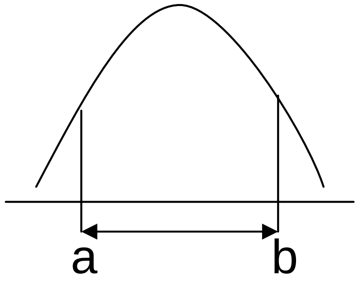



※ $\Gamma(\alpha)=(\alpha-1)!$

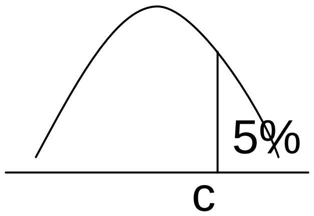
※ By integrating by parts, $\Gamma(\alpha)=(\alpha-1)\cdot\Gamma(\alpha-1)$

- Confidence Intervals (信賴區間)



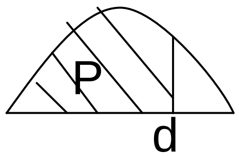

- Confidence Levels (信賴水準)

(given 5%, find c, c為95%之confidence level (有95%不會超過c) )




- Percentile

(given p, 算出d, 則d為p%ile )



- Correlation, Covariance

$$\text{If } y = \sum_{i=1}^{n} x_i,\ \bar{y} = \sum_{i=1}^{n} \bar{x}_i \text{ and } \sigma_y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{i,j}\sigma_{x_i}\sigma_{x_j}$$

$$\text{*if } \rho_{i,j} = 0 \Rightarrow \sigma_y^2 = n\sigma_x^2 \quad \Rightarrow \sigma_y = \sqrt{n}\sigma_x$$

  - Matrix Representation of Standard Deviation

$$S = [\sigma_{x_1} \quad \sigma_{x_2} \quad \ldots\ldots \quad \sigma_{x_n}]$$

$$R = \begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} & \cdots & \rho_{1,n} \\ \rho_{2,1} & 1 & & & \vdots \\ \rho_{3,1} & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ \rho_{n,1} & \cdots & \cdots & \cdots & 1 \end{bmatrix} \qquad \Rightarrow \sigma_y^2 = \underset{1\times n}{S}\,\underset{n\times n}{R}\,\underset{n\times 1}{S}^T$$

- Random Process (Stochastic Process)
  - For Stock Price

$$S_{t+\Delta t} = S_t + \Delta S_t$$

$$(1)\ \Delta S_t = \sigma \sqrt{\Delta t}\, z_t, \text{ where } z_t \sim N(0,1)$$

$$(2)\ \Delta S_t = S_t \sigma \sqrt{\Delta t}\, z_t, \text{ where } z_t \sim N(0,1)$$

$$(3)\ \Delta S_t = S_t \mu \Delta t + S_t \sigma \sqrt{\Delta t}\, z_t, \text{ where } z_t \sim N(0,1)$$

  - For Interest Rate (Cox-Ingersoll-Ross (CIR) model)

$$r_{t+\Delta t} = r_t + c(r_\mu - r_t) + \sigma_r \sqrt{r_t}\, z_t, \text{ where } z_t \sim N(0,1)$$