## Chapter 7

## Net Present Value

 and OtherInvestment Criteria

## Topics Covered

ONet Present Value
OOther Investment Criteria
OMutually Exclusive Projects
Capital Rationing

## Net Present Value

## Net Present Value - Present value of cash flows minus initial investments

Opportunity Cost of Capital - Expected rate of return given up by investing in a project

## Net Present Value

## Example

Suppose we can invest $\$ 50$ today \& receive $\$ 60$ later today. What is our increase in value?

$$
\begin{aligned}
\text { Profit } & =-\$ 50+\$ 60 \\
& =\$ 10
\end{aligned}
$$

| $\$ 10$ | Added Value |
| :--- | :--- |
| $\$ 50$ | Initial Investment |

## Net Present Value

## Example

Suppose we can invest $\$ 50$ today and receive $\$ 60$ in one year. What is our increase in value given a $10 \%$ expected return?

$$
\text { Profit }=-50+\frac{60}{1.10}=\$ 4.55
$$

| $\$ 4.55$ | Added Value |
| ---: | :--- | :--- |
| $\$ 50$ | Initial Investment |

## Net Present Value

## NPV = PV - required investment

$$
N P V=C_{0}+\frac{C_{t}}{(1+r)^{t}}
$$

$$
N P V=C_{0}+\frac{C_{1}}{(1+r)^{1}}+\frac{C_{2}}{(1+r)^{2}}+\ldots+\frac{C_{t}}{(1+r)^{t}}
$$

## Net Present Value

## Terminology

$C=$ Cash Flow
$t=$ time period of the investment
$r=$ "opportunity cost of capital"

The Cash Flow could be positive or negative at any time period

## Net Present Value

## Net Present Value Rule

Managers increase shareholders’ wealth by accepting all projects that are worth more than they cost.

Therefore, they should accept all projects with a positive net present value.

## Net Present Value

## Example

You have the opportunity to purchase an office building. You have a tenant lined up that will generate $\$ 16,000$ per year in cash flows for three years. At the end of three years you anticipate selling the building for $\$ 450,000$. How much would you be willing to pay for the building?

## Net Present Value

## Example - continued



## Net Present Value



Example - continued If the building is being offered for sale at a price of \$350,000, would you buy the building and what is the added value generated by your purchase and management of the building?

## Net Present Value

## Example - continued

If the building is being offered for sale at a price of $\$ 350,000$, would you buy the building and what is the added value generated by your purchase and management of the building?

$$
N P V=-350,000+\frac{16,000}{(1.07)^{1}}+\frac{16,000}{(1.07)^{2}}+\frac{466,000}{(1.07)^{3}}
$$

$N P V=\$ 59,323$

## Payback Method

Payback Period - Time until cash flows recover the initial investment of the project.

The payback rule specifies that a project be accepted if its payback period is less than the specified cutoff period. The following example will demonstrate the absurdity of this statement.

## Payback Method

## Example

The three project below are available. The company accepts all projects with a 2 year or less payback period. Show how this policy will impact our decision.

## Cash Flows

| Project | $\underline{\mathbf{C}_{\mathbf{0}}}$ | $\underline{\mathbf{C}_{\mathbf{1}}}$ | $\underline{\mathbf{C}_{2}}$ | $\underline{\mathbf{C}_{\mathbf{3}}}$ |
| :--- | :--- | :---: | :---: | :---: |
| A | -2000 | +1000 | +1000 | +10000 |
| B | -2000 | +1000 | +1000 | 0 |
| C | -2000 | 0 | +2000 | 0 |


| Payback | NPV@10\% |
| :---: | :---: |
| 2 | $+7,249$ |
| 2 | $-\quad 264$ |
| 2 | -347 |

## Payback Method

The limitation of payback method:
$\rightarrow$ Payback does not consider any cash flows that arrive after the payback period
$\rightarrow$ Payback gives equal weight to all cash flows arriving before the cutoff period (an improved method is to calculate the discounted payback period)
$\rightarrow$ Usually the large construction projects inevitably have long payback periods

* Therefore, payback method is most commonly used when the capital investment is small when the merits of the project is so obvious that formal analysis is unnecessary.


## Internal Rate of Return

Rate of Return Rule - Invest in any project offering a rate of return that is higher than the opportunity cost of capital.

$$
\text { Rate of Return }=\frac{C_{1} \text {-investment }}{\text { investment }}
$$

Internal Rate of Return (IRR) - An average discount rate at which $\mathrm{NPV}=0$.

## Internal Rate of Return

## Example

You can purchase a building for $\$ 350,000$. The investment will generate $\$ 16,000$ in cash flows (i.e. rent) during the first three years. At the end of three years you will sell the building for $\$ 450,000$. What is the IRR on this investment?

## Internal Rate of Return

## Example

You can purchase a building for $\$ 350,000$. The investment will generate $\$ 16,000$ in cash flows (i.e. rent) during the first three years. At the end of three years you will sell the building for $\$ 450,000$. What is the IRR on this investment?

$$
0=-350,000+\frac{16,000}{(1+I R R)^{1}}+\frac{16,000}{(1+I R R)^{2}}+\frac{466,000}{(1+I R R)^{3}}
$$

IRR = 12.96\%

## Internal Rate of Return

| Calculating IRR by using a spreadsheet |  |  |  |  |  |  |
| :---: | ---: | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |
| Year | Cash Flow |  |  |  | Formula |  |
| 0 | $(350,000.00)$ |  | IRR $=$ | $12.96 \%$ | IRR(B3:B7) |  |
| 1 | $16,000.00$ |  |  |  |  |  |
| 2 | $16,000.00$ |  |  |  |  |  |
| 3 | $466,000.00$ |  |  |  |  |  |
|  |  |  |  |  |  |  |

## Internal Rate of Return

Calculating the IRR can be a laborious task. Fortunately, financial calculators can perform this function easily. Note the previous example.

| HP-10B |  | EL-733A |  | BAII Plus |
| :---: | :---: | :---: | :---: | :---: |
| -350,000 | CFj | -350,000 | CFi | CF |
| 16,000 | CFj | 16,000 | CFi | 2nd \{CLR Work |
| 16,000 | CFj | 16,000 | CFi | -350,000 ENTER |
| 466,000 | CFj | 466,000 | CFi | 16,000 ENTER |
|  |  | IRR |  | 16,000 ENTER |
|  |  |  |  | 466,000 ENTER |
|  | uce | $\mathrm{R}=12.96$ |  | IRR CPT |

## Internal Rate of Return



## IRR and NPV

© The rate of return rule will give the same answer as the NPV rule as long as the NPV of a project declines smoothly (as the case in the previous slide) as the discounted rate increases

## IRR vs. Opportunity Cost of Capital

- Internal Rate of Return measures the profitability of the project and only depends on the project's own cash flows
- The opportunity cost of capital is the standard for deciding whether to accept the project and is equal to the return offered by equivalent-risk investments in the capital market


## Internal Rate of Return

## Pitfall 1 - Mutually Exclusive Projects

- IRR sometimes ignores the magnitude of the project

The following two projects illustrate that problem

## Internal Rate of Return

## Example

You have two proposals to choice between. The initial proposal has a cash flow that is different than another one, which the cash inflow is brought by selling the building for $\$ 400,000$ at the end of the first year. Using IRR, which do you prefer?

$$
\begin{aligned}
N P V & =-350+\frac{16}{(1+I R R)^{1}}+\frac{16}{(1+I R R)^{2}}+\frac{466}{(1+I R R)^{3}}=0 \\
& =12.96 \%
\end{aligned}
$$

$$
\begin{aligned}
N P V & =-350+\frac{400}{(1+I R R)^{1}}=0 \\
& =14.29 \%
\end{aligned}
$$

## Internal Rate of Return

## Example

You have two proposals to choice between. The initial proposal has a cash flow that is different than another one, which the cash inflow is brought by selling the building for $\$ 400,000$ at the end of the first year. Using IRR, which do you prefer?

| Project | $\mathrm{C}_{0}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ | IRR | NPV@14\% | NPV@7\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| initial | -350000 | 16000 | 16000 | 466000 | $12.96 \%$ | $-\$ 8000$ | $\$ 59000$ |
| another | -350000 | 400000 |  |  | $14.29 \%$ | $\$ 770$ | $\$ 24000$ |

## Internal Rate of Return



## Internal Rate of Return

If you want to maximize the value of your firm, projects that earn a good rate of return for a long time often have higher NPVs than those offer high percentage rates of return but die young

## Internal Rate of Return

## Pitfall 2 - Lending or Borrowing?

© With some cash flows (ex. for borrowing) the NPV of the project increases as the discount rate increases, that is contrary to the normal relationship between NPV and discount rates

| Project | $\mathrm{C}_{0}$ | $\mathrm{C}_{1}$ | IRR | NPV@10\% |
| :---: | :---: | :---: | :---: | :---: |
| Lending | -100 | 150 | $50.00 \%$ | $\$ 36.4$ |
| Borrowing | 100 | -150 | $50.00 \%$ | $-\$ 36.4$ |

* When NPV is higher as the discount rate increases, a project is acceptable only if its internal rate of return is less than the opportunity cost of capital


## Internal Rate of Return

## Pitfall 3 - Multiple Rates of Return

- Certain cash flows can generate NPV=0 at two different discount rates
© The following cash flow generates NPV=0 at both $6 \%$ and $28 \%$ (p. 194 Figure 7-4)

| $\mathrm{C}_{0}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ | $\mathrm{C}_{4}$ | $\mathrm{C}_{5}$ | IRR | NPV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -22 | 15 | 15 | 15 | 15 | -40 | $6.00 \%$ | $\$ 0$ |
|  |  |  |  |  |  | $28.00 \%$ | $\$ 0$ |

*When there are multiple changes in the sign of the cash flows, the IRR rule does not work, but the NPV rule always does

## Mutually Exclusive Projects

When you need to choose between mutually exclusive projects, the decision rule is simple. Calculate the NPV of each project, and from those options that have a positive NPV, choose the one whose NPV is highest.

## Mutually Exclusive Projects

## Example

Select one of the two following projects, based on highest NPV.

| System | $C_{0}$ | $C_{1}$ | $C_{2}$ | $C_{3}$ | $N P V$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Faster | -800 | 350 | 350 | 350 | +118.5 |
| Slower | -700 | 300 | 300 | 300 | +87.3 |

assume 7\% discount rate

## Investment Timing

Sometimes you have the ability to defer an investment and select a time that is more ideal at which to make the investment decision. A common example involves a tree farm. You may defer the harvesting of trees. By doing so, you defer the receipt of the cash flow, yet increase the cash flow.

## Investment Timing

## Example

You may purchase a computer anytime within the next five years. While the computer will save your company money, the cost of computers continues to decline. If your cost of capital is $10 \%$ and given the data listed below, when should you purchase the computer?

## 

Example
You may purchase a computer anytime within the next five years. While the computer will save your company money, the cost of computers continues to decline. If your cost of capital is $10 \%$ and given the data listed below, when should you purchase the computer?

| Year | Cost | PV Savings* | NPV at Purchase | NPV Today |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 50 | 70 | 20 | 20.0 |
| 1 | 45 | 70 | 25 | 22.7 |
| 2 | 40 | 70 | 30 | 24.8 |
| 3 | 36 | 70 | 34 Date to purchase | 25.5 |
| 4 | 33 | 70 | 37 | 25.3 |
| 5 | 31 | 70 | 39 | 24.2 |

## Long- vs. Short-Lived Equipment

Equivalent Annual Annuity - The cash flow per period which is with the same present value as the cost of buying and operating a machine.

Equivalent annual annuity $=\frac{\text { present value of cash flows }}{\text { annuity factor }}$

## Equivalent Annual Annuity

## Example

Given the following costs of operating two machines and a $6 \%$ cost of capital, select the lower cost machine using the lowest equivalent annual annuity method.

## Year

| Mach | 1 | 2 | 3 | 4 | PV@6\% | EAA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D | -15 | -4 | -4 | -4 | -25.69 | -9.61 |
| => |  | -9.61 | -9.61 | -9.61 |  |  |
| E | -10 | -6 | -6 |  | -21.00 | -11.45 |
| => |  | -11.45 | -11.45 |  |  |  |

## Equivalent Annual Annuity

## Example (with a twist)

Select one of the two following projects, based on highest "equivalent annual annuity" ( $r=9 \%$ ).

| Project | $C_{0}$ | $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $N P V$ | $E A A$ |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| $A$ | -15 | 4.9 | 5.2 | 5.9 | 6.2 | 2.82 | .87 |
| $B$ | -20 | 8.1 | 8.7 | 10.4 |  | 2.78 | 1.10 |

## Capital Rationing

Capital Rationing - Limit set on the amount of funds available for investment.

Soft Rationing - Limits on available funds imposed by management.

Hard Rationing - Limits on available funds imposed by the unavailability of funds in the capital market.

## Profitability Index

## Profitability Index - Ratio of present value to

 initial investment (NPV per dollar spent)| Project | C0 (Investment) | C1 | C2 | NPV@10\% Profitability Index |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L | -3 | 2.2 | 2.42 | 1 | $1 / 3=.33$ |
| M | -5 | 2.2 | 4.84 | 1 | $1 / 5=.20$ |
| N | -7 | 6.6 | 4.84 | 3 | $3 / 7=.43$ |
| O | -6 | 3.3 | 6.05 | 2 | $2 / 6=.33$ |
| P | -4 | 1.1 | 4.84 | 1 | $1 / 4=.25$ |

* $\mathrm{N} \rightarrow \mathrm{O}(=\mathrm{L}) \rightarrow \mathrm{P} \rightarrow \mathrm{M}$
* If there is no soft or hard capital rationing, more NPVs will be preferred even when more dollars are spent
* This rule cannot rank mutually exclusive projects


## Summary

## - A Comparison of Investment Decision Rules

(p. 201 Table 7-3)

## © A recent survey found that

$\rightarrow 75 \%$ of firms either always or almost always use both NPV and IRR to evaluate projects
$\rightarrow$ Just over half of corporations will always or almost always compute a project's payback period
$\rightarrow$ Profitability index is routinely computed by about 12 \% of firms

|  | Percentage of Firms <br> Investment <br> Criterion | That Always or Almost <br> Always Use Criterion | Average Score on 0-4 Scale <br> $(0=$ never use; 4 $\mathbf{0}=$ always use $)$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| All Firms | Small Firms | Large Firms |  |  |  |
| Internal rate of return | 76 | 3.1 | 2.9 | 3.4 |  |
| Net present value | 75 | 3.1 | 2.8 | 3.4 |  |
| Payback period | 57 | 2.5 | 2.7 | 2.3 |  |
| Profitability index | 12 | 0.8 | 0.9 | 0.8 |  |

[^0]
[^0]:    Source: J. R. Graham and C. R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," Journal of Financial Economics, May 2001, pp. 187-243.

