Chapter 4

The Time Value of Money

Topics Covered

- Future Values and Compound Interest
- Present Values
- Multiple Cash Flows
- Perpetuities and Annuities
- Inflation and Time Value
- Effective Annual Interest Rate

Future Values

Future Value - Amount to which an investment will grow after earning interest.

Compound Interest - Interest earned on interest.

Simple Interest - Interest earned only on the original investment.

Future Values

<u>Example - Simple Interest</u>

Interest earned at a rate of 6% for five years on a principal balance of $100.

Interest Earned Per Year = 100 x .06 = $ 6

Future Values

Example - Simple Interest

Interest earned at a rate of 6% for five years on a principal balance of $100.

	Today	Future Years				
		1	2	3	4	5
Interest Earned		6	6	6	6	6
Value	100	106	112	118	124	**130**

Value at the end of Year 5 = $130

Future Values

Example - Compound Interest

Interest earned at a rate of 6% for five years on the previous year's balance.

Interest Earned Per Year =Prior Year Balance x .06

Future Values

Example - Compound Interest

Interest earned at a rate of 6% for five years on the previous year's balance.

	Today	*Future Years*				
		1	*2*	*3*	*4*	*5*
Interest Earned		6.00	6.36	6.74	7.15	7.57
Value	100	106.00	112.36	119.10	126.25	133.82

Value at the end of Year 5 = $133.82 (p.75 Figure 4-1)

Future Values

Future Value of $100 = FV

$$FV = \$100 \times (1+r)^t$$

* FVIF$_{r,t}$=$(1+r)^t$ (Future Value Interest Factor for *r* and *t*) (Table A-1)

Future Values

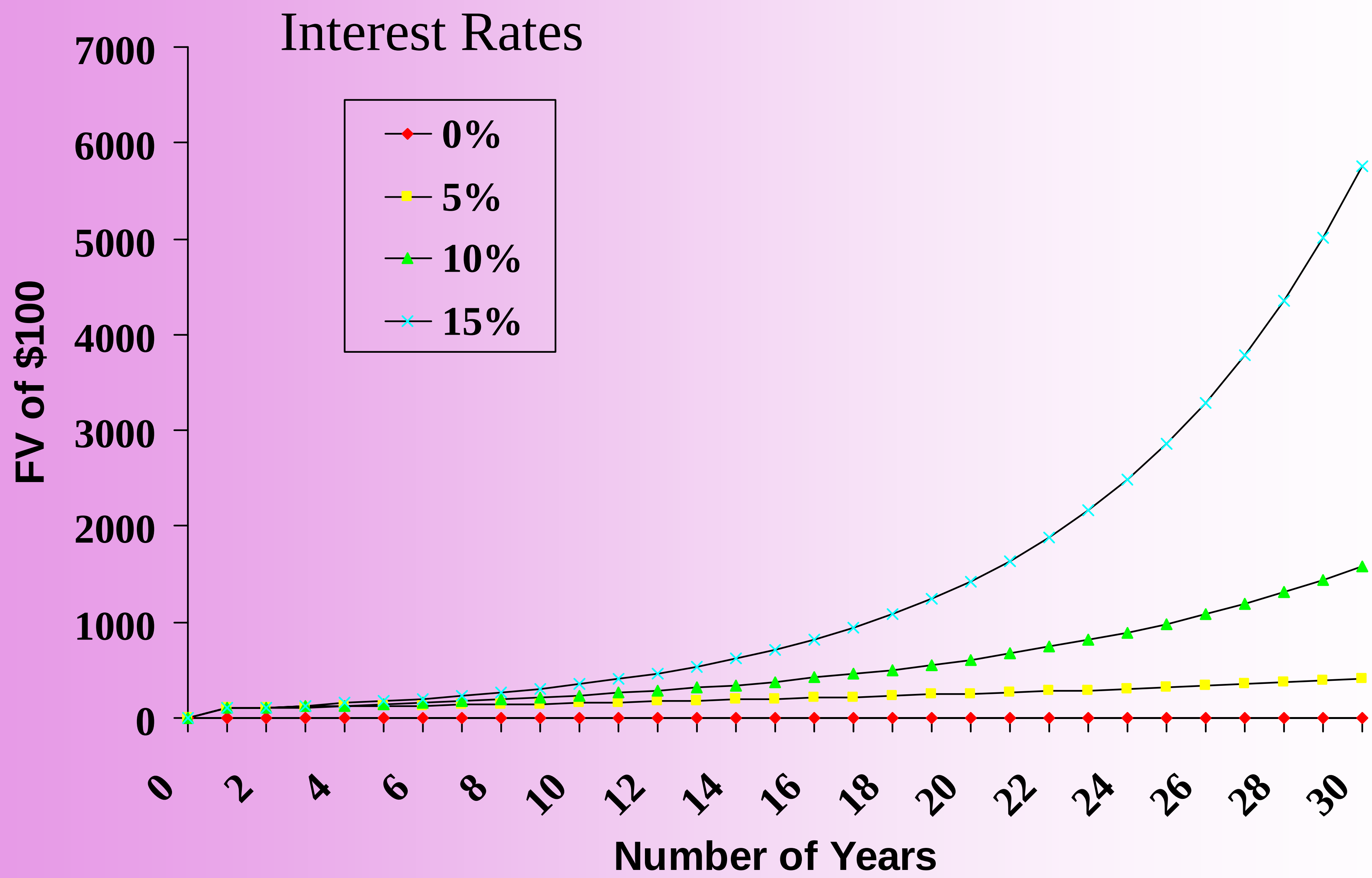
$$FV = \$100 \times (1 + r)^t$$

Example - FV

What is the future value of $100 if interest is compounded annually at a rate of 6% for five years?

$$FV = \$100 \times (1 + .06)^5 = \$133.82$$

Future Values with Compounding



Manhattan Island Sale

Peter Minuit bought Manhattan Island for $24 in 1626.
Was this a good deal?

To answer, determine $24 is worth in the year 2006, compounded at 8%.

$$FV = \$24 \times (1 + .08)^{380}$$

$$= \$120.57 \text{ } trillion$$

FYI - The value of Manhattan Island land is well below this figure.

Present Values

Present Value

Value today of a future cash flow

Discount Factor

Present value of a $1 future payment

Discount Rate

Interest rate used to compute present values of future cash flows

Present Values

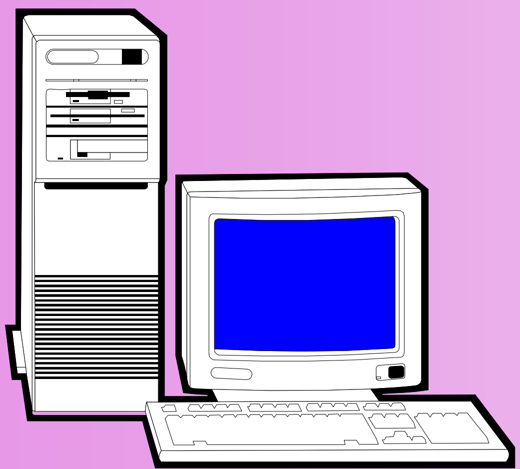
Present Value=PV

$$PV = \frac{\text{Future Value after t periods}}{(1+r)^{t}}$$

Present Values

Example

You want to buy a new computer for $3,000 2 years later. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?



$$PV = \frac{3000}{(1.08)^2} = \$2,572$$

Present Values

Discount Factor = DF = PV of $1

$$\boxed{DF = \frac{1}{(1+r)^t}}$$

➲ Discount Factors can be used to compute the present value of any cash flow

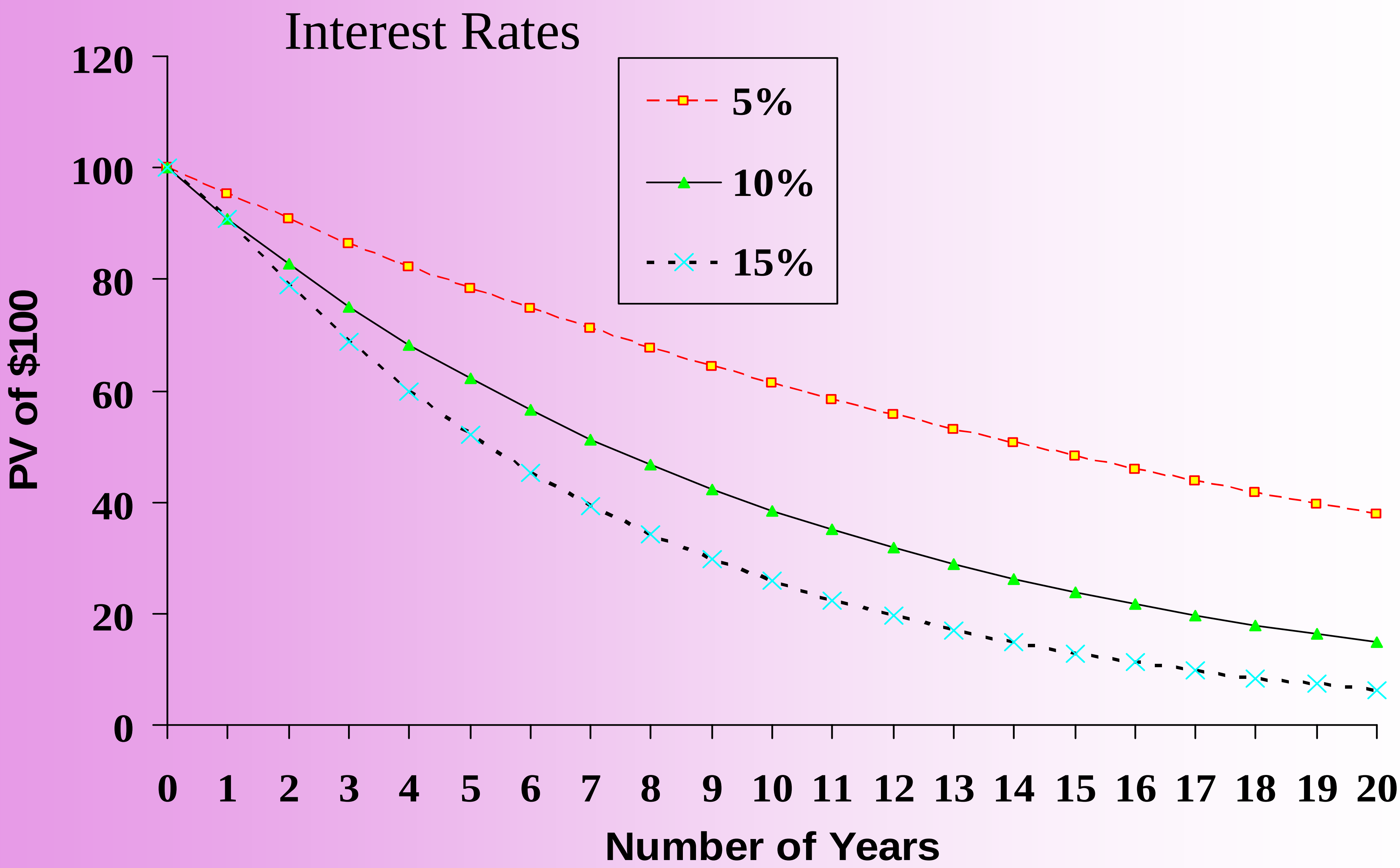
* $PVIF_{r,t} = 1/(1+r)^t$ (Present Value Interest Factor for *r* and *t*) (Table A-2)

Time Value of Money
(applications)

- The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable.

$$\boxed{PV = FV \times \frac{1}{(1+r)^t}}$$

Present Values with Compounding



Time Value of Money
(applications)

- Value of Free Credit (p.81 Example 4.4)
- Implied Interest Rates (p.83~84)
- Time necessary to accumulate funds (p.84 Example 4.5)

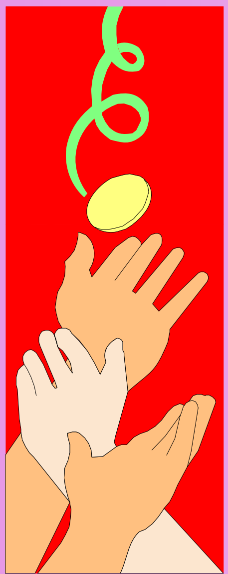
Time Value of Money
(applications)

- An introduction to financial calculators (p.80)
  - HP-10B
  - TI BA II Plus
  - Sharp EL-733A
  - Casio FC-200V

PV of Multiple Cash Flows

Example

Your auto dealer gives you the choice to pay $15,500 cash now, or make three payments: $8,000 now and $4,000 at the end of the following two years. If your cost of money is 8%, which do you prefer?



Immediate payment 8,000.00

$$PV_1 = \frac{4,000}{(1+.08)^1} = 3,703.70$$

$$PV_2 = \frac{4,000}{(1+.08)^2} = 3,429.36$$

Total PV = $15,133.06

PV of Multiple Cash Flows

- PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$

PV of Multiple Cash Flows

Finding the present value of multiple cash flows by using a spreadsheet				
Time until CF	**Cash flow**	**Present value**	**Formula in Column C**	
0	8000	$8,000.00	=PV(B11,A4,0,-B4)	
1	4000	$3,703.70	=PV(B11,A5,0,-B5)	
2	4000	$3,429.36	=PV(B11,A6,0,-B6)	
SUM:		$15,133.06	=SUM(C4:C6)	
Discount rate:	0.08			

PV(Rate, Nper, Pmt, FV, Type) (p. 83)

Rate: 每期利率

Nper: 期數

Pmt: 除了最後一期之每期現金流量

FV: 最後一期之現金流量

Type: 1為期初給付，0或省略為期末給付

Perpetuities & Annuities

Perpetuity

A stream of level cash payments that never ends.

Annuity

Equally spaced level stream of cash flows for a limited period of time.

Perpetuities & Annuities

PV of Perpetuity Formula

$$\boxed{PV = \frac{C}{r}}$$

C = cash payment per period

r = interest rate per period

Perpetuities & Annuities

Example - Perpetuity

In order to create an endowment, which pays \$100,000 per year, forever, how much money must be set aside today in the rate of interest is 10%?

$$PV = \frac{100,000}{0.1} = \$1,000,000$$

Perpetuities & Annuities

Example - continued

If the first perpetuity payment will not be received until three years from today, how much money needs to be set aside today?

$$PV = 100,000\,\frac{1}{0.1}\,\frac{1}{(1+0.1)^3} = \$751,315$$

Perpetuities & Annuities

The Formula of the PV of Annuity

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

C = cash payment

r = interest rate

t = number of years cash payment is received

Perpetuities & Annuities

<u>PV Annuity Factor (PVAF)</u> - The present value of $1 a year for each of *t* years

$$PVAF = \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

* PVIFA$_{r,t}$ (Present Value Interest Factor for Annuity) (Table A-3)

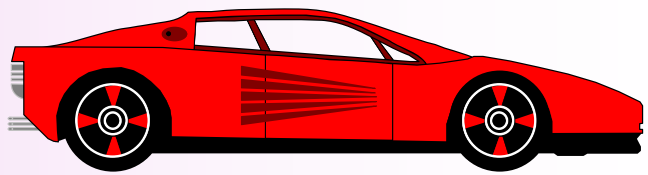
Perpetuities & Annuities

Example - Annuity

You are purchasing a car. You are scheduled to make 3 annual installments of $4,000 per year. Given a rate of interest of 10%, what is the price you are paying for the car (i.e. what is the PV)?

$$PV = 4{,}000\left[\frac{1}{.10} - \frac{1}{.10(1+.10)^3}\right]$$

$$PV = \$9{,}947.41$$



Perpetuities & Annuities

Applications

- ➲Value of payments (p.90-91 Example 4.9, 4.10)
- ➲Implied interest rate for an annuity (car loan)
- ➲Calculation of periodic payments
  - ➔Mortgage payment (p.91-92 Table 4.5)
  - ➔Annual income from an investment payout（房東）
- ➲Future Value of annual payments

$$FV = \left[C \times PVAF \right] \times (1 + r)^t$$

Perpetuities & Annuities

<u>Example - Future Value of annual payments</u>

You plan to save $4,000 every year for 20 years and then retire. Given a 10% rate of interest, what will be the FV of your retirement account?

$$FV = 4{,}000\left[\frac{1}{.10} - \frac{1}{.10(1+.10)^{20}}\right] \times (1+.10)^{20}$$

$$FV = \$229{,}100$$

* $\text{FVIFA}_{r,t}$ (Future Value Interest Factor for an Annuity) (Table A-4)

Perpetuities & Annuities

Annuity Due

Level stream of cash flows starting immediately

Present value of an annuity due

= (1+r) x present value of an annuity

Inflation

Inflation - Rate at which prices as a whole are increasing.

Nominal Interest Rate - Rate at which money invested grows.

Real Interest Rate - Rate at which the purchasing power of an investment increases.

Inflation

$$1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

* Approximation formula is as follows:

$$\text{real interest rate} \approx \text{nominal interest rate} - \text{inflation rate}$$

Inflation

Example

If the interest rate on one year government bonds is 5.0% and the inflation rate is 2.2%, what is the real interest rate?

$$1 + \text{real interest rate} = \frac{1+.050}{1+.022}$$

$$1 + \text{real interest rate} = 1.027$$

$$\text{real interest rate} = .027 \text{ or } 2.7\%$$

$$\text{Approximation} = .050 - .022 = .028 \text{ or } 2.8\%$$

(p.99 Example 4.14, Table 4-7 and Figure 4-13 for CPI)

(p.103 Example 4.16 vs. p.91 Example 4.10)

Effective Interest Rates

Effective Annual Interest Rate - Interest rate that is annualized using compound interest.

Annual Percentage Rate - Interest rate that is annualized using simple interest.

Effective Interest Rates

Example

Given a monthly rate of 1%, what is the Effective Annual Rate (EAR)? What is the Annual Percentage Rate (APR)?

$$\text{EAR} = (1+.01)^{12} - 1 = r$$

$$\text{EAR} = (1+.01)^{12} - 1 = .1268 \text{ or } 12.68\%$$

$$\text{APR} = .01 \times 12 = .12 \text{ or } 12.00\%$$

Effective Interest Rates

Period per year (m)	Final Sum	EAR
1	$1.06	6.0000%
2	$\$1.03^2 = 1.0609$	6.0900%
4	$\$1.015^4 = 1.061364$	6.1364%
12	$\$1.005^{12} = 1.061678$	6.1678%
365	$\$1.0001644^{365} = 1.061831$	6.1831%
∞	$e^{0.06} = 1.061837$	6.1837%