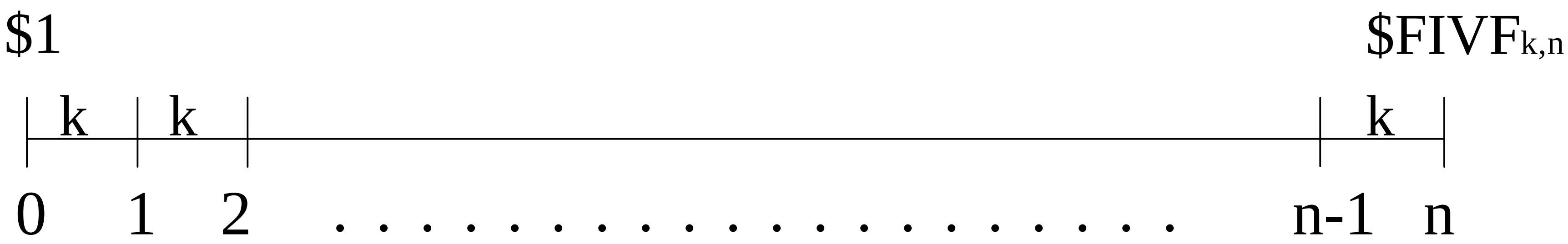
Time Value of Money

Future Value (FV)



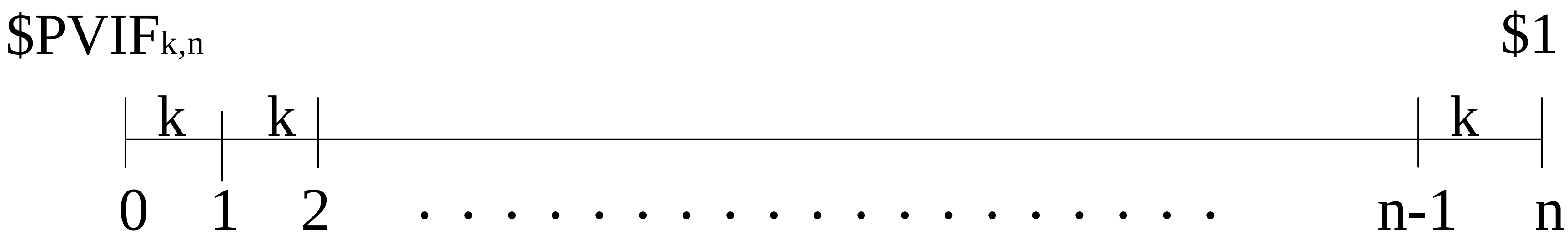
$$FVIF_{k,n} = (1 + k)^{n}$$

(Future Value Interest Factor for k and n)

EX1.(k=8%) FV Calculation

Year	CF	$\times$	$FVIF_{k,n}$	$=$	FV
0	100				
1	300				
2	200				
3	500				
			Total		

Present Value (PV)



$$PVIF_{k,n} = \frac{1}{(1+k)^n}$$

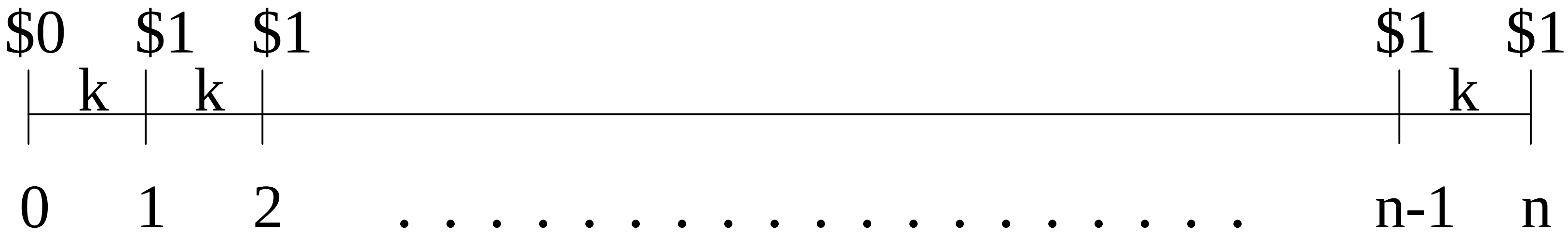
(Present Value Interest Factor for k and n)

EX2.(k=8%) PV Calculation

-1000	200	1500	2000
0	1	2	3

Year	CF	$\times$	$PVIF_{k,n}$	$=$	PV
0	-1000				
1	200				
2	1500				
3	2000				
			Total		

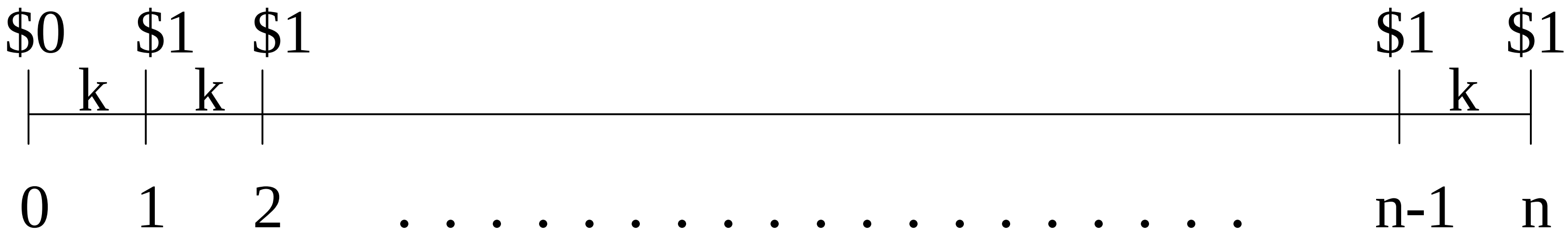
FV of Annuity



$$FVIFA_{k,n} = \sum_{t=1}^{n} (1+k)^{n-t} = \frac{(1+k)^n - 1}{k}$$

(Future Value Interest Factor for an Annuity)

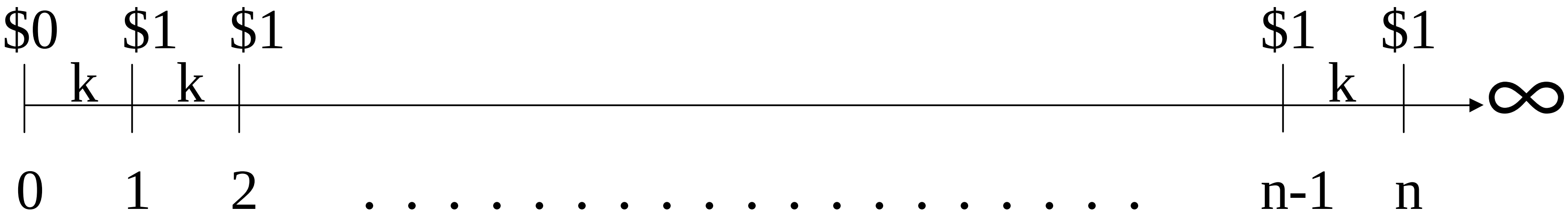
PV of Annuity



$$PVIFA_{k,n} = \sum_{t=1}^{n} \left(\frac{1}{1+k} \right)^{t} = \frac{1 - \frac{1}{(1+k)^{n}}}{k}$$

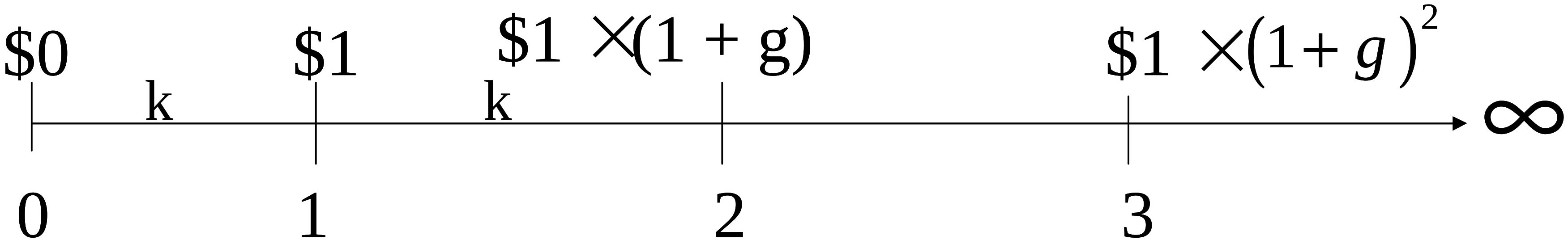
(Present Value Interest Factor for an Annuity)

PV of Perpetuity



$$PV = \frac{1}{k} \quad \leftarrow PVIFA_{k,\infty}$$

Constant Growth Perpetuity



$$PV = \frac{1}{(k-g)}$$

$$Note: k > g$$

$$PV = \sum_{t=1}^{\infty} \frac{1 \cdot (1+g)^{t-1}}{(1+k)^{t}}$$

$$= \frac{1}{1+k} \sum_{t=1}^{\infty} \left(\frac{1+g}{1+k}\right)^{t-1}$$

$$= \frac{1}{1+k} \frac{1}{1-\frac{1+g}{1+k}}$$

$$= \frac{1}{1+k} \frac{1}{\frac{1+k-1-g}{1+k}}$$

$$= \frac{1}{k-g}$$

EX 3.

Find the amount of money that you will have if you invest $1,000 per year (starting at the end of this year) for next 30years and then withdraw it. (suppose k=12%)

EX 4.

Suppose that you want to guarantee yourself $1M when you retire 30 years from now. How much must you invest each year, starting at the end of this year? (suppose k=8%)

EX 5.

Suppose that you want to know how much you need to put away in order to guarantee yourself payments of $5,000 per year for the next 25 years ,starting at the end of this year? (k=9%)

EX 6.

Suppose that you attend an expensive business school &you have been forced to take out $20,000 in loan at 9%. You want to know what your yearly payment will be, given that you have 10 years to pay back the loan.

EX 7.

Suppose that you want to buy a car and need to finance $20,000. The terms of the loan are 2 and half years with a 24% annual interest rate payable in monthly installments.

(a) What are your monthly payments?

(b) What is the remaining principal after one and half years of payment?

EX 8.

Suppose that you are 25 years old .You figure that your income will be such that you will be able to save for the next 25 years, until age 50. At age 50, your income will just cover your expenses. Finally, you expect to retire at age 60 and live until age 80. Suppose that you want to guarantee yourself an income of $30,000 per year after retirement. How much should you put away every year, for the next 25 years, starting at the end of this year. Assume that the interest rate is k=12%.

EX 9.

Suppose that you put away $1,000 per year for the next 20 years. After 20 years, how much can you withdraw per year for the following 30 years?

(a) Suppose that the interest rate is 10%.

(b) Suppose that the interest rate is 10% for the next 15 years and then 12% thereafter.

EX 10.

To see how the perpetuity can be a useful approximation to a long-lived annuity. Let's look at the following two cases : (k=10%)

(a) A perpetuity of $100.

(b) A 30 year annuity of $100.

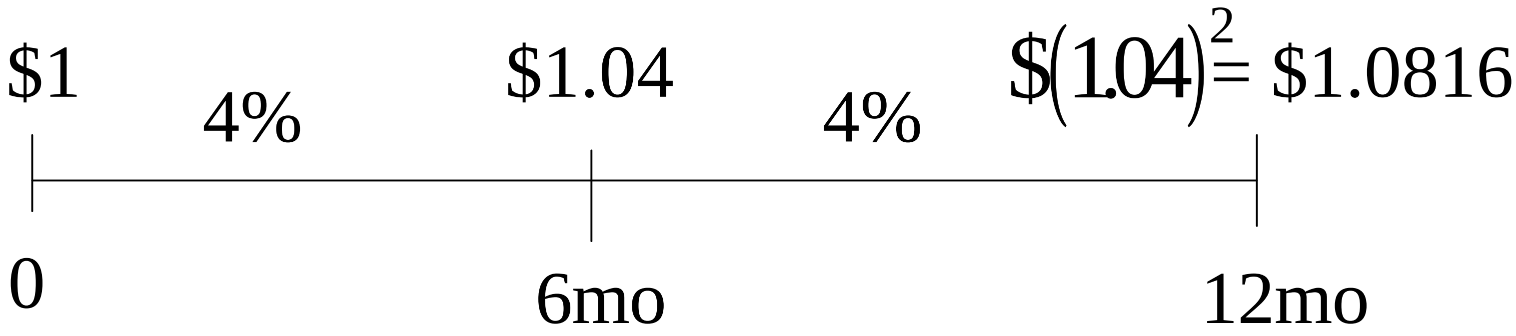
EX 11.

Suppose that you want to find a PV of a perpetuity that pays $100 next year, and a cash flow that increases at 5% per year forever. If the discount rate is 10%.

Compounding IR

Annually:

Semi-annually:



Compounding IR (Cont.)

$$\$1 \quad \left(1+\frac{8\%}{12}\right) \quad \left(1+\frac{8\%}{12}\right)^{2} \quad \cdots \quad \left(1+\frac{8\%}{12}\right)^{12} = \$1.082999$$

$\frac{8}{12}\%$ $\quad$ $\frac{8}{12}\%$

0 $\quad$ 1mo $\quad$ 2mo 12mo

$$\$1 \quad \left(1+\frac{8\%}{365}\right) \quad \left(1+\frac{8\%}{365}\right)^{2} \quad \cdots \quad \left(1+\frac{8\%}{365}\right)^{365} = \$1.08327$$

$\frac{8}{365}\%$ $\quad$ $\frac{8}{365}\%$

0 $\quad$ 1days $\quad$ 2days 365days

Compounding Frequency	Final Sum
Annually	1.08
Semi-annually	1.0816
Monthly	1.082999
Daily	1.08327
$\frac{1}{1000}$ second	?

In General

$$\$1 \times \left(1 + \frac{k}{m}\right)^{m}$$

Where

m=number of compounding intervals per year

k=annual interest rate

Ex 12.

Suppose that you wish to investment $200 for 20 years at 12% with monthly compounding. Then, at the end of 20 years, how much do you have?

Continuously Compounding

Let m $\rightarrow \infty$

$$\lim_{m \to \infty} \left(1 + \frac{k}{m}\right)^m = e^k$$

where e=2.718281828

Effective APR (or EAY,effective annual yield)

Define an interest rate k' such that

$$k' = \left(1 + \frac{k}{m}\right)^m - 1$$

Ex13. EAY k=12%

m	Final Sum	EAY
1	$1.1200	12%
2	$1.1236	12.36%
12	$1.126825	12.6825%
365	$1.1274746	12.74746%
∞	$1.1274969	12.74969%

Ex 14.

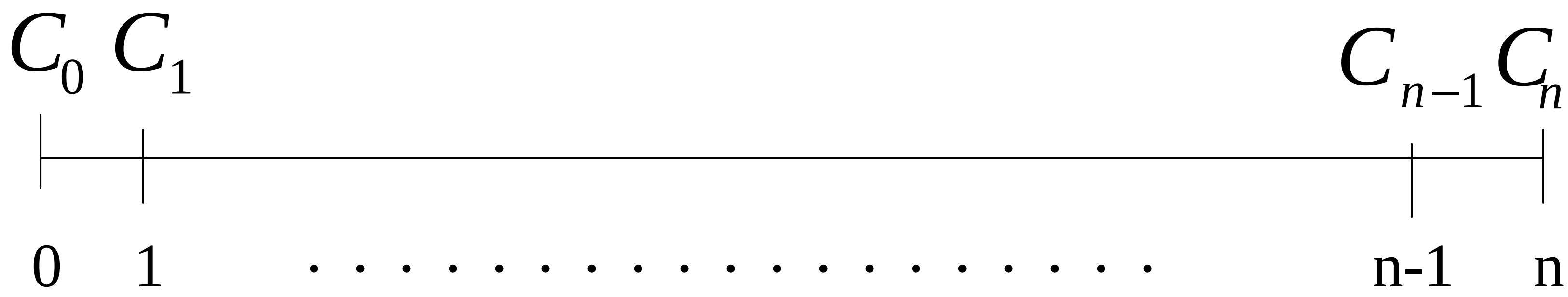
	US	TAIWAN
GNP	$30,000	$15,000
Growth rate	2%	6%

If the growth rates stay at the same,

where can Taiwan catch up with Uncle Sam?

Internal Rate of Return (IRR)

<u>Defn'</u> : The IRR is the interest rate such that the Net Present Value (NPV) of a stream of cash flows equals to zero.



$$0 = C_0 + \frac{C_1}{(1+IRR)} + \cdots + \frac{C_n}{(1+IRR)^n}$$

Ex 15.

Suppose that you know that we will get two cash flows from a $410 investment today.

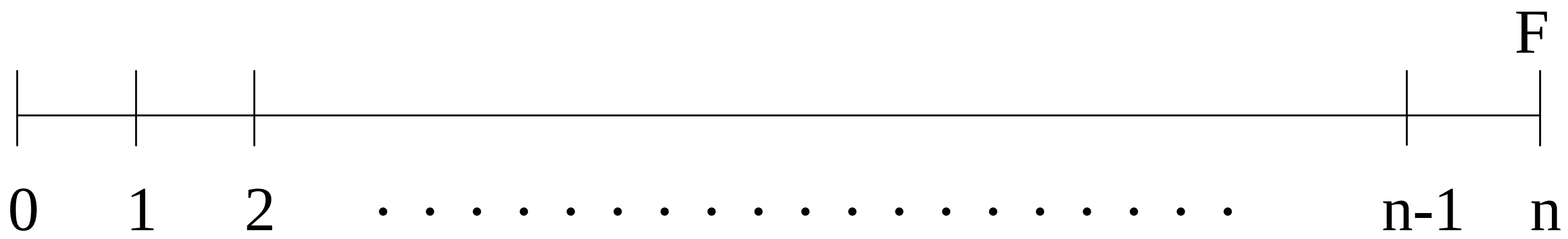
(A)$100,5 years out

(B)$900,10 years out

What is the IRR?

Bond Pricing

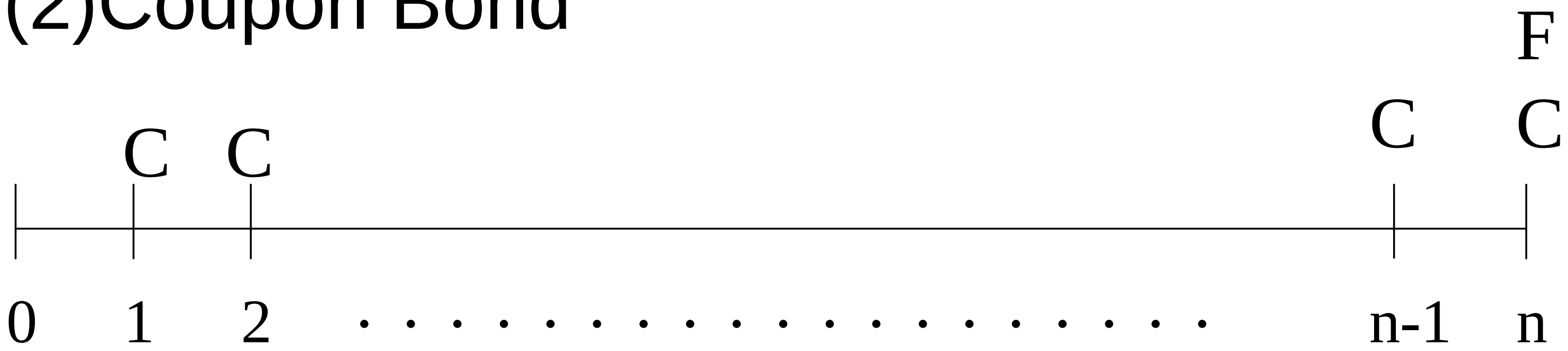
(1)Zero-Coupon Bond



$$P = \frac{F}{(1+k)^n} = F \times PVIF_{k,n}$$

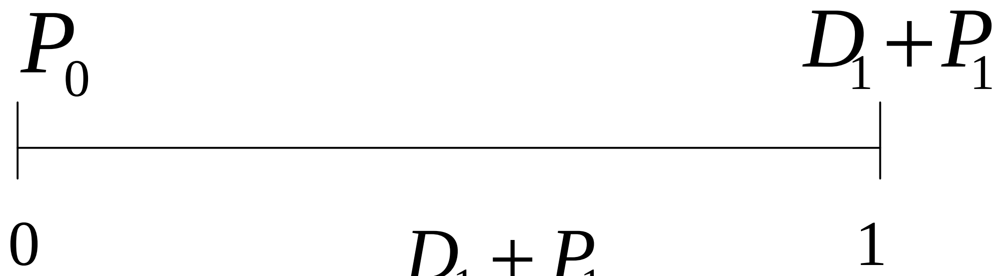
Bond Pricing (Cont.)

(2)Coupon Bond

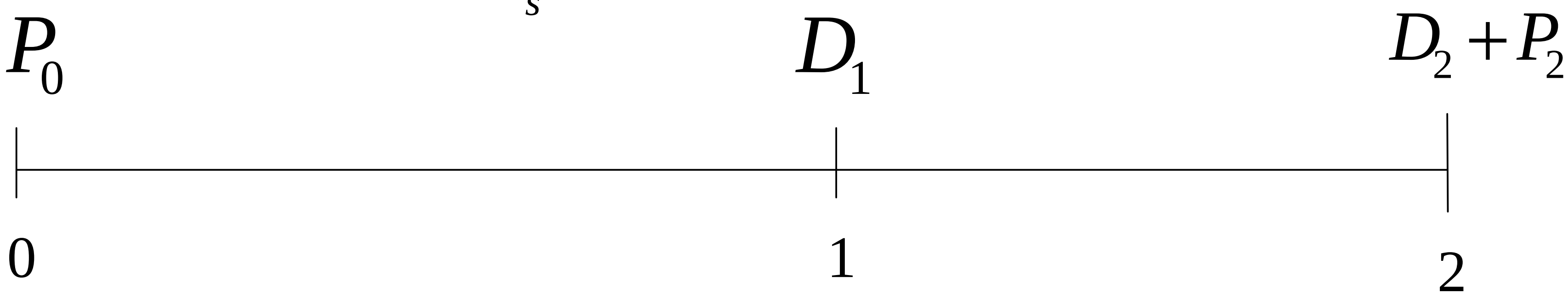


$$P = C \times PVIFA_{k,n} + F \times PVIF_{k,n}$$

Stock Pricing



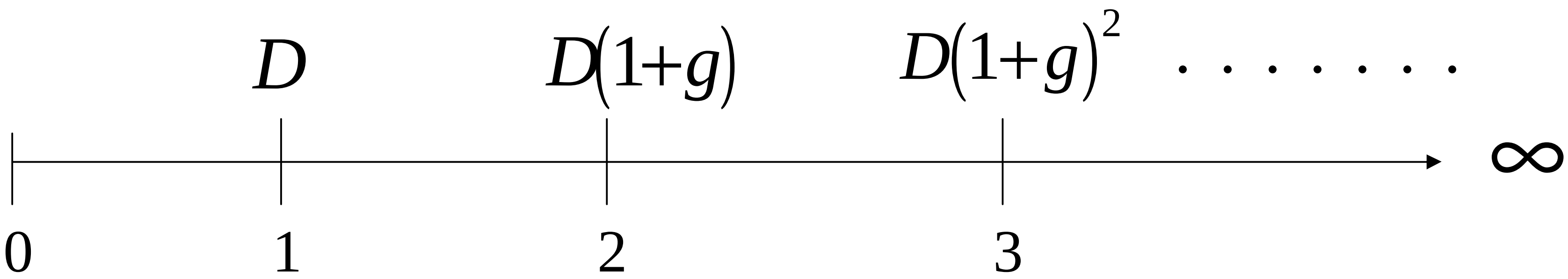
$$P_0 = \frac{D_1 + P_1}{1 + k_s}$$



$$P_0 = \frac{D_1}{(1 + k_s)} + \frac{D_2 + P_2}{(1 + k_s)^2}$$

$$P_0 = \sum_{i=1}^{\infty} \frac{D_i}{\left(1 + k_s\right)^i}$$

Gordon Model



$$P_0 = \frac{D}{K_s - g}$$