Chapter 7

Net Present Value and Other Investment Criteria
Topics Covered

- Net Present Value
- Other Investment Criteria
- Mutually Exclusive Projects
- Capital Rationing
Net Present Value

Net Present Value - Present value of cash flows minus initial investments

Opportunity Cost of Capital - Expected rate of return given up by investing in a project
Net Present Value

Example

Suppose we can invest $50 today & receive $60 later today. What is our increase in value?

Profit = - $50 + $60
= $10
Net Present Value

Example

Suppose we can invest $50 today and receive $60 in one year. What is our increase in value given a 10% expected return?

\[
\text{Profit} = -50 + \frac{60}{1.10} = \$4.55
\]

This is the definition of NPV
Net Present Value

NPV = PV - required investment

\[ NPV = C_0 + \frac{C_t}{(1 + r)^t} \]

\[ NPV = C_0 + \frac{C_1}{(1 + r)^1} + \frac{C_2}{(1 + r)^2} + \ldots + \frac{C_t}{(1 + r)^t} \]
Net Present Value

Terminology

$C = \text{Cash Flow}$

$t = \text{time period of the investment}$

$r = \text{“opportunity cost of capital”}$

깐 The Cash Flow could be positive or negative at any time period
Net Present Value

**Net Present Value Rule**

Managers increase shareholders’ wealth by accepting all projects that are worth more than they cost.

Therefore, they should accept all projects with a positive net present value.
Net Present Value

Example

You have the opportunity to purchase an office building. You have a tenant lined up that will generate $16,000 per year in cash flows for three years. At the end of three years you anticipate selling the building for $450,000. How much would you be willing to pay for the building?
Net Present Value

Example - continued

Present Value

<table>
<thead>
<tr>
<th>Year</th>
<th>$16,000</th>
<th>$16,000</th>
<th>$466,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>$450,000</td>
</tr>
<tr>
<td>1</td>
<td>$16,000</td>
<td></td>
<td>$16,000</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$16,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>$409,323</td>
</tr>
</tbody>
</table>

$409,323
Example - continued

If the building is being offered for sale at a price of $350,000, would you buy the building and what is the added value generated by your purchase and management of the building?
**Net Present Value**

*Example - continued*

If the building is being offered for sale at a price of $350,000, would you buy the building and what is the added value generated by your purchase and management of the building?

\[
NPV = -350,000 + \frac{16,000}{(1.07)^1} + \frac{16,000}{(1.07)^2} + \frac{466,000}{(1.07)^3}
\]

\[
NPV = $59,323
\]
Payback Method

Payback Period - Time until cash flows recover the initial investment of the project.

The *payback rule* specifies that a project be accepted if its payback period is less than the specified cutoff period. The following example will demonstrate the absurdity of this statement.
Payback Method

Example

The three projects below are available. The company accepts all projects with a 2 year or less payback period. Show how this policy will impact our decision.

Cash Flows

<table>
<thead>
<tr>
<th>Project</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>Payback</th>
<th>NPV@10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-2000</td>
<td>+1000</td>
<td>+1000</td>
<td>+10000</td>
<td>2</td>
<td>+ 7,249</td>
</tr>
<tr>
<td>B</td>
<td>-2000</td>
<td>+1000</td>
<td>+1000</td>
<td>0</td>
<td>2</td>
<td>- 264</td>
</tr>
<tr>
<td>C</td>
<td>-2000</td>
<td>0</td>
<td>+2000</td>
<td>0</td>
<td>2</td>
<td>- 347</td>
</tr>
</tbody>
</table>
Payback Method

The limitation of payback method:

- Payback does not consider any cash flows that arrive after the payback period.
- Payback gives equal weight to all cash flows arriving before the cutoff period (an improved method is to calculate the discounted payback period).
- Usually the large construction projects inevitably have long payback periods.

* Therefore, payback method is most commonly used when the capital investment is small when the merits of the project is so obvious that formal analysis is unnecessary.
Rate of Return Rule - Invest in any project offering a rate of return that is higher than the opportunity cost of capital.

\[
\text{Rate of Return} = \frac{C_1 - \text{investment}}{\text{investment}}
\]

Internal Rate of Return (IRR) – An average discount rate at which NPV = 0.
Example

You can purchase a building for $350,000. The investment will generate $16,000 in cash flows (i.e. rent) during the first three years. At the end of three years you will sell the building for $450,000. What is the IRR on this investment?
Internal Rate of Return

Example

You can purchase a building for $350,000. The investment will generate $16,000 in cash flows (i.e. rent) during the first three years. At the end of three years you will sell the building for $450,000. What is the IRR on this investment?

\[
0 = -350,000 + \frac{16,000}{(1 + IRR)^1} + \frac{16,000}{(1 + IRR)^2} + \frac{466,000}{(1 + IRR)^3}
\]

\[
IRR = 12.96\%
\]
## Internal Rate of Return

Calculating IRR by using a spreadsheet

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(350,000.00)</td>
<td>IRR = 12.96%</td>
</tr>
<tr>
<td>1</td>
<td>16,000.00</td>
<td>=IRR(B3:B7)</td>
</tr>
<tr>
<td>2</td>
<td>16,000.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>466,000.00</td>
<td></td>
</tr>
</tbody>
</table>
# Internal Rate of Return

Calculating the IRR can be a laborious task. Fortunately, financial calculators can perform this function easily. Note the previous example.

<table>
<thead>
<tr>
<th>HP-10B</th>
<th>EL-733A</th>
<th>BAII Plus</th>
</tr>
</thead>
<tbody>
<tr>
<td>-350,000 CFj</td>
<td>-350,000 CFi</td>
<td>CF</td>
</tr>
<tr>
<td>16,000 CFj</td>
<td>16,000 CFi</td>
<td>2nd {CLR Work}</td>
</tr>
<tr>
<td>16,000 CFj</td>
<td>16,000 CFi</td>
<td>-350,000 ENTER</td>
</tr>
<tr>
<td>466,000 CFj</td>
<td>466,000 CFi</td>
<td>16,000 ENTER</td>
</tr>
<tr>
<td>{IRR/YR}</td>
<td>IRR</td>
<td>16,000 ENTER</td>
</tr>
<tr>
<td></td>
<td></td>
<td>466,000 ENTER</td>
</tr>
</tbody>
</table>

All produce IRR=12.96
Internal Rate of Return

IRR = 12.96%
The rate of return rule will give the same answer as the NPV rule as long as the NPV of a project declines smoothly (as the case in the previous slide) as the discounted rate increases.
Internal Rate of Return measures the profitability of the project and only depends on the project’s own cash flows.

The opportunity cost of capital is the standard for deciding whether to accept the project and is equal to the return offered by equivalent-risk investments in the capital market.
Internal Rate of Return

Pitfall 1 - Mutually Exclusive Projects

- IRR sometimes ignores the magnitude of the project
- The following two projects illustrate that problem
Internal Rate of Return

**Example**

You have two proposals to choice between. The initial proposal has a cash flow that is different than another one, which the cash inflow is brought by selling the building for $400,000 at the end of the first year. Using IRR, which do you prefer?

\[
NPV = -350 + \frac{16}{(1 + IRR)^1} + \frac{16}{(1 + IRR)^2} + \frac{466}{(1 + IRR)^3} = 0
\]

\[= 12.96\%
\]

\[
NPV = -350 + \frac{400}{(1 + IRR)^1} = 0
\]

\[= 14.29\%
\]
Internal Rate of Return

**Example**

You have two proposals to choice between. The initial proposal has a cash flow that is different than another one, which the cash inflow is brought by selling the building for $400,000 at the end of the first year. Using IRR, which do you prefer?

<table>
<thead>
<tr>
<th>Project</th>
<th>C₀</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>IRR</th>
<th>NPV@14%</th>
<th>NPV@7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial</td>
<td>-350000</td>
<td>16000</td>
<td>16000</td>
<td>466000</td>
<td>12.96%</td>
<td>-$8000</td>
<td>$59000</td>
</tr>
<tr>
<td>another</td>
<td>-350000</td>
<td>400000</td>
<td></td>
<td>14.29%</td>
<td></td>
<td>$770</td>
<td>$24000</td>
</tr>
</tbody>
</table>
Internal Rate of Return

NPV $, 1,000s

Discount rate, %

initial proposal

another proposal

IRR = 12.26%

IRR = 12.96%

IRR = 14.29%
Internal Rate of Return

If you want to maximize the value of your firm, projects that earn a good rate of return for a long time often have higher NPVs than those offer high percentage rates of return but die young.
Internal Rate of Return

Pitfall 2 - Lending or Borrowing?

With some cash flows (ex. for borrowing) the NPV of the project increases as the discount rate increases, that is contrary to the normal relationship between NPV and discount rates.

<table>
<thead>
<tr>
<th>Project</th>
<th>C₀</th>
<th>C₁</th>
<th>IRR</th>
<th>NPV@10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lending</td>
<td>-100</td>
<td>150</td>
<td>50.00%</td>
<td>$36.4</td>
</tr>
<tr>
<td>Borrowing</td>
<td>100</td>
<td>-150</td>
<td>50.00%</td>
<td>-$36.4</td>
</tr>
</tbody>
</table>

* When NPV is higher as the discount rate increases, a project is acceptable only if its internal rate of return is less than the opportunity cost of capital.
Pitfall 3 - Multiple Rates of Return

- Certain cash flows can generate NPV=0 at two different discount rates
- The following cash flow generates NPV=0 at both 6% and 28%

<table>
<thead>
<tr>
<th>C₀</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>IRR</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>-22</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>-40</td>
<td>6.00%</td>
<td>$0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>28.00%</td>
<td>$0</td>
</tr>
</tbody>
</table>

*When there are multiple changes in the sign of the cash flows, the IRR rule does not work, but the NPV rule always does.
Mutually Exclusive Projects

When you need to choose between mutually exclusive projects, the decision rule is simple. Calculate the NPV of each project, and from those options that have a positive NPV, choose the one whose NPV is highest.
Mutually Exclusive Projects

Example

Select one of the two following projects, based on highest NPV.

<table>
<thead>
<tr>
<th>System</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faster</td>
<td>−800</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>+118.5</td>
</tr>
<tr>
<td>Slower</td>
<td>−700</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>+ 87.3</td>
</tr>
</tbody>
</table>

assume 7% discount rate
Sometimes you have the ability to defer an investment and select a time that is more ideal at which to make the investment decision. A common example involves a tree farm. You may defer the harvesting of trees. By doing so, you defer the receipt of the cash flow, yet increase the cash flow.
Investment Timing

Example

You may purchase a computer anytime within the next five years. While the computer will save your company money, the cost of computers continues to decline. If your cost of capital is 10% and given the data listed below, when should you purchase the computer?
Investment Timing

**Example**

You may purchase a computer anytime within the next five years. While the computer will save your company money, the cost of computers continues to decline. If your cost of capital is 10% and given the data listed below, when should you purchase the computer?

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost</th>
<th>PV Savings*</th>
<th>NPV at Purchase</th>
<th>NPV Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
<td>70</td>
<td>20</td>
<td>20.0</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
<td>70</td>
<td>25</td>
<td>22.7</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>70</td>
<td>30</td>
<td>24.8</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>70</td>
<td>34</td>
<td>Date to purchase 25.5</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>70</td>
<td>37</td>
<td>25.3</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>70</td>
<td>39</td>
<td>24.2</td>
</tr>
</tbody>
</table>

* The PV of the savings for the company at the time of purchase
Long- vs. Short-Lived Equipment

Equivalent Annual Annuity - The cash flow per period which is with the same present value as the cost of buying and operating a machine.

Equivalent annual annuity = \frac{\text{present value of cash flows}}{\text{annuity factor}}
Equivalent Annual Annuity

Example

Given the following costs of operating two machines and a 6% cost of capital, select the lower cost machine using the lowest equivalent annual annuity method.

<table>
<thead>
<tr>
<th>Year</th>
<th>Mach.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>PV@6%</th>
<th>EAA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D</td>
<td>-15</td>
<td>-4</td>
<td>-4</td>
<td>-4</td>
<td>-25.69</td>
<td>-9.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>=&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>-10</td>
<td>-6</td>
<td>-6</td>
<td></td>
<td>-21.00</td>
<td>-11.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>=&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Equivalent Annual Annuity

Example (with a twist)

Select one of the two following projects, based on highest “equivalent annual annuity” (r=9%).

<table>
<thead>
<tr>
<th>Project</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>NPV</th>
<th>EAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>−15</td>
<td>4.9</td>
<td>5.2</td>
<td>5.9</td>
<td>6.2</td>
<td>2.82</td>
<td>.87</td>
</tr>
<tr>
<td>$B$</td>
<td>−20</td>
<td>8.1</td>
<td>8.7</td>
<td>10.4</td>
<td></td>
<td>2.78</td>
<td>1.10</td>
</tr>
</tbody>
</table>
Capital Rationing

Capital Rationing - Limit set on the amount of funds available for investment.

Soft Rationing - Limits on available funds imposed by management.

Hard Rationing - Limits on available funds imposed by the unavailability of funds in the capital market.
Profitability Index

Profitability Index – Ratio of present value to initial investment (NPV per dollar spent)

<table>
<thead>
<tr>
<th>Project</th>
<th>C0 (Investment)</th>
<th>C1</th>
<th>C2</th>
<th>NPV@10%</th>
<th>Profitability Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>-3</td>
<td>2.2</td>
<td>2.42</td>
<td>1</td>
<td>1/3 = .33</td>
</tr>
<tr>
<td>M</td>
<td>-5</td>
<td>2.2</td>
<td>4.84</td>
<td>1</td>
<td>1/5 = .20</td>
</tr>
<tr>
<td>N</td>
<td>-7</td>
<td>6.6</td>
<td>4.84</td>
<td>3</td>
<td>3/7 = .43</td>
</tr>
<tr>
<td>O</td>
<td>-6</td>
<td>3.3</td>
<td>6.05</td>
<td>2</td>
<td>2/6 = .33</td>
</tr>
<tr>
<td>P</td>
<td>-4</td>
<td>1.1</td>
<td>4.84</td>
<td>1</td>
<td>1/4 = .25</td>
</tr>
</tbody>
</table>

* N → O (=L) → P → M

* If there is no soft or hard capital rationing, more NPVs will be preferred even when more dollars are spent.

* This rule cannot rank mutually exclusive projects.
Summary

A Comparison of Investment Decision Rules (p.201 Table 7-3)

A recent survey found that

- 75% of firms either always or almost always use both NPV and IRR to evaluate projects
- Just over half of corporations will always or almost always compute a project’s payback period
- Profitability index is routinely computed by about 12% of firms

<table>
<thead>
<tr>
<th>Investment Criterion</th>
<th>Percentage of Firms That Always or Almost Always Use Criterion</th>
<th>Average Score on 0–4 Scale (0 = never use; 4 = always use)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal rate of return</td>
<td>76</td>
<td>3.1 2.9 3.4</td>
</tr>
<tr>
<td>Net present value</td>
<td>75</td>
<td>3.1 2.8 3.4</td>
</tr>
<tr>
<td>Payback period</td>
<td>57</td>
<td>2.5 2.7 2.3</td>
</tr>
<tr>
<td>Profitability index</td>
<td>12</td>
<td>0.8 0.9 0.8</td>
</tr>
</tbody>
</table>