# The Use of Strategy Methods in Experimental Pivotal-Voting Game

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**Abstract**

We use the strategy method to conduct laboratory experiments on the nine-player heterogeneous-cost voting game of Levine and Palfrey (2007). We replicate the underdog effect and competition effect, but find significantly higher voter turnout rates that are only partially explained by the logit quantal response equilibrium. Using cut-offs elicited by the strategy method, we examine round-by-round changes in behavior and find voters are highly responsive to historical pivotal events. Voters also respond to past winning and tying, but only as minority (upsetting the majority), demonstrating an "underdog winning effect," or receiving extra subjective utility when winning as minority. An equilibrium with such asymmetry in utility (and its corresponding two-parameter logit quantal response equilibrium) explains the high minority turnout, as well as the high majority turnout as a best response to it.

J.E.L. classification codes: C71, C91

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## 1. Introduction

Why do some people vote and others don't? As Dhillon and Peralta (2002) note, this question gives economists and political scientists a chance to examine the power of their theories. Many theories have been developed to solve this problem. The rational choice theory (or the pivotal voter model), originally formulated by Downs (1957), caught economists' attention with its purest instrumental rationality approach, and became the most extensively used framework in explaining voter turnout problem.

But it is difficult to reconcile the pivotal voting model with empirical evidence. As noted by Aldrich (1997), "*The rationality of voting is the Achilles' heel of rational choice theory in political science*." The theory predicts that a voter only obtains utility if and only if its vote changes the result. Since the probability of casting the pivotal vote decreases when the size of electorate increases, a rational voter should not vote in a large size election. This prediction is contrary to what we observe in the real world, resulting in the "paradox of voter turnout."

To avoid the difficulty of controlling other extraneous factors in field data, such as preferences, cost distribution, etc., researchers have employed laboratory experiments to investigate voter turnout (Schram and Sonnemans, 1996; Levine and Palfrey, 2007; Duffy and Tavits, 2008). In particular, Levine and Palfrey (2007) conduct a series of simple plurality voting games based on Palfrey and Rosenthal's (1983) model with heterogeneous costs, and find that theory works well at both the aggregate level and the individual level. They confirm nearly all comparative static predictions of Palfrey and Rosenthal (1983) and find that individual subjects follow cut-off strategies but with some error, which could be captured by a logit quantal response equilibrium (logit-QRE) model. In particular, their empirical turnout rates are higher than Nash for large elections ($N$=27, 51), but close to Nash for smaller elections ($N$=9).[1]

[1] For very small elections ($N$=3), voter turnout rate is slightly higher than Nash for the minority group, but much lower than Nash for the majority group.

Given Levine and Palfrey's (2007) success with small elections, we seek to explore the robustness of their result when eliciting voting decision using the strategy method that explicitly elicits cut-off strategies. Eliciting a cut-off is interesting for the following four reasons:

First, many decisions regarding to vote or not (especially when it comes to planning and scheduling) are made prior to knowing the exact voting cost, which depends on unforeseen events like the weather on election day. Hence, these decisions are in fact contingent plans, and indeed change depending on actual rainfall and/or temperature, as shown in Gomez, Hansford and Krause (2007) and Eisinga, Te Grotenhuis, and Pelzer (2012). A simple cut-off strategy is definitely an over-simplification of such plans, but is nonetheless the first step in understanding this complicated decision process.

What is more, the strategy method allows the experimenter to directly observe round-by-round change of individual cut-offs. This is of particular interest because Levine and Palfrey (2007) find that many subjects do not adhere exactly to a fixed cut-off. Instead of attributing this to random noise, one could hypothesize that subjects are changing their strategies as a response to past pivotal events, but this is only testable when we directly observe individual strategies.

Thirdly, the strategy method itself could have an impact on subject behavior since it imposes a monotonic strategy that favors Nash equilibrium, but is more complicated and could be difficult for subjects to understand. The survey of Brandts and Charness (2011) suggests that the strategy method and the direct-response method produce similar results, except in environments involving the use of punishment, and situations involving a lower number of decisions, which do not exist in our experimental design. Nonetheless, experimental subjects do employ different strategies when facing strategically equivalent game, such as ascending English auctions and sealed-bid second price auctions (Kagel, Harstad and Levin, 1987). Thus, it is not clear whether the strategy method would yield the same results as direct response.

Lastly, using the strategy method allows researchers to collect more data that could be used to construct a database for future subjects to play against. However, before pursuing this possibility, one has to know whether results of Levine and Palfrey (2007) replicate under the strategy method.

In this paper, we adopt the strategy method instead of the direct response method used by Levine and Palfrey (2007), and force subjects to use a monotonic cut-off strategy. This modification allows us to observe subjects' cut-offs for each round directly and provides us a chance to investigate subjects' behavior further, especially regarding round-by-round changes in the cut-offs. To ensure comparability, we use the same parameters and terms as the nine-player game in Levine and Palfrey (2007). We find that the underdog effect and competition effect are supported by our data, but we cannot replicate the Levine and Palfrey's (2007) result regarding voter turnout for $N$=9. Instead, voter turnout rates are higher than predicted by the symmetric Nash equilibrium, which is also what Levine and Palfrey (2007) found for $N$=27 and 51.[2] A noisier logit-QRE model can partially explain the excessive turnout. We also find evidence indicating that subjects are highly responsive to historical pivotal events, which is the most important implication of the rational choice model. Finally, we find that subjects are responsive to past winning and tying, but only when in the minority group. This suggests our subjects exhibit an "underdog winning effect," assigning extra subjective utility to winning/tying as a minority, (i.e. upsetting the majority). In fact, an equilibrium with asymmetric winning/tying utility can generate voter turnouts that are higher than 50%. Estimating a logit-QRE model with asymmetric winning/tying utility yields an underdog winning utility of 16.71 and explains the excessive voter turnouts for both the minority group and the majority group (as an equilibrium best-response).

The remainder of this paper is organized as follows. In Section 2, we formulate some

[2] We observe higher turnout rates even in the direct-response training sessions, but these sessions are designed for training purposes and do not have enough observations.

theoretical predictions and propose a set of hypotheses. Section 3 explains the design of our experiments. Section 4 analyzes our experimental results and section 5 concludes.

## 2. The Pivotal-Voting Game

We adopt Levine and Palfrey's (2007) model of simple plurality voting game with heterogeneous costs. The game is played by two groups, the minority group $A$ and the majority group $B$ with each group supporting different candidates. The size of group $A$ is denoted by $N_A$. The size of group $B$ is denoted by $N_B$ where $N_B > N_A$. All voters simultaneously decide whether to vote (for their candidate) or abstain. The candidate who receives more votes wins the election. If both candidates receive the same amount of votes, the winner of the election is decided by flipping a fair coin. The supporters of the winner will receive a payoff of $H$, and supporters of the loser will receive a payoff of $L<H$. Voting is costly. The cost for each voter is drawn independently from the same distribution and the cost is always positive. The group size, the payoff, and the density function of the cost distribution are common knowledge to all voters. Each voter knows his real cost privately before making his decision.

Levine and Palfrey (2007) derive the symmetric Nash equilibrium, $(\tau_A^*(c),\ \tau_B^*(c))$, which involves cut-off turnout strategies: $\tau_j^*$=0 (abstain) if $c > c_j^*$, and $\tau_j^*$=1 (vote) otherwise. In other words, voters vote if and only if their voting cost is no greater than cut-offs $(c_A^*, c_B^*)$. Levine and Palfrey's (2007) equilibrium characterization involves the following six equations: First, the aggregate voting probabilities for each group are

$$(2.1)\quad p_{\mathrm{A}}^* = \int_{-\infty}^{c_{\mathrm{A}}^*} \tau_A^*(c) f(c) dc = F(c_A^*) \qquad (2.2)\quad p_{\mathrm{B}}^* = \int_{-\infty}^{c_{\mathrm{B}}^*} \tau_B^*(c) f(c) dc = F(c_B^*).$$

Next, voters with voting costs equal to the cut-offs are indifferent between voting and abstaining if and only if

$$(2.3)\quad \frac{H-L}{2} \cdot \pi_A^* = c_A^* \qquad (2.4)\quad \frac{H-L}{2} \cdot \pi_B^* = c_B^*$$

where $\pi_A^*$ ($\pi_B^*$) is the probability that a member of group A (B) is pivotal (makes or breaks a tie) given that others are following the equilibrium strategies, or

$$(2.5)\quad \pi_A^* = \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k} (p_{\mathrm{A}}^*)^k (1-p_{\mathrm{A}}^*)^{N_A-1-k} (p_{\mathrm{B}}^*)^k \, (1-p_{\mathrm{B}}^*)^{N_B-k}$$

$$+ \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k+1} (p_{\mathrm{A}}^*)^k (1-p_{\mathrm{A}}^*)^{N_A-1-k} (p_{\mathrm{B}}^*)^{k+1} \, (1-p_{\mathrm{B}}^*)^{N_B-1-k}$$

$$(2.6)\quad \pi_B^* = \sum_{k=0}^{N_A} \binom{N_A}{k} \binom{N_B-1}{k} (p_{\mathrm{A}}^*)^k (1-p_{\mathrm{A}}^*)^{N_A-k} (p_{\mathrm{B}}^*)^k \, (1-p_{\mathrm{B}}^*)^{N_B-1-k}$$

$$+ \sum_{k=0}^{N_A} \binom{N_A}{k} \binom{N_B-1}{k-1} (p_{\mathrm{A}}^*)^k (1-p_{\mathrm{A}}^*)^{N_A-k} (p_{\mathrm{B}}^*)^{k-1} \, (1-p_{\mathrm{B}}^*)^{N_B-k}$$

We adopt the same parameters as Levine and Palfrey (2007) to ensure comparability. In particular, each election has 9 voters, with $(N_A, N_B)$ being either (3, 6) for the election to be a "landslide", or (4, 5), a "toss-up." Members in the winning group received payoff $H = 105$, and members in the losing group received payoff $L = 5$. When a tie occurs, everyone receives a payoff of $\frac{H-L}{2} = 50$. The costs were drawn independently from the uniform distribution between 0 and 55.

Since in our experiment we do not vary the total participants in an election, we can only use turnout rates and average cut-offs to test for the underdog effect (minority group has higher turnout rates) and competition effect (turnout rates are higher in close elections), but not group size effects. In particular, we examine the following four hypotheses:

**H1. (Competition Effect)** The turnout rates and average cut-offs in toss-up elections are higher than that in landslide ones.

1. $p_A^T > p_A^L$ and $p_B^T > p_B^L$
2. $c_A^T > c_A^L$ and $c_B^T > c_B^L$

**H2. (Underdog Effect)** The turnout rates and average cutoffs of the minority group are higher than that of the majority group.

1. $p_A^L > p_B^L$ and $p_A^T > p_B^T$
2. $c_A^L > c_B^L$ and $c_A^T > c_B^T$

**H3. (Competition Effect on the Frequency of Pivotal Events)** The probability of pivotal events is higher in toss-up elections than in landslide ones.

$$\pi^T > \pi^L$$

**H4. (Upset Rate)** The upset rate is lower in landslide elections than in toss-up ones.

$$Q^T > Q^L$$

where $P_A^L(P_A^T)$ is the minority turnout rate in a landslide (toss-up) election, $c_A^L(c_A^T)$ is the minority group's average cut-off, $\pi^L(\pi^T)$ is the frequency of pivotal events (the outcome is either a tie or one vote away from a tie), and $Q^L(Q^T)$ is the upset rate (in which the minority group ties or wins the election).

## 3. Experimental Design and Procedure

All experiments were conducted in Chinese using Z-Tree (Zurich Toolbox for Readymade Economic Experiments, developed by Fischbacher, 2007) at the Taiwan Social Science Experimental Laboratory (TASSEL) in National Taiwan University (NTU). Subjects were recruited via TASSEL's online recruiting website. Announcements were made via online flyers posted on BBS and via email sent to NTU students who registered on TASSEL's website. A total of 108 NTU undergraduate/graduate students participated in the experiments. Average earnings (including show-up fee NT$100) were NT$547.6 (approximately US$18.27), ranging from NT$434.95 to NT$744.55 (approximately US$14.51 to US$24.85), and the exchange rate is 20 Experimental Standard Currency (ESC) for NT$1.

We conduct 4 experiments with 9 subjects for the following three types of experiments: Baseline, Quiz and Quiz+Training. All experiments consist of two 50-round sessions employing the strategy method, one session with toss-up elections and the other with landslide. Half of the experiments (within each type) start with the landslide elections, while the other half start with the toss-up. In "Quiz" experiments, a quiz related to the strategy method is conducted before starting the first session. In the "Quiz+Training" experiment, an initial direct-response training session is added to the "Quiz" experiment, in which we conduct the same toss-up or landslide elections as the first strategy-method session for 50 rounds, but eliciting direct responses.[3]

The three types of experiments were designed to foster better and better understanding of the strategy method, since it is more complicated than direct response. If subjects fail to follow the Nash equilibrium prediction due to insufficient understanding of the strategy method, we should see incremental improvement across the three types of experiments. Each subject received a copy of the experimental instructions that were also read aloud to the subjects to ensure that the information contained in the instructions is induced as common knowledge among the subjects, and screenshots of the experimental software interface were projected along the way.

For a given session, in each of the 50 rounds, subjects were randomly assigned to either group $A$ (minority) or group $B$ (majority). Then, subjects were asked to choose $X$ (vote) or $Y$ (abstain). The group with more subjects choosing $X$ would win the election. Each member in the winning group would receive 105 Experimental Standard Currency (ESC), and each member in the losing group would receive 5. When a tie occurs, all subjects receive 55.

Following Levine and Palfrey (2007), the opportunity cost of voting was referred to as a "Y bonus," drawn independently from a uniform distribution between 0 and 55. If a subject chooses $Y$, he receives a payoff equal to his "Y bonus." In the direct response training sessions, subjects see their "Y bonus" before making a decision, while in the

[3] To familiarize the participants with the direct response method, we also administer a quiz regarding the direct response method at the beginning of this training session.

strategy method sessions, subjects are required to enter a cut-off (termed “Baseline Value”) before learning their “Y bonus.” The “Y bonus” was shown after entering the cut-off, and the computer program makes the decision for the subject by comparing the cut-off and the “Y bonus.” If the “Y bonus” is smaller or equal to the cut-off, the computer will choose *X*. If the “Y bonus” is larger than the cut-off, the computer will choose *Y*. Therefore, the cut-off elicited can be viewed as the subject’s cut-off strategy.

# 4. Experimental Results

## 4.1 Aggregate Results

We compare the results across different experiments. In particular, we focus on average cut-offs, observed turnout rates, probabilities of pivotal events and upset rates and see if these numbers fit the theoretical predictions.

Table 1 presents the average cut-offs for each group in each treatment for each experiment type to investigate the influence of quizzes and training sessions on the results of strategy method sessions. The average cut-offs change in opposite directions when moving from “Baseline” to “Quiz” and from “Quiz” to “Quiz+Training,” which is inconsistent with incremental improvements in comprehension. We treat each subject’s average cut-off for each group in each treatment as a single observation, and employ the Kruskal-Wallis test to examine the effect of quizzes and training direct response sessions on strategy method sessions. We fail to reject the null hypothesis (p-value=0.832). Even if we drop the incremental assumption and conduct binary comparisons with the Mann-Whitney rank sum test treating average cut-offs for each 9-voter experiment in each treatment as a single observation, we still fail to reject the null hypotheses that “Baseline = Quiz”, “Quiz = Quiz+Training” and “Baseline = Quiz+Training” for both the majority and minority groups (all 12 $p$-values $> 0.342$). These results indicate that quizzes and training sessions have little effect on subject behavior, either because subjects already understand the strategy method from the instructions, or because these additions are still insufficient for subjects to understand the strategy method. In any case, this allows us to pool all data together in the following analysis.

Table 1. Comparison of Average Cut-offs Across Experiment Types

| Treatment | Landslide | | Toss-up | |
|---|---|---|---|---|
| Group | Minority | Majority | Minority | Majority |
| Baseline | 29.790 (.849) | 22.733 (.567) | 32.078 (.739) | 27.170 (.651) |
| Quiz | 26.107 (.788) | 24.073 (.518) | 31.371 (.626) | 30.092 (.548) |
| Quiz+Training | 27.678 (.833) | 22.068 (.508) | 32.704 (.643) | 27.911 (.625) |

Table 2 displays the average cut-off and turnout rates of the 12 experiments with their standard errors in parentheses. Nash equilibrium values for each group and treatment are also reported. We find significantly high voter turnout in our data. In fact, average cut-offs and turnout rates are all significantly higher than the theoretical predictions (three of the four average turnout rates are above 50%). Using subject's average cut-off and turnout rates for each group in each treatment as a single observation to conduct t-tests, we obtain $p$-values that are all below 0.01 except for the majority group in landslide elections ($p<0.1$). Nonetheless, the hypotheses of underdog effect (H1) and competition effect (H2) are all supported by our data under a paired t-test using every subject's average cut-off for each treatment and group as a single observation (p-values are 0.007 and 0.012 for underdog effects and 0.0001 and 0.007 for competition effects).

We also find higher than theoretical prediction turnout rates in the four direct response training sessions (Table 3), which does not exhibit underdog effect ($p$-values are 0.8107 and 0.2761 for Mann-Whitney rank sum tests with wrong sign for Toss-up) or competition effect ($p$-values are 0.8493 and 0.1210 for Mann-Whitney rank sum tests).

This hints that higher-than-Nash turnout rates could be present even under direct response. However, these sessions were designed for training purposes, and have small sample size, so we refrain from drawing firm conclusions.[4]

Table 2. Average Cut-offs and Turnout Rates

| Average Cut-offs | | | | |
|---|---|---|---|---|
| Treatment | $c_A$ | $c_A^{Nash}$ | $c_B$ | $c_B^{Nash}$ |
| Landslide | 27.85** (0.476) | 22.715 | 22.958* (0.307) | 20.615 |
| Toss-up | 32.05*** (0.39) | 25.300 | 28.39** (0.353) | 24.860 |
| Turnout Rates | | | | |
| Treatment | $p_A$ | $p_A^{Nash}$ | $p_B$ | $p_B^{Nash}$ |
| Landslide | 0.515*** (0.025) | 0.413 | 0.417 (0.023) | 0.374 |
| Toss-up | 0.594*** (0.024) | 0.460 | 0.520** (0.022) | 0.452 |

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$ for t-tests against the prediction of Nash equilibrium.

Table 3. Turnout Rates of Direct Response Sessions

| Treatment | $p_A$ | $p_A^{Nash}$ | $p_B$ | $p_B^{Nash}$ |
|---|---|---|---|---|
| Landslide | 0.468 (0.032) | 0.413 | 0.449 (0.052) | 0.374 |
| Toss-up | 0.501 (0.040) | 0.460 | 0.541 (0.036) | 0.452 |

Table 4 displays observed proportions and theoretical probabilities of pivotal events and upset rates (standard errors in parentheses). The observed proportions of pivotal events are close to the theoretical prediction for the two treatments, but the upset rates in both treatments are significantly higher than what theory predicts. This is due to the 9.8% and 7.4% turnout rate differences of the two groups (for landslide and toss-up, respectively) which are much higher than the 3.9% and 0.8% predicted by theory. We

[4] In particular, Levine and Palfrey (2007) report 18 direct response sessions (9 landslide and 9 toss-up sessions), while we have only 4 direct response training sessions (2 landslide and 2 toss-up sessions).

conduct a t-test (using every single round as an observation) to examine the competition effect on pivotal events and upset rates (H3, H4) and they are supported by our data ($p$-values are 0.0042 and 0.0000).

Table 4. Probability of Pivotal Events and Upset Rates

| Probabilities of Pivotal Events | | |
|---|---|---|
| Treatment | $\pi$ | $\pi^{Nash}$ |
| Landslide | 0.598 (0.007) | 0.599 |
| Toss-up | 0.672 (0.006) | 0.666 |
| Upset Rates | | |
| Treatment | Q | $Q^{Nash}$ |
| Landslide | 0.378*** (0.007) | 0.151 |
| Toss-up | 0.598*** (0.007) | 0.270 |

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$ for t-tests against the prediction of Nash equilibrium.

## 4.2 Individual Results

After examining the data at the aggregate level, we turn to examine how the rational choice model works at the individual level. When subject's behavior satisfies the following conditions, we classify him/her as complying with the corresponding effect:

1. *Within-Subject Competition Effect*: A subject exhibits individual competition effect if his/her average cut-off in the toss-up treatment is higher than that in the landside treatment (for each group).
2. *Within-Subject Underdog Effect*: A subject exhibits individual underdog effect if his/her average cut-off as a minority is higher than that as a majority in both landslide and toss-up treatments.

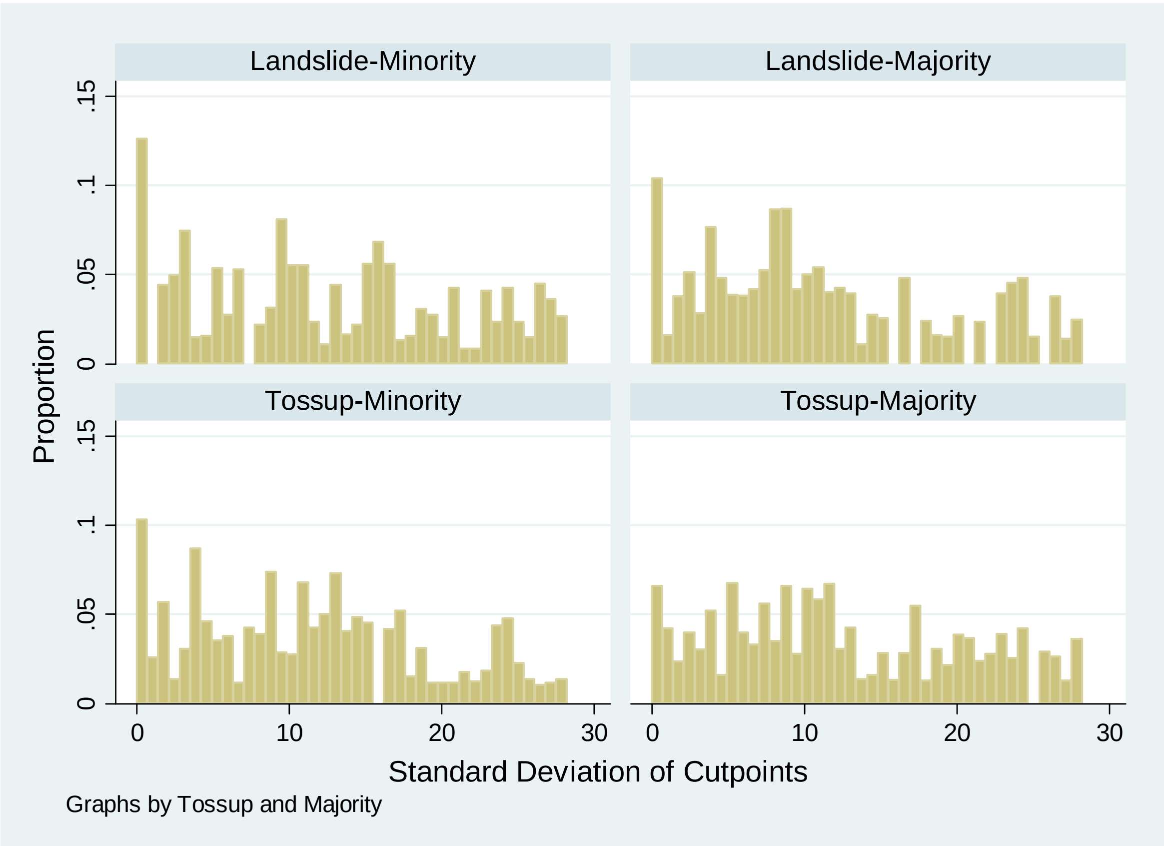
Table 5 shows the proportions of subjects satisfying each condition. 57-68% of the subjects exhibit individual competition or underdog effects, which is comparable to the 54-65% found by Levine and Palfrey (2007). This result shows that the rational choice model works well even at the individual level for more than half of the subjects.

Table 5. Proportion of Subjects Exhibiting Individual Competition or Underdog Effects

| Within-Subject Competition Effect | | Within-Subject Underdog Effect | |
|---|---|---|---|
| Minority | Majority | Toss-up | Landslide |
| 0.574 | 0.676 | 0.583 | 0.611 |

To examine whether each subject uses a fixed cut-off strategy, we calculated the standard deviation of their cut-offs in each treatment and group. The results are shown in Figure 1. As can be seen, most subjects frequently change their cut-offs, though the modes are zero. This indicates that many subjects do not literally use one single deterministic cut-off strategy across the 50 rounds of a session. Moreover, changes in cut-offs would allow us to investigate how subjects adjust their cut-offs round-by-round, say, responding to history.

Figure 1 Standard Deviation of Cut-offs



## 4.3 Multivariate Analysis

Using the strategy method allows us to observe how subjects adjust their cut-offs round-by-round. Hence, we conduct a series of regressions with random effects to investigate changes in subjects' behavior, and report the results in Table 6. We consider the following two questions: First, do subjects respond to being pivotal? Second, if they do, how do they respond?

In Table 6, model 1 is the baseline model and tests for competition and underdog effects. We include three independent variables in Model 1, which are *Tossup* (for treatments that are toss-ups), *Majority* (for subjects in the majority group) and *Round* (for the round number). From column (1) of Table 6, we find strong support for the competition effect and underdog effect confirming the aggregate results. Moreover, the round number has a positive effect on subjects' cut-offs, which could be consistent with a learning story. However, given the aggregate high turnout rates, a positive coefficient for *Round* indicates that subjects' behaviors do not converge toward equilibrium. Instead, they drift away. Since *Round* matters, we use Model 2 and 3 to investigate how subjects adjust their cut-offs after the occurrence of a pivotal event. We define the variable *IsPivotal*(t-1) as whether the voter was pivotal last time he/she was in the same group. *IsPivotal*(t-2) indicates whether the voter was pivotal in the "next-to-last" time, and so on. Consistent with Duffy and Tavits (2008), we find subjects increasing their cut-offs after being pivotal. This result indicates that subjects are indeed responding to history throughout the experiment.

In Model 4, we add the variable *PivotalFreq*(t-1), or the historical frequency of being pivotal conditional on group. Consistent with the rational choice model, we find that subjects indeed increase their cut-offs when they perceive that they are more likely to be pivotal, which is contrary to what Duffy and Tavits (2008) find.

Table 6. Random Effect Model of Cut-offs

| | (1) Cut-off | (2) Cut-off | (3) Cut-off | (4) Cut-off |
|---|---|---|---|---|
| *Tossup* | 4.949*** | 4.551*** | 4.054*** | 3.592*** |
| | (0.338) | (0.344) | (0.359) | (0.346) |
| *Majority* | -4.301*** | -4.364*** | -4.563*** | -4.330*** |
| | (0.349) | (0.355) | (0.371) | (0.351) |
| *Round* | 0.0345** | 0.0337** | 0.0260 | 0.0350** |
| | (0.0116) | (0.0122) | (0.0137) | (0.0121) |
| *IsPivotal*(t-1) | | 3.467*** | 3.432*** | |
| | | (0.345) | (0.357) | |
| *IsPivotal*(t-2) | | | 2.718*** | |
| | | | (0.357) | |
| *IsPivotal*(t-3) | | | 1.949*** | |
| | | | (0.356) | |
| *PivotalFreq*(t-1) | | | | 15.90*** |
| | | | | (0.863) |
| *Constant* | 26.58*** | 25.18*** | 23.63*** | 19.92*** |
| | (0.887) | (0.915) | (0.989) | (0.972) |
| *N* | 10800 | 10368 | 9504 | 10368 |

Standard errors are clustered at the subject level and reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, given excessive upset rates in Table 4, we consider how the different groups of voters respond to upsetting (i.e. winning or tying with) the other group. In particular, we conduct random effect regressions predicting the minority (majority) group cut-offs using *NotLoss*(t-1), the dummy for winning or tying with the other group, controlling for *Tossup*, *Round* and *IsPivotal*(t-1). As shown in Column (1) and (2) of Table 7, minority voters respond to upsets by significantly increasing their cut-offs by 4.35, but majority voters do not. Other coefficients are still all significant, demonstrating the robustness of the underdog effect and response to history.[5] In other words, subjects

[5] In contrast, little is found using data from direct response, as shown in Colum (3) and (4) of Table 7. However, this is possibly due to insufficient observations.

have differential responses to winning or tying for the two groups. This suggests that Taiwanese students exhibit an "underdog winning effect," assigning extra (non-monetary) utility to winning (or tying) as a minority. In fact, if one plugs empirical turnout rates $p_A$ and $p_B$ and the average cut-off $c_A$ into Equations (2.5) and (2.3), one may solve for the subjective utility for the minority group, $u^{minority}\left(\frac{H-L}{2}\right)$ = 63.28 for landslide and 62.65 for toss-up, both much higher than 50.[6] Replacing $\left(\frac{H-L}{2}\right)$ with $\left(\frac{H-L}{2}\right)+D$, $D>0$, in Equation (2.3), one can solve the equilibrium where the minority experience additional winning/tying utility. For $D = 63 - 50 = 13$, the minority turnout rates are $p_A^{UD}=$ 52.2% and 59.5% for landslide and toss-up, respectively, and majority turnout rates are $p_B^{UD}=$ 40.5% and 47.6%, all closer to empirical turnout rates than the symmetric Nash equilibrium without additional underdog winning/tying utility (Table 8).

Table 7. Random Effect Models for Minority and Majority Groups

| | (1)<br>Minority<br>Cut-off | (2)<br>Majority<br>Cut-off | (3)<br>Minority<br>Vote | (4)<br>Majority<br>Vote |
|---|---|---|---|---|
| *Tossup* | 3.131*** | 4.948*** | 0.068 | 0.388 |
| | (0.526) | (0.401) | (0.221) | (0.285) |
| *Round* | 0.0752*** | -0.000693 | 0.0009926 | -0.01* |
| | (0.0182) | (0.0140) | (0.0058) | (0.0047) |
| *NotLoss(t-1)* | 4.351*** | -0.166 | 0.221 | -0.278 |
| | (0.612) | (0.475) | (0.203) | (0.2) |
| *IsPivotal(t-1)* | 1.921** | 2.855*** | 0.179 | 0.206 |
| | (0.584) | (0.398) | (0.198) | (0.137) |
| *Constant* | 23.54*** | 22.00*** | -0.284 | 0.18 |
| | (1.155) | (1.122) | (0.2311) | (0.301) |
| *N* | 3984 | 6384 | 664 | 1064 |

Standard errors are clustered at the subject level and reported in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

[6] A similar procedure yields subjective utility for the majority group, $u^{majority}\left(\frac{H-L}{2}\right)$ = 53.77 for landslide and 54.76 for toss-up, both close to 50.

## 4.4 Quantal Response Equilibrium

In this section, we employ the logit quantal response equilibrium (logit-QRE) model to account for deviations from Nash equilibrium. In particular, Levine and Palfrey (2007) show that by replacing voter's turnout probabilities with continuous functions of voting costs

$$\tau_j(c; \lambda) = \frac{1}{1+e^{\lambda\left(\frac{c}{H-L}-\frac{\pi_j}{2}\right)}},\ j = A,\ B,$$

one could derive voters' ex ante turnout probabilities and pivotal probabilities using equations similar to (2.1), (2.2), (2.5) and (2.6), and compute the logit-QRE for any given $\lambda$. We estimate this free parameter $\lambda$ to fit the outcome data of all of our experiments (including both landslide and toss-up elections) using maximum likelihood and find $\hat{\lambda} = 5.98$.[7] This estimated parameter is smaller than what Levine and Palfrey (2007) obtain using all of their data ($\hat{\lambda} = 7$), indicating that we require more noise to account for deviations in our data. The turnout rates $p_A^{\lambda=5.98}$ and $p_B^{\lambda=5.98}$ for the logit-QRE model are reported in the third column of Table 8. They are all on the right track, but the magnitudes of improvement are modest. In particular, the logit-QRE model cannot account for the minority turnout rates, which are both above 50% ($p$<0.001 for both t-tests using each group in each treatment as a single observation). We can also reject the logit-QRE prediction for the majority turnout rate in toss-up elections ($p$<0.05, similar t-test), but not in landslide elections.

We also consider the equilibrium where minority voters experience additional utility $D > 0$ during an upset. By estimating a two-parameter logit-QRE model (again using all of our outcome data), we obtain a logit error parameter of $\hat{\lambda} = 7.12$ and an additional underdog winning/tying utility of $\hat{D} = 16.71$. Table 8 reports the resulting voter turnout rates $p_A^{D=16.71,\lambda=7.12}$ and $p_B^{D=16.71,\lambda=7.12}$ for the two groups under each

[7] We estimate the model using realized voting costs and turnout outcomes with the same turnout quantal response function as Levine and Palfrey (2007) to maintain comparability.

treatment. All turnout rates are closer to the empirical data than the original one-parameter logit-QRE model. In fact, the likelihood ratio test statistic is LR = 282.47 with $df = 1$, strongly rejecting the restricted one-parameter model.[8]

Table 8. Empirical and Estimated Turnout Rates

| Minority Turnout Rates ($p_A$) | | | | | |
|---|---|---|---|---|---|
| Treatment | Data | $p_A^{Nash}$ | $p_A^{\lambda=5.98}$ | $p_A^{D=13}$ | $p_A^{D=16.71,\lambda=7.12}$ |
| Landslide | 0.515 (0.025) | 0.413 | 0.424 | 0.522 | 0.521 |
| Toss-up | 0.594 (0.024) | 0.460 | 0.470 | 0.595 | 0.558 |
| **Majority Turnout Rates ($p_B$)** | | | | | |
| Treatment | Data | $p_B^{Nash}$ | $p_B^{\lambda=5.98}$ | $p_B^{D=13}$ | $p_B^{D=16.71,\lambda=7.12}$ |
| Landslide | 0.417 (0.023) | 0.374 | 0.401 | 0.405 | 0.418 |
| Toss-up | 0.520 (0.022) | 0.452 | 0.466 | 0.476 | 0.478 |

## 5. Conclusion

There are five main findings from our experiment.

First, we find the underdog effect and competition effect predicted by the pivotal voting model supported by using average cut-offs, probabilities of pivotal outcome, upset rates and with panel regressions. This confirms Levine and Palfrey (2007). Second, unlike Levine and Palfrey's (2007) result for nine players, our average cut-offs and turnout

[8] Note that plugging in $D = 13$ alone yields turnout rates similar to the two-parameter logit-QRE model for the minority in landslides and the majority in toss-ups, predicts worse for the majority in landslides, but is closer to data for the minority in toss-ups. This is likely due to our attempt (following Levine and Palfrey, 2007) to fit one set of parameters on *all* of our data, which features more observations for the majority group (in landslides) than minority group (in toss-ups).

rates are all significantly higher than theoretical predictions. This can be partially explained by the logit-QRE model where voters make small mistakes. Third, we find evidences which affirm that rational choice model works well at the individual level for the majority of the subjects follow the underdog effect and competition effect despite using the strategy method. Fourth, our results indicate that subjects are highly responsive to historical pivotal events. This result shows that a subject raises his/her cut-off when he/she perceives the probability of being the pivotal voter increasing, which is the most important implication of the rational choice model. Lastly, our subjects also respond asymmetrically to past winning or tying, exhibiting an "underdog winning effect." The equilibrium where minority voters assign much higher subjective utility to an upset (and its corresponding two-parameter logit-QRE model) explains the high minority turnout rate and the high upset rate, as well as the high majority turnout rate as a best response to high minority turnout.

There could be other explanations for the observed excessive turnouts. One possible reason is that there are group oriented subjects (Feddersen and Sandroni, 2006) in our experiments. They may put more value on group utility than their individual benefits, and make turnout rates higher than predictions of rational choice model. Another possibility is that Taiwanese subjects are more enthusiastic in politics, since the political culture in Taiwan is generally enthusiastic, having voter turnout rate 74-82% in the past few presidential elections.[9] However, both explanations cannot easily account for the same subjects' asymmetric responses to winning/tying as minority and majority, and require some form of "underdog winning effect" where subjects have higher subjective utility of winning/tying as a minority group to rationalize both higher turnouts and asymmetric responses.

[9] Turnout rates of the 1996, 2000, 2004, 2008, 2012 presidential elections were 76.04%, 82.69%, 80.28%, 76.33%, and 74.38%, respectively.

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