Micro Theory I: Midterm

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Total Score: 30%, plus bonus 4% point (and 2% for those who missed Crawford's lectures).

1 Allocating Rights to Enroll in Classes [10%]

Many classes at National Taiwan University face an overflow of students who want to enroll. Now, the university decides to give students the rights to take whatever class they enroll in initially. The university has to either find classrooms that are large enough to host all students, or "buy them out"— pay students scholarships so they are willing to forego the rights to take the course.

Consider a classroom with capacity size q_0, scheduled n classes per week, and each class i has initial enrollment $\overline{x}_i$. Suppose the building cost for capacity q_0 is $C(q_0)$, and $p_i(x_j)$ is the willingness-to-forego for student x_j (to give up class i). The students are lined up so that $p_i(x_j)$ is a decreasing function, so that the total scholarship payment is

$$\begin{aligned} B(q_i) &= \int_{q_i}^{\overline{x}_i} p_i(x_j)dx_j && \text{if } q_i < \overline{x}_i \\ &= 0 && \text{if } q_i \geq \overline{x}_i \end{aligned}$$

NTU chooses capacity q_0, and actual enrollments $q_1, \cdots, q_n$ to minimize its cost (scholarship payout).

1. (1%) Write down NTU's cost minimization problem as a constrained maximization and state its corresponding Lagrangian.

2. (1%) What assumption would you need to have a unique solution to this problem?

3. (2%) What are the first order conditions, and when would the equalities hold?

4. (1%) What is the shadow price when a class does not hit its capacity ($\overline{x}_i < q_0$)?

5. (2%) When would NTU choose to increase the capacity of the classroom? When would NTU choose to hand out scholarships? What capacity would NTU set?

6. (2%) Assume that $C(q_0) = q_0^2$, $n = 3$, and $p_i(x_j) = 10000 - a_i \cdot x_j$, $a_i = 100, 80, 50$. Solve for the optimal q_0, q_1, q_2, q_3 and NTU's total cost.

7. (1%) Suppose now NTU assignes the property rights so that students are required to pay extra tuition to enroll in classes with excess demand. If the capacity q_0 and final enrollments q_1, q_2, q_3 are the same as above, would students who enroll be the same as above? Why or why not?

2 Roy's Identity? [8%]

1. (2%) Consider two consumers: A and B, having utility functions (for $x_i^h \geq 0$)

$$
\begin{aligned}
u_A(x_1^A, x_2^A) &= -\frac{A_1}{x_1^A} - \frac{A_2}{x_2^A} && \text{if } x_1^A \cdot x_2^A > 0, \\
&= -\infty && \text{if } x_1^A \cdot x_2^A = 0. \\
u_B(x_1^B, x_2^B) &= \min\{2x_1^B, 3x_2^B\}.
\end{aligned}
$$

   Draw the income expansion path for the two utility functions.

2. (4%) Derive the indirect utility function $V_A(p, I)$ and $V_B(p, I)$ for a given price vector p and income I. Can you use the Roy's Identity to derive each consumer's demand? Why or why not?

3. (2%) Hence, or otherwise, derive $x_i^{h*}(p, I)$, consumer h's demand functions for consumer $h = A, B$ and commodity $i = 1, 2$.

3 2x2 Exchange Economy [6%]

(Hint: You should use what you have learned in the previous section.)

1. (2%) Consider two consumers: A and B, having utility functions

$$\begin{aligned} u_A(x_1, x_2) &= \min\{4x_1, 6x_2\}, \\ u_B(y_1, y_2) &= -\frac{1}{B_1 y_1} - \frac{1}{B_2 y_2} && \text{if } y_1 \cdot y_2 > 0, \\ &= -\infty && \text{if } y_1 \cdot y_2 = 0. \end{aligned}$$

   Suppose consumer A and B both have endowment $(\omega_1, \omega_2) = (30, 30)$. Draw the Edgeworth box of this 2-person economy and indicate the Pareto efficient allocations.

2. (2%) What is the Walrasian equilibrium for these two consumers?

3. (2%) Are all of the Pareto efficient allocations implementable as Walrasian Equilibrium? Why or why not?

4 "Silent Sit-In" as Public Goods Contribution [6% + 6%]

There are a group of students who want to protest for human right violation by performing a "silent sit-in" at Liberty Square. However, it is midterm week, so it is very costly to participate in the sit-in (since you cannot study for the midterm as well as at home). The effect of the sit-in will be determined by the total sit-in hours of the whole group.

In particular, let the hours of sit-in for student i be g_i for $i = 1, \cdots, n$, and the total sit-in hours be $G = \frac{1}{n}\sum_i g_i$. A student i's leisure hours is

$$x_i = 24 - g_i + m_i \sum_{k=1}^{n} g_k$$

where m_i is the value ratio the student assigns to the effect of the sit-in, in terms of leisure hours he is willing to forego.

1. (2%) Assume $m_i = m < 1$ for all $i = 1, \cdots, n$, and each student cares only about his or her leisure hours. What is the Nash equilibrium contribution level g_i?

2. (2%) Now assume each student has a "guilt-envy" utility function (Fehr and Schmidt, 1999):

$$u_i(x) = x_i - \frac{\alpha_i}{n-1} \sum_{k \neq i} \max(x_i - x_k, 0) - \frac{\beta_i}{n-1} \sum_{k \neq i} \max(x_k - x_i, 0)$$

where $0 \le \beta_i \le 1$, $\beta_i \le \alpha_i$.

For a given set of g_k ($k \neq i$), show that students with $\beta_i < 1 - m_i$ will not go to the sit-in at all ($g_i = 0$).

3. (2%) Suppose $g_1 \le g_2 \le \cdots \le g_n$. If there are k students who have $\beta_i < 1 - m_i$ (so $g_1 = g_2 = \cdots = g_k = 0$), and the rest have $\beta_i > 1 - m_i$ and

$$\frac{m_i + \beta_i - 1}{\alpha_i + \beta_i} > \frac{i-1}{n-1}.$$

Show that it is a Nash equilibrium for the k students to not go at all ($g_i = 0$), and the rest go for certain hours ($g_i > 0$).

4. (**bonus** 2%) Furthermore, show that everyone will not go to the sit-in at all ($g_i = 0$) if they believe that k other students are not going at all, in which $k > \max\left\{\frac{m_i(n-1)}{2}\right\}$.

5. (**bonus** 2%) How would you take this theory to data?

6. (**bonus** 2%) **[This is for those who could not attend the level-k thinking lectures and want to make it up.]**
Suppose on the first day one cannot observe the amount of hours others have decided to sit in (when they make their own sit-in decision). If a L0 student randomly chooses $g_i \in [0, 24]$. What is the best response for a L1 student (believe everyone else is L0), under the assumption that L1 student cares only about his or her leisure hours? What is the best response for a L2 student (who believes every else is L1) under the same assumption?

   Can a level-k thinking model explain the heterogeneious sit-in behavior you observe in the data? Why or why not?

5 Suggested Answers

5.1 Allocating Rights to Enroll in Classes

1. The minimization problem can be written as maximizing negative cost:

$$\begin{aligned}
&\min_{q_0,q_i} \left\{ C(q_0) + \sum_{i=1}^{n} B(q_i) \right\} \\
&\quad \text{s.t. } q_i \leq q_0 \\
&= \max_{q_0,q_i} \left\{ -C(q_0) - \sum_{q_i \leq \overline{x}_i} \int_{q_i}^{\overline{x}_i} p_i(x_j) dx_j \right\} \\
&\quad \text{s.t. } q_i \leq q_0
\end{aligned}$$

   The Lagrangian is

$$\mathcal{L} = -C(q_0) - \sum_{q_i \leq \overline{x}_i} \int_{q_i}^{\overline{x}_i} p_i(x_j) dx_j + \sum_{i=1}^{n} \lambda_i (q_0 - q_i)$$

2. The constraints are all linear, so we only need the objective function to be strictly quasi-concave to get a unique solution.

3. The first order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q_0} &= -C'(q_0) + \sum_{i=1}^{n} \lambda_i \leq 0, \text{ with equality if } q_0 > 0 \\
\frac{\partial \mathcal{L}}{\partial q_i} &= p_i(q_i) - \lambda_i \leq 0, \text{ with equality if } q_i > 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda_i} &= q_0 - q_i \geq 0, \text{ with equality if } \lambda_i > 0
\end{aligned}$$

4. First, $q_0 > 0$ (or we will get zero capacity, and $q_i = 0$, trivial). Moreover, for all $q_i < q_0$, the shadow price $\lambda_i = 0$.

5. Since for $q_i > 0$, $p_i(q_i) = \lambda_i$, you will not hit capacity only if $p_i(q_i) = 0$. In other words, for any q_0, NTU should choose $q_i = q_0$ for all $\lambda_i = p_i(q_0) > 0$. Furthermore, NTU should choose q_0 such that $\sum_{q_0 \leq \overline{x}_i} p_i(q_0) = C'(q_0)$.

6. Plug in the numbers... (TBA)

7. Things will be the same due to the Coase Theorem (ignoring transaction cost, since both systems should have similar transaction costs because they have the same set of people transfering between them, just the opposite direction.