Microeconomic Theory I Final [1/15/2010]

Please Note: You have 3 hours (2:20-5:20pm); there are a total of 80 points plus bonus 30 points (and would count for "40% + bonus 15%" toward your final grade). Allocate your time wisely.

Part A (10%): Properties of the Utility/Production Function

1. (5%) The degree of relative risk aversion is $RRA(c) = -cv''(c)/v'(c)$. Show that if $v'(c) = c^{-1/\sigma}, \sigma > 0$, the individual exhibits constant relative risk aversion.
2. (5%) The production function for some good is given by $y = x_2\left[\sin\left(\frac{x_1}{x_2} - \frac{\pi}{2}\right) + 1\right]$ for $0 \le x_1 \le 2\pi$ and $0 < x_2 \le 2\pi$ and $y = 0$ when $x_2 = 0$. What are the scale properties of this technology?

Part B (60%): Blackboard Economics, Dooms Day Prophecy, FTA, and Asset Markets

First, consider Professor W's class teaching "blackboard economics." In the class, Professor W introduces an exchange economy with *H* consumers. Suppose consumer $h, h = 1, ..., H$ has a logarithmic utility function $u(x) = \sum_{i=1}^{n} \alpha_i \ln x_i$.

1. (4%) Show that if this is an endowment economy with aggregate endowment $(\omega_1, ..., \omega_n)$, the Walrasian equilibrium prices satisfy
$$\frac{p_i}{p_1} = \frac{\alpha_i \omega_1}{\alpha_1 \omega_i}.$$

Now some more realistic scenarios… **Note that you should use what you learned in question 1 to answer the following questions.** First, suppose a "prophet" comes to town and claims that Daiwan will face judgment day exactly after three years.

2. (6%) Believing this prophecy, each individual has the utility function (of yearly consumption):
$$U(c) = \ln c_1 + \delta \ln c_2 + \delta^2 \ln c_3.$$
The aggregate endowment in the three years is $(\omega_1, \omega_2, \omega_3) = \left(k, (1+\theta)k, (1+\theta)^2 k\right)$.
There is no storage. Solve for the Walrasian equilibrium prices. Hence, show that the equilibrium interest rate between year 1 and year 2 is the same as the interest rate between year 2 and year 3.

After three years, nothing happens and people forget about the prophecy. Instead, consider Daiwan's (one-period) economy recovering from last year's recession. This year, the economy could be in three states, depending on whether important trade agreements could be signed. In particular, state 1 happens if Daiwan only signed one agreement, the EKHA with Dailiok; state 3 happens if Daiwan could sign both EKHA and the FTA with Beekok. State 2 is the status quo.

3. (5%) Given the probabilities of each state (π_1, π_2, π_3), each individual has a utility function $u(c) = \sum_{i=1}^{n} \pi_i \ln c_i$. The aggregate supply of state claims is (100, 200, 300). Solve for the state claims equilibrium prices.

Now assume that the states are equally likely. Suppose that the "endowment" is really two assets, "ELSE" and "GLDW." The riskless asset ELSE has a yield of 100 in each state. Risky asset GLDW yields nothing in state 1, 100 in state 2, and 200 in state 3.

4. (4%) What will be the prices of these two assets?
5. (4%) Explain why, in general, trading in asset markets alone cannot be a perfect substitute for trading in state claim markets.

Suppose Daiwan has two groups of individuals, each accounting for half of the population. Group 1 initially owns asset ELSE while group 2 owns asset GLDW.

6. (8%) What is the state claims equilibrium allocation? If there were no state claims markets, and the asset market prices were the same as in Question 4, could each group trade to their allocation in the state claims equilibrium?
7. (4%) Draw a conclusion about the efficiency of the asset market equilibrium in this case.

Now assume there is a third asset, "CJZK" which yields 300 in state 1, 200 in state 2, and 100 in state 3. Suppose 50% of asset CJZK is owned by group 1, and the rest is owned by group 2.

8. (10%) What is the new aggregate endowment? Solve for the new state claims equilibrium prices. What are the prices of the three assets?`
9. (8%) What is the new state claims equilibrium allocation? Are consumers fully insured? why or why not?
10. (7%) Can the asset markets substitute for the state claims markets and achieve this allocation? Why or why not?

Part C (10%): H1N1 Vaccine Production

The biotech firm Koh-Keng produces H1N1 swine flu vaccines to meet the demands of Daiwan. The price it can charge for vaccines is fixed at p = NT$100, and it must meet all demand at that price. It turns out that the amount of vaccines demanded is always the same over every 1-year period, but demand differs from flu season (September to February) to non-flu season (March to August). During the fall-winter flu season, four million shots are demanded, whereas during the spring-summer non-flu season only three million shots are demanded. Total output for each 1-year period is thus always equal to seven million shots. Koh-Keng produces vaccines according to the production function

$$y_i = (KF_i)^{1/2},\ i = \text{flu season, non-flu season } (y_i \text{ measured in millions}),$$

Where K is the size of the generating plant, and F_i is million dozens of eggs. The firm must build a single plant; it cannot change plant size from flu season to non-flu season. If a unit of plant size costs w_k per 1-year period and a million dozens of eggs costs w_f, what size plant will Koh-Keng build?

Part D (Bonus 30%): Input Price Equalization

Consider firm DW, which produces a high tech commodity y_1 and a low tech commodity y_2, with Cobb-Douglas production functions $y_1 = K_1^{\alpha} L_1^{1-\alpha}$ and $y_2 = K_2^{\beta} L_2^{1-\beta}$ using inputs of capital $K = K_1 + K_2$ and labor $L = L_1 + L_2$.

1. (4%) Verify that the high tech commodity y_1 is more capital intense if $\alpha > \beta$. In other words, $MRTS_1(K, L) > MRTS_2(K, L)$ for all inputs (K, L).
2. (6%) Now assume $\alpha > \beta$ and the firm takes output prices (P_1, P_2) as given. If the firm owns input endowments (ω_K, ω_L) and cannot buy or sell inputs elsewhere, what is the profit maximizing allocation input ratio $\frac{L_1^*}{K_1^*}$ if both commodities are produced?
   (Hint: You do NOT have to solve for each amount separately; the ratio is sufficient.)
   (Hint2: To get rid of ω_K, ω_L, or equivalently, L_2^*, K_2^*, appeal to the equality derived from $MRTS_1(K_1, L_1) = MRTS_2(K_2, L_2)$, which is true because "both goods are produced.)
3. (10%) Suppose there are only two industries in Daiwan, high tech and low tech, each having numerous firms producing either y_1 or y_2 with the same technology of firm DW: $y_1 = K_1^{\alpha} L_1^{1-\alpha}$ or $y_2 = K_2^{\beta} L_2^{1-\beta}$ (and nothing else). The total amount of skilled and unskilled labor in Daiwan is (ω_K, ω_L). Explain why we can view the aggregate of all

firms in Daiwan as the single firm DW. Hence or otherwise, Solve for the equilibrium input price ratio $\frac{r_K}{r_L}$ that supports the input allocation $\frac{L_1^*}{K_1^*}$.

4. (3%) Use your results to derive a one-to-one mapping between the equilibrium output price ratio $\frac{P_1}{P_2}$ and the input price ratio $\frac{r_K}{r_L}$. Does this depend on total input endowment?
5. (4%) Hence or otherwise, establish the **Proposition of Input Price Equalization**: Suppose two countries have the same Cobb-Douglas technologies to produce two goods with capital and labor. Suppose also that at the aggregate endowment for each country, commodity 1 is more capital intense. Then if the two countries trade at the same world prices and both commodities are produced, the Walrasian Equilibrium input prices are also the same.
6. (3%) How would your answers apply to the ongoing debate regarding how ECFA would affect the wages in Taiwan? What assumptions are likely to be false in reality? How would your answers change if these assumptions fail?