Robinson Crusoe Economy

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(Lecture 12, Micro Theory I)
Chapter Overview

- Ch. 3: Equilibrium in an Exchange Economy;
- This Chapter: generalize to include production
- **Section 5.1:** Simplest economy possible:
  - Robinson Crusoe Economy (1 person only)
- **Section 5.2:** General equilibrium model with production and the 1st & 2nd Welfare Theorem
- **Section 5.3:** Existence of a Walrasian Equil.
- **Sec. 5.4-6:** Examples--time, public goods, CRS
  - In this course, only have time for CRS example
One Person Economy: Robinson Crusoe Economy

- The simplest case: Robinson Crusoe Economy
- Robinson the manager (price-taker)
  - Decides how much output to produce
- Crusoe the consumer (price-taker)
  - Decides how many hours to work and how much output to consume
- Walrasian Equilibrium: Market clearing price
- Does the Walrasian Equilibrium always exist?
  - Not if the production set is not convex…
Why do we care about this?

- Equilibrium is the central concept in economics
  - Where forces of supply and demand balance out
- Empirically used to predict outcome
- Robinson Crusoe Economy: See how it works in the simplest example (one person economy)
  - Get intuition about how it works in this “toy model”
  - Then generalize to other cases…
- What if you happen to be in an island alone?
- Also, some macro models have only one agent!
One Person Economy: Robinson Crusoe Economy

- **2 Commodities**: Labor hours (good 1) & corn (2)
- **Consumers**: $U^h(x^h) = U^h(x^h_1, x^h_2)$ \( h = 1, \ldots, H \)
  - Endowment: $\omega^h = (\omega^h_1, \omega^h_2)$
  - As if ONE representative agent:
  - Robinson Crusoe with endowment: $\omega = \sum_{h=1}^{H} \omega^h$
- **Single Firm**: with convex production set $\gamma$
Produce Corn with Labor Hours

Commodity 1 (labor hours) is an input to produce commodity 2 (corn).

\[ y_2 > 0 \]
\[ y_1 < 0 \]
\[ y_1 = -z_1 \]
The Optimum

Feasible consumption bundle $\gamma + \omega$

Endowment $\omega$

$x_1, y_1$

$\max_x \{U(x) | x \in \gamma + \omega\}$

$x^* = y^* + \omega$
Example:

\[ U(x) = \ln x_1 + \ln x_2 \]
\[ \gamma = \{(y_1, y_2) | y_1 \leq 0, y_2^2 + y_1 \leq 0\}, \omega = (144, 3) \]

Since \( x = y + \omega \) (to maximize utility)

\[ U(y + \omega) = \ln(\omega_1 + y_1) + \ln(\omega_2 + y_2) \]
\[ = \ln(144 - y_2^2) + \ln(3 + y_2) \]

(Since utility increasing implies \( y_1 = -y_2^2 \))

FOC:

\[ \frac{dU}{dy_2} = -\frac{2y_2}{144 - y_2^2} + \frac{1}{3 + y_2} = \frac{144 - 6y_2 - 3y_2^2}{(144 - y_2^2)(3 + y_2)} \]
\[ = \frac{3(6 - y_2)(8 + y_2)}{(144 - y_2^2)(3 + y_2)} \geq 0 \text{ if } y_2 \leq 6 \]
\[ > 0 \text{ if } y_2 > 6 \]

Hence, \( y^* = (-36, 6) \) and \( x^* = y^* + \omega = (108, 9) \).
Walrasian Equilibrium: Prices $p \geq 0$

Robinson the Manager

Choose $y(p)$

\[ \Pi(p) = p \cdot y(p) = \max_y \{p \cdot y | y \in \gamma^f\} \]

\[ p \cdot y = \Pi(p) \]
Example (Continued):

\[
\gamma = \{(y_1, y_2) | y_1 \leq 0, y_2^2 + y_1 \leq 0\}, \omega = (144, 3)
\]

Robinson the Manager solves

\[
\max_y \{p \cdot y | y \in \gamma\} = \max_y \{p \cdot y | y_1 \leq 0, y_1 + y_2^2 \leq 0\}
\]

\[
\pi(y_2) = p_1 y_1 + p_2 y_2 = -p_1 \cdot y_2^2 + p_2 \cdot y_2
\]

(Constraint binds at optimum)

FOC yields \(y_2(p) = \frac{p_2}{2p_1}\), \(y_1(p) = -y_2^2 = -\frac{p_2^2}{4p_1^2}\)

Hence, \(\Pi(p) = \frac{p_2^2}{4p_1}\)
Walrasian Equilibrium: Crusoe the Consumer

Prices $p \geq 0$
Choose $x(p)$

$$\max_x \left\{ U(x) \left| x \geq 0, \quad p \cdot x \leq p \cdot \omega + \Pi(p) \right. \right\}$$

Excess Demand for Leisure
Excess Supply for Corn

$x_2, y_2$
$\omega + y(p)$

$y(p)$
$Y + \omega$

$\Pi(p)$
Walrasian Equilibrium: Markets Clear at Optimum

\[ x(\bar{p}) = y(\bar{p}) + \omega = x^* \]

\[ U(x) = U(x^*) \]

\[ \bar{p} \cdot x = \bar{p} \cdot \omega + \Pi(\bar{p}) \]
Crusoe the Consumer solves

$$\max_x \{ \ln x_1 + \ln x_2 | p \cdot x \leq \Pi(p) + \omega \}$$

(B.C. binds since utility strictly increasing)

\[ \begin{align*}
\text{FOC: } & \quad \frac{\partial U}{\partial x_1} = \frac{\partial U}{\partial x_2} = \frac{1}{p_1 x_1} = \frac{1}{p_2 x_2} = \frac{2}{p \cdot \omega + \Pi(p)} \\
\text{So, } x_2(p) = & \quad \frac{\Pi(p) + p \cdot \omega}{2p_2} = \frac{1}{2} \left( \frac{p_2}{4p_1} + 144 \cdot \frac{p_1}{p_2} + 3 \right)
\end{align*} \]

(Recall $\omega = (144, 3)$ and $\Pi(p) = \frac{p_2}{4p_1}$.)
Example (Continued): Recall \( y_2(p) = \frac{p_2}{2p_1} \),

Markets clear when

\[
e_2(p) = x_2(p) - y_2(p) - 3
\]

\[
= \frac{1}{2} \left( \frac{p_2}{4p_1} + 144 \cdot \frac{p_1}{p_2} + 3 \right) - \frac{p_2}{2p_1} - 3
\]

\[
= \frac{1}{2} \left( 144 \cdot \frac{p_1}{p_2} - \frac{3p_2}{4p_1} - 3 \right)
\]

\[
= \frac{1}{2} \cdot \frac{p_1}{p_2} \left( 12 - \frac{p_2}{p_1} \right) \cdot \left( 12 + \frac{3p_2}{4p_1} \right)
\]

\( = 0 \) at \( \frac{p_2}{p_1} = 12 \)
Existence Problems: No Walrasian Equilibrium with Large Fixed Cost

\[ \bar{p} \cdot y = \bar{p} \cdot y^* = \Pi(\bar{p}) < 0 \]

\[ x^* = y^* + \omega \]

\[ U(x) = U(x^*) \]

\[ \bar{p} \cdot y = \bar{p} \cdot y^* < 0 \]
Summary of 5.1

- Robinson the Manager
  - Maximize Profit taking prices given
- Crusoe the Consumer
  - Maximize Utility taking prices given
- Walrasian Equilibrium
  - Prices where markets clear
- Homework: Exercise 5.1-1~4
- Why would Robinson Crusoe be a price-taker?
  - Doesn’t he have market power in this economy?