Robinson Crusoe Economy

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(Lecture 12, Micro Theory I)

Chapter Overview

- Ch.3: Equilibrium in an Exchange Economy;
- This Chapter: generalize to include production
- Section 5.1: Simplest economy possible:
  - Robinson Crusoe Economy (1 person only)
- Section 5.2: General equilibrium model with production and the 1st & 2nd Welfare Theorem
- Section 5.3: Existence of a Walrasian Equil.
- Sec. 5.4-6: Examples--time, public goods, CRS
  - In this course, only have time for CRS example

One Person Economy: Robinson Crusoe Economy

- The simplest case: Robinson Crusoe Economy
- Robinson the manager (price-taker)
  - Decides how much output to produce
- Crusoe the consumer (price-taker)
  - Decides how many hours to work and how much output to consume
- Walrasian Equilibrium: Market clearing price
- Does the Walrasian Equilibrium always exist?
  - Not if the production set is not convex…

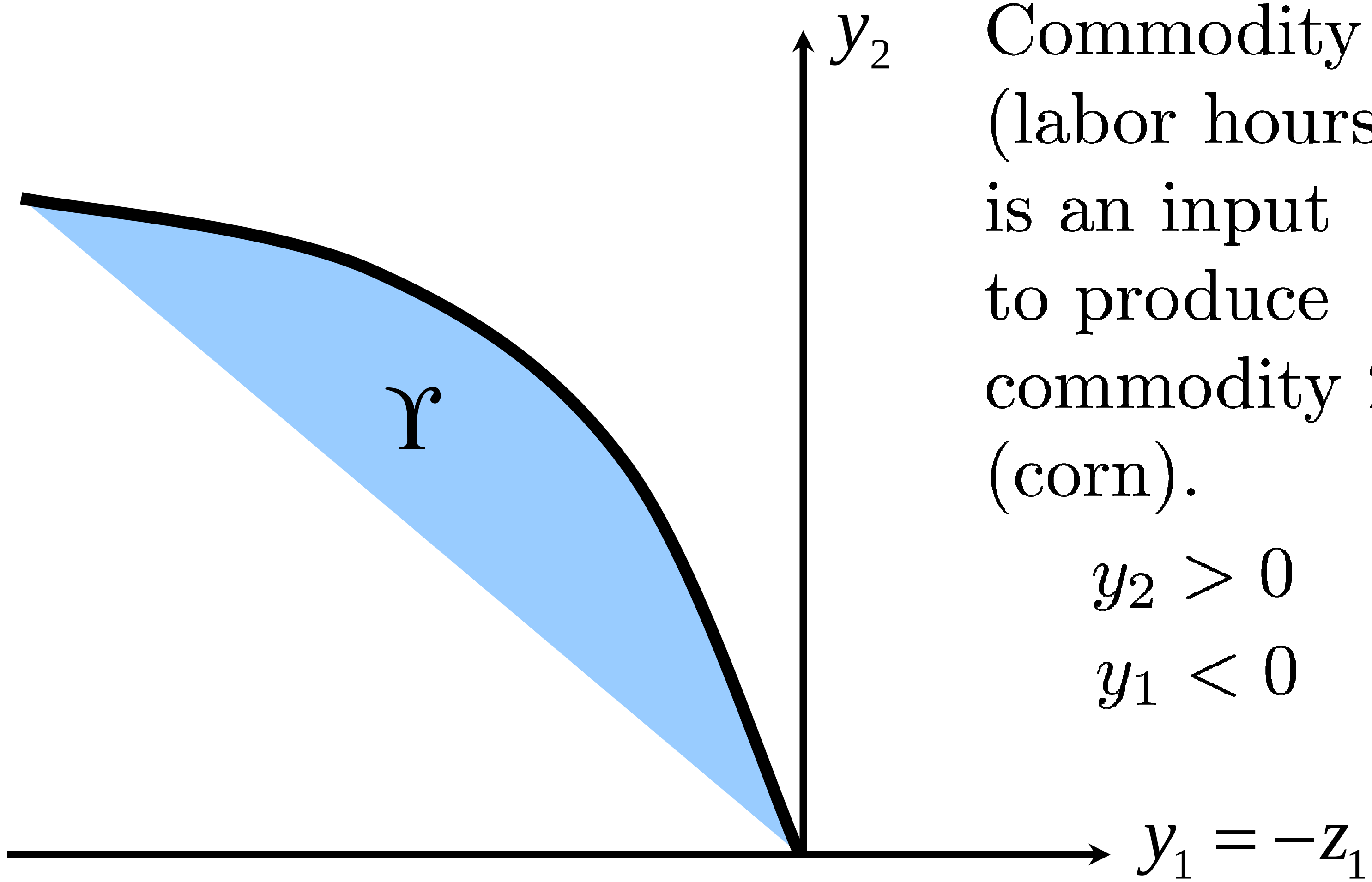
Why do we care about this?

- Equilibrium is the central concept in economics
  - Where forces of supply and demand balance out
- Empirically used to predict outcome
- Robinson Crusoe Economy: See how it works in the simplest example (one person economy)
  - Get intuition about how it works in this “toy model”
  - Then generalize to other cases…
- What if you happen to be in an island alone?
- Also, some macro models have only one agent!

One Person Economy: Robinson Crusoe Economy

- 2 Commodities: Labor hours (good 1) & corn (2)
- Consumers: $U^h(x^h) = U^h(x_1^h, x_2^h) \quad h = 1, \cdots, H$
  - Endowment: $\omega^h = (\omega_1^h, \omega_2^h)$
  - As if ONE representative agent:
  - Robinson Crusoe with endowment: $\omega = \sum_{h=1}^{H} \omega^h$
- Single Firm: with convex production set γ

Produce Corn with Labor Hours

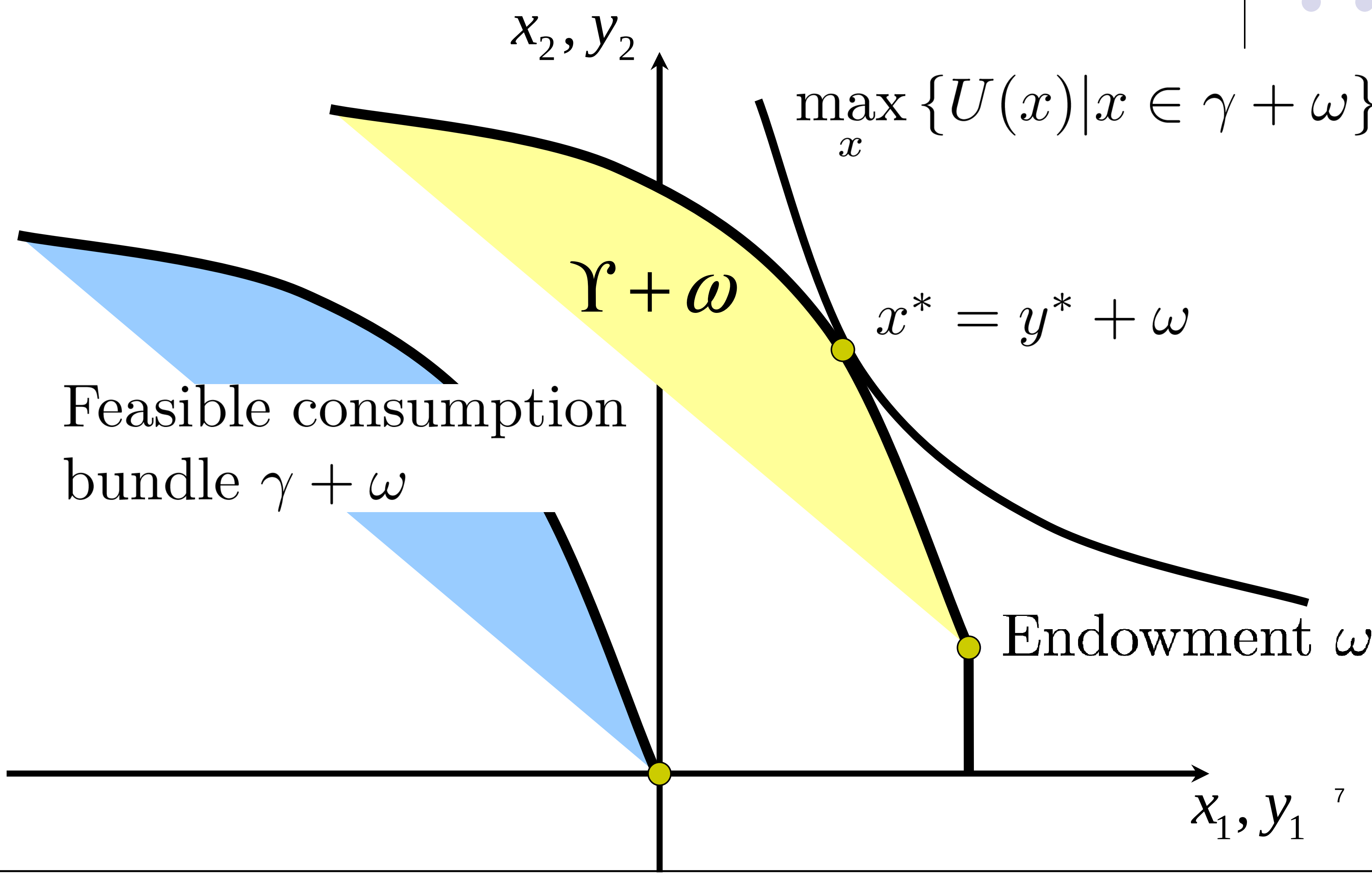


Commodity 1 (labor hours) is an input to produce commodity 2 (corn).

$$y_2 > 0$$

$$y_1 < 0$$

The Optimum



x_2, y_2
$\max_x \{U(x) | x \in \gamma + \omega\}$
$\Upsilon + \omega$
$x^* = y^* + \omega$
Feasible consumption bundle $\gamma + \omega$
Endowment ω
x_1, y_1

Example: $U(x) = \ln x_1 + \ln x_2$

$$\gamma = \left\{(y_1, y_2) | y_1 \le 0, y_2^2 + y_1 \le 0\right\}, \omega = (144, 3)$$

Since $x = y + \omega$ (to maximize utility)

$$U(y+\omega) = \ln(\omega_1 + y_1) + \ln(\omega_2 + y_2)$$
$$= \ln(144 - y_2^2) + \ln(3 + y_2)$$

(Since utility increasing implies $y_1 = -y_2^2$)

FOC: $$\frac{dU}{dy_2} = \frac{-2y_2}{144 - y_2^2} + \frac{1}{3 + y_2} = \frac{144 - 6y_2 - 3y_2^2}{(144 - y_2^2)(3 + y_2)}$$

$$= \frac{3(6 - y_2)(8 + y_2)}{(144 - y_2^2)(3 + y_2)} \gtreqless 0 \text{ if } y_2 \lesseqgtr 6$$

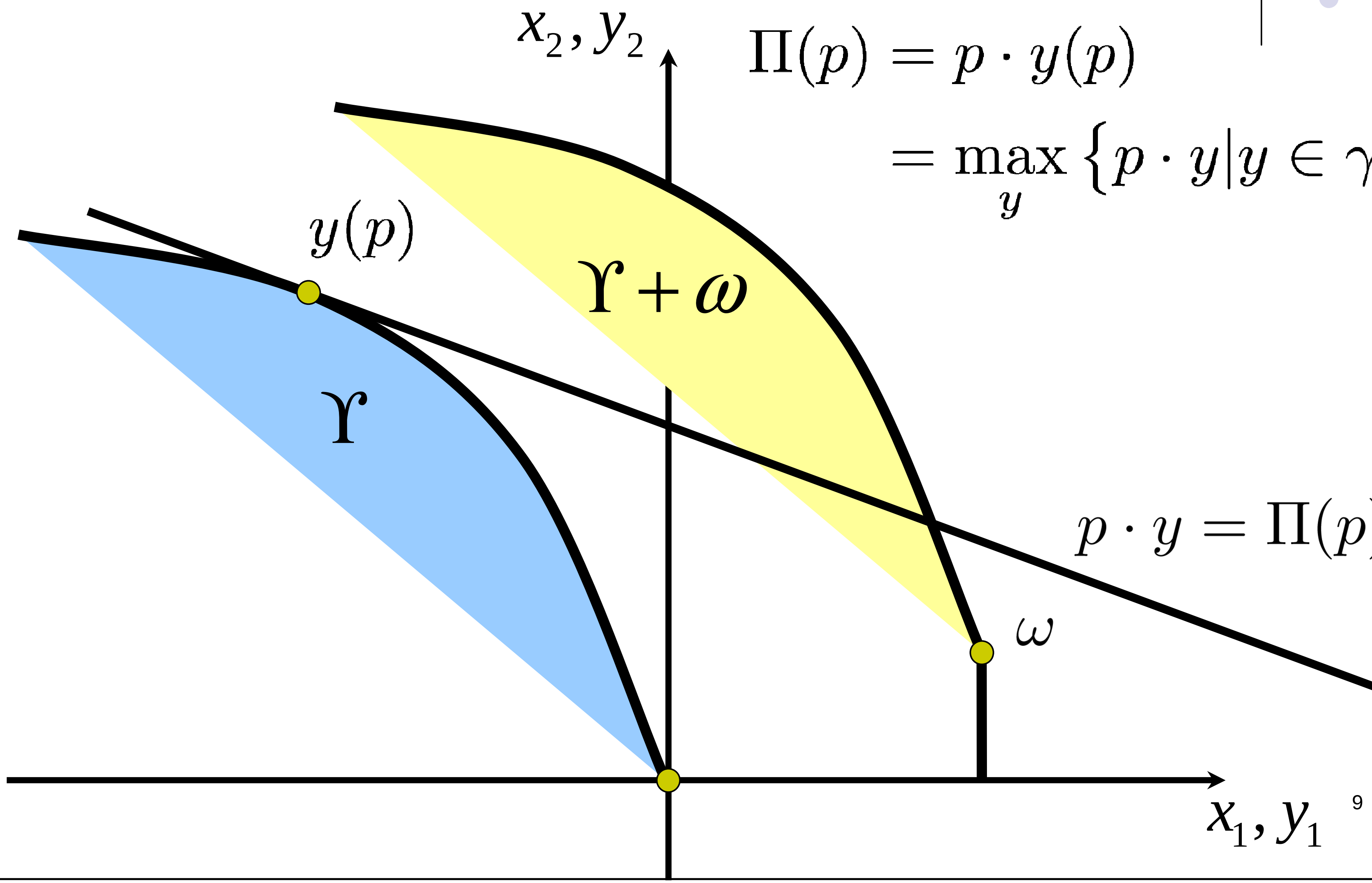
Hence, $y^* = (-36, 6)$ and $x^* = y^* + \omega = (108, 9)$.

Walrasian Equilibrium: Robinson the Manager

Prices $p \geq 0$

Choose $y(p)$

$$\Pi(p) = p \cdot y(p)$$
$$= \max_y \{p \cdot y | y \in \gamma^f\}$$



Example (Continued):

$$\gamma = \left\{(y_1, y_2) | y_1 \le 0, y_2^2 + y_1 \le 0\right\}, \omega = (144, 3)$$

Robinson the Manager solves

$$\max_y \{p \cdot y | y \in \gamma\} = \max_y \left\{p \cdot y | y_1 \le 0, y_1 + y_2^2 \le 0\right\}$$

$$\pi(y_2) = p_1 y_1 + p_2 y_2 = -p_1 \cdot y_2^2 + p_2 \cdot y_2$$

(Constraint binds at optimum)

FOC yields $y_2(p) = \dfrac{p_2}{2p_1}$, $y_1(p) = -y_2^2 = -\dfrac{p_2^2}{4p_1^2}$

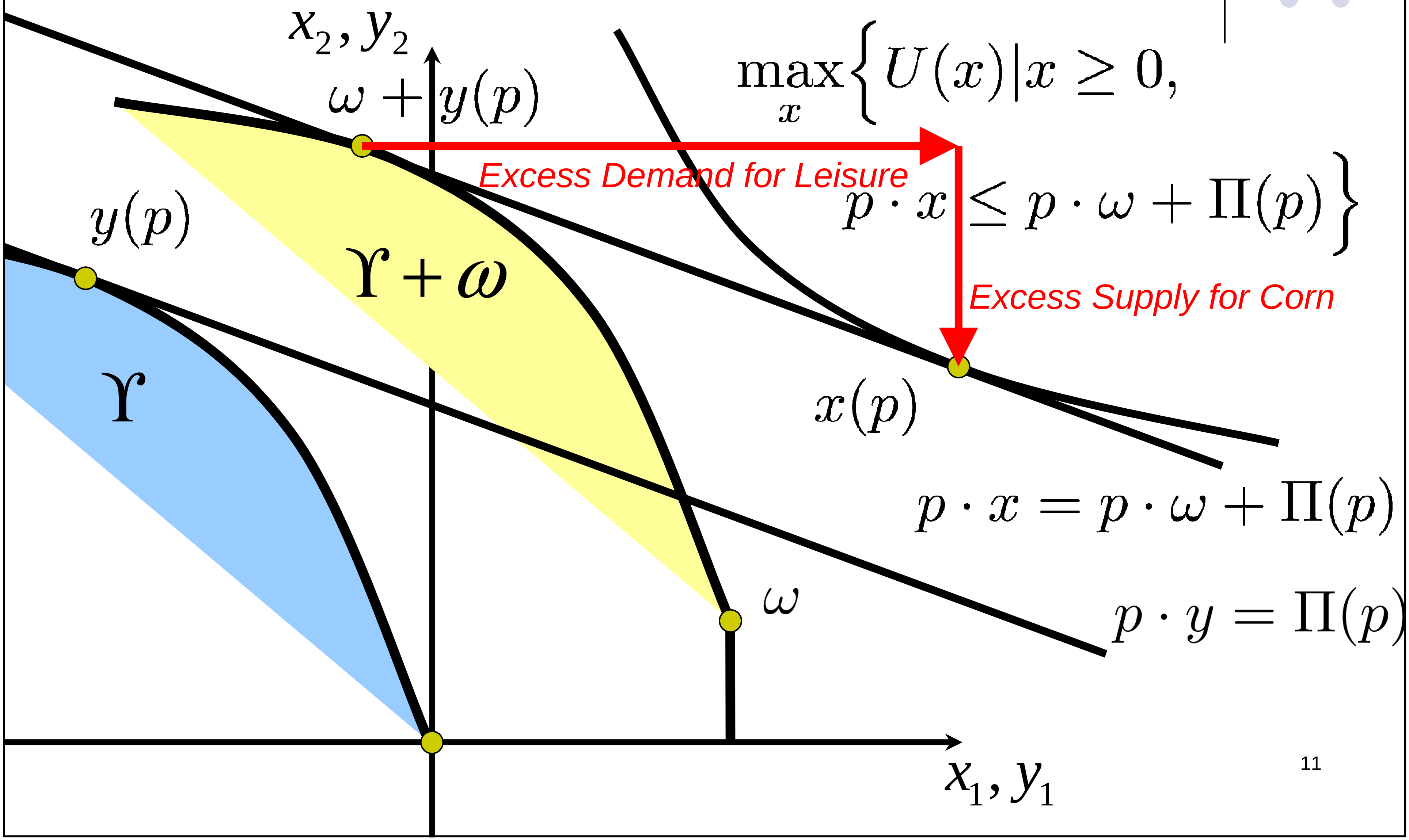
Hence, $\Pi(p) = \dfrac{p_2^2}{4p_1}$

Walrasian Equilibrium: Crusoe the Consumer

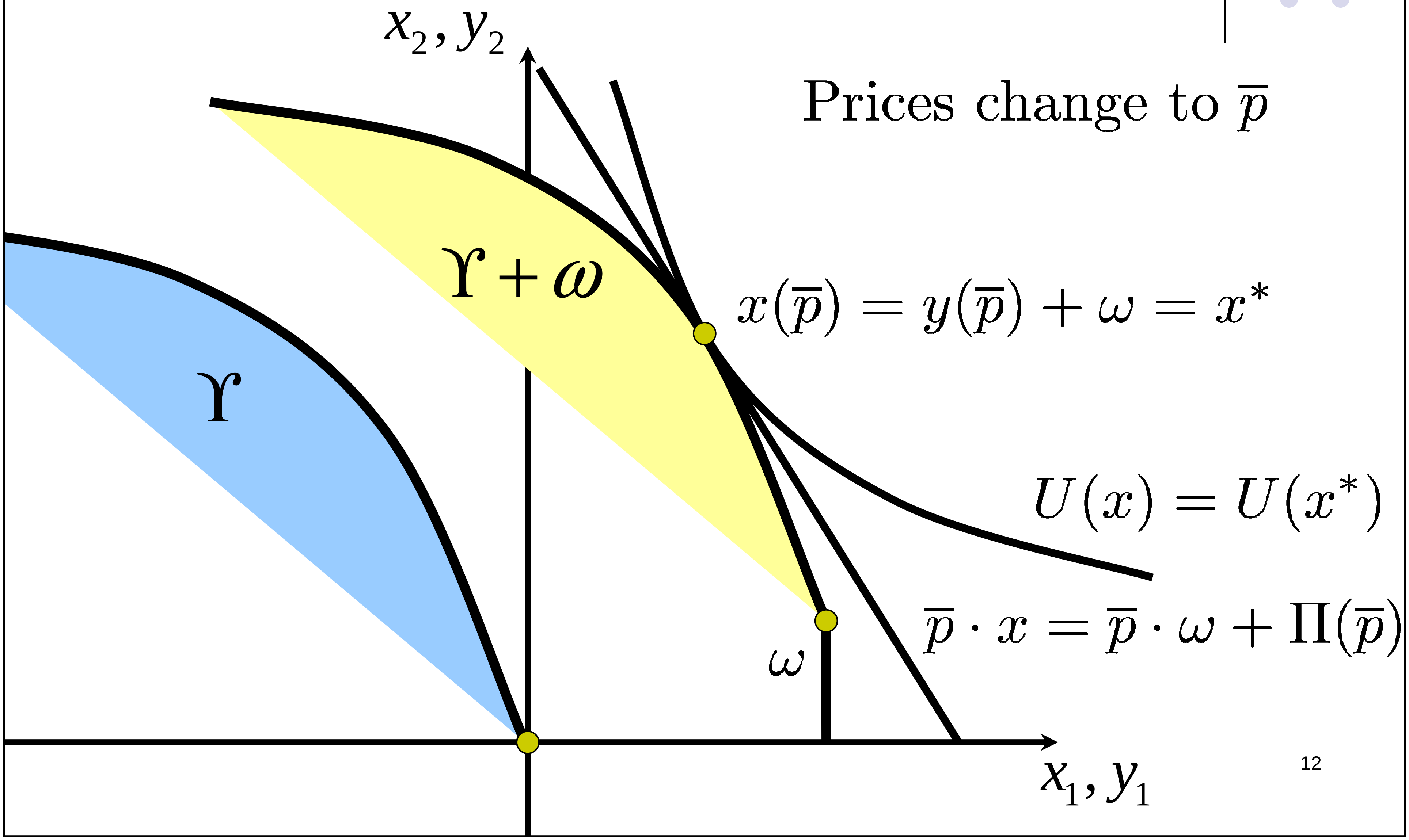
Prices $p \geq 0$

Choose $x(p)$

$$\max_x \left\{ U(x) | x \geq 0, p \cdot x \leq p \cdot \omega + \Pi(p) \right\}$$



Walrasian Equilibrium: Markets Clear at Optimum



Example (Continued):

Crusoe the Consumer solves

$$\max_x \{\ln x_1 + \ln x_2 | p \cdot x \leq \Pi(p) + \omega\}$$

(B.C. binds since utility strictly increasing)

$$\text{FOC: } \frac{\frac{\partial U}{\partial x_1}}{p_1} = \frac{\frac{\partial U}{\partial x_2}}{p_2} = \frac{1}{p_1 x_1} = \frac{1}{p_2 x_2} = \frac{2}{p \cdot \omega + \Pi(p)}$$

$$\text{So, } x_2(p) = \frac{\Pi(p) + p \cdot \omega}{2p_2} = \frac{1}{2}\left(\frac{p_2}{4p_1} + 144 \cdot \frac{p_1}{p_2} + 3\right)$$

(Recall $\omega = (144, 3)$ and $\Pi(p) = \frac{p_2^2}{4p_1}$.)

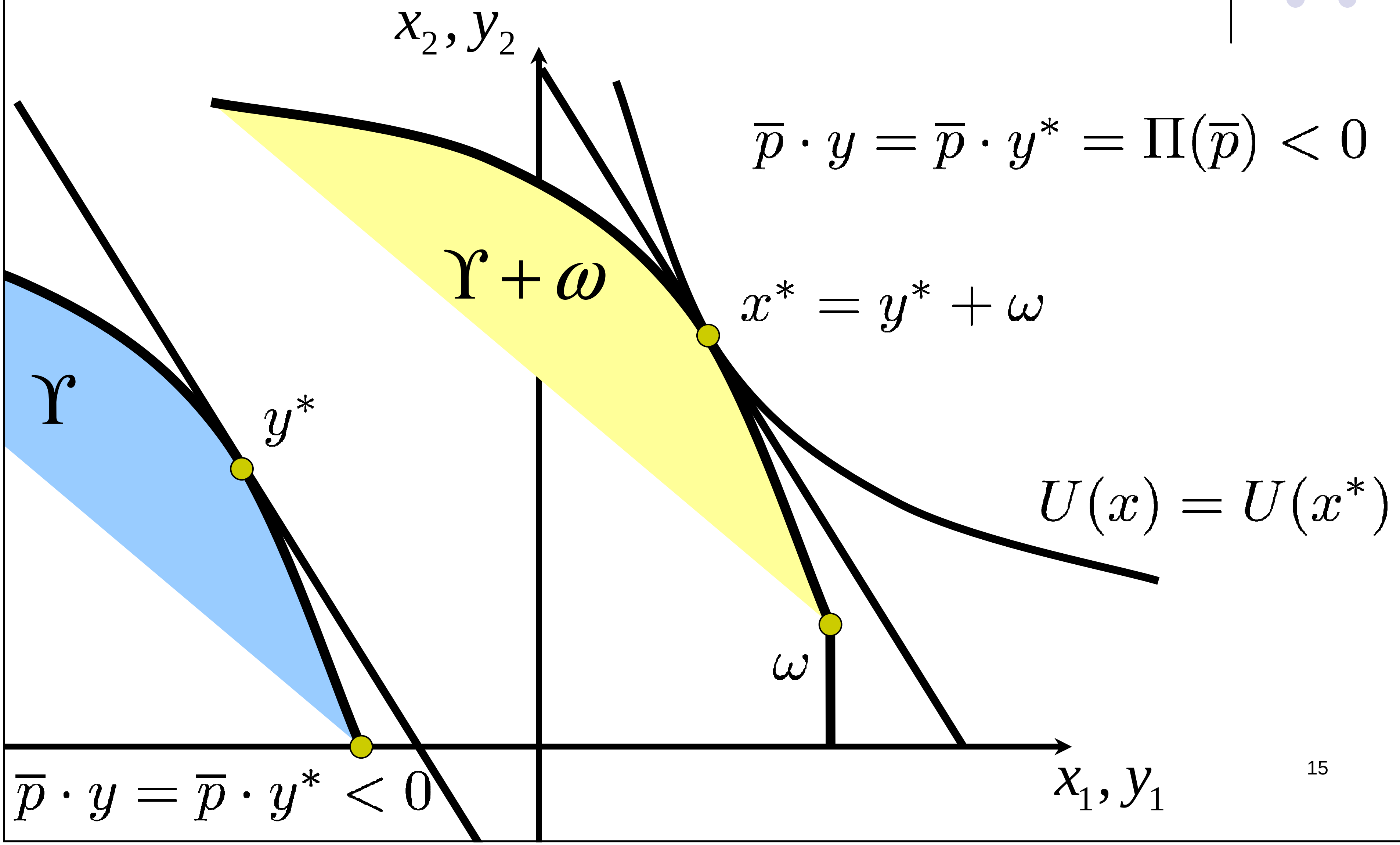
Example (Continued):

$$\text{Recall } y_2(p) = \frac{p_2}{2p_1},$$

Markets clear when

$$\begin{aligned}
e_2(p) &= x_2(p) - y_2(p) - 3 \\
&= \frac{1}{2}\left(\frac{p_2}{4p_1} + 144 \cdot \frac{p_1}{p_2} + 3\right) - \frac{p_2}{2p_1} - 3 \\
&= \frac{1}{2}\left(144 \cdot \frac{p_1}{p_2} - \frac{3p_2}{4p_1} - 3\right) \\
&= \frac{1}{2} \cdot \frac{p_1}{p_2}\left(12 - \frac{p_2}{p_1}\right) \cdot \left(12 + \frac{3}{4}\frac{p_2}{p_1}\right) \\
&= 0 \text{ at } \frac{p_2}{p_1} = 12
\end{aligned}$$

Existence Problems: No Walrasian Equilibrium with Large Fixed Cost



Summary of 5.1

- Robinson the Manager
  - Maximize Profit taking prices given
- Crusoe the Consumer
  - Maximize Utility taking prices given
- Walrasian Equilibrium
  - Prices where markets clear
- Homework: Exercise 5.1-1~4
- Why would Robinson Crusoe be a price-taker?
  - Doesn't he have market power in this economy?