

## Supporting Price

Name  
Major  
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1. Robinson Crusoe lives alone on an island off the coast of New Zealand. He has a production set  $\Upsilon = \{(-z_1, y_2) | y_2 \leq 16z_1^{1/3}, z_1 \geq 0\}$  and an endowment vector  $\omega = (32, 0)$ . His preferences are represented by the utility function  $U(x) = \ln x_1 + \ln x_2$ .
  - (a) Solve for his optimal choice of input and hence his optimal production plan and consumption plan  $x^*$ .
  - (b) Depict the production set and the set  $\Upsilon + \omega$  in a neat figure and indicate the optimal production and consumption plans. Explain what it means for the optimal production plan to be supported by a price vector  $p = (p_1, p_2)$ .
  - (c) Solve for the price vector that supports the optimal production plan.
  - (d) Depict this supporting price line, Crusoe's budget set and indifference curve through  $x^*$ .
  - (e) Hence explain why the supporting price vector is a Walrasian equilibrium price vector if Robinson Crusoe is a price-taker.
  
2. Robinson Crusoe has a production set  $\Upsilon = \{(-z, y_3) | y_3 \leq z_1^{1/3} z_2^{2/3}, z \geq 0\}$  and an endowment vector  $\omega = (32, 160, 0, 16)$ . His preferences are represented by the utility function  $U(x) = \sum_{j=1}^4 \ln x_j$ 
  - (a) Solve for the optimal production plan and hence the optimal consumption vector  $x^*$ .
  - (b) Obtain a price vector  $(p_1, p_2, p_3)$ , where  $p_2 = 1$ , that supports the optimal production plan.
  - (c) What must be the price of commodity 4 if  $x^*$  is Crusoe's optimal consumption bundle in his budget set? HINT: There is only one consumer so Crusoe must consume his endowment.