The 2x2 Exchange Economy

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(Lecture 8, Micro Theory I)

Road Map for Chapter 3

- Pareto Efficiency Allocation (PEA)
  - Cannot make one better off without hurting others
- Walrasian (Price-taking) Equilibrium (WE)
  - When Supply Meets Demand
  - Focus on Exchange Economy First
- 1st Welfare Theorem:
  - Any WE is PEA (Adam Smith Theorem)
- 2nd Welfare Theorem:
  - Any PEA can be supported as a WE with transfers

Why Should We Care About This?

- Professor L, "Students told me you finished what Professor H taught in three weeks?!"
- Me, "Yes and no. I try to show the essence and move quickly through theory of choice and consumer theory, so I can get to equilibrium ASAP since it's a core concept in economics."
- General Equilibrium underlies nearly all modern macroeconomic models
  - Professor Y has to teach it in macro theory...

2x2 Exchange Economy

- 2 Commodities: Good 1 and 2
- 2 Consumers: Alex and Bev - $h = A, B$
  - Endowment: $\vec{\omega}^h = (\omega_1^h, \omega_2^h)$, $\omega_i = \omega_i^A + \omega_i^B$
  - Consumption Set: $\vec{x}^h = (x_1^h, x_2^h) \in \mathbb{R}_+^2$
  - Strictly Monotonic Utility:

$$U^h(\vec{x}^h) = U^h(x_1^h, x_2^h), \quad \frac{\partial U^h}{\partial x_i^h}(\vec{x}^h) > 0$$

- Edgeworth Box
  - These consumers could be representative agents, or literally TWO people (bargaining)

Why do we care about this?

- The Walrasian (Price-taking) Equilibrium (W.E.) is (a candidate of) Adam Smith's "Invisible Hand"
  - Are real market rules like Walrasian auctioneers?
  - Is Price-taking the result of competition, or competition itself?
- Illustrate W.E. in more general cases
  - Hard to graph "N goods" as 2D
- Two-party Bargaining
  - This is what Edgeworth himself really had in mind

Why do we care about this?

- Consider the following situation: You company is trying to make a deal with another company
  - You have better technology, but lack funding
  - They have plenty of funding, but low-tech
- There are "gives" and "takes" for both sides
- Where would you end up making the deal?
  - Definitely not where "something is left on the table."
- What are the possible outcomes?
  - How did you get there?

Social Choice and Pareto Efficiency

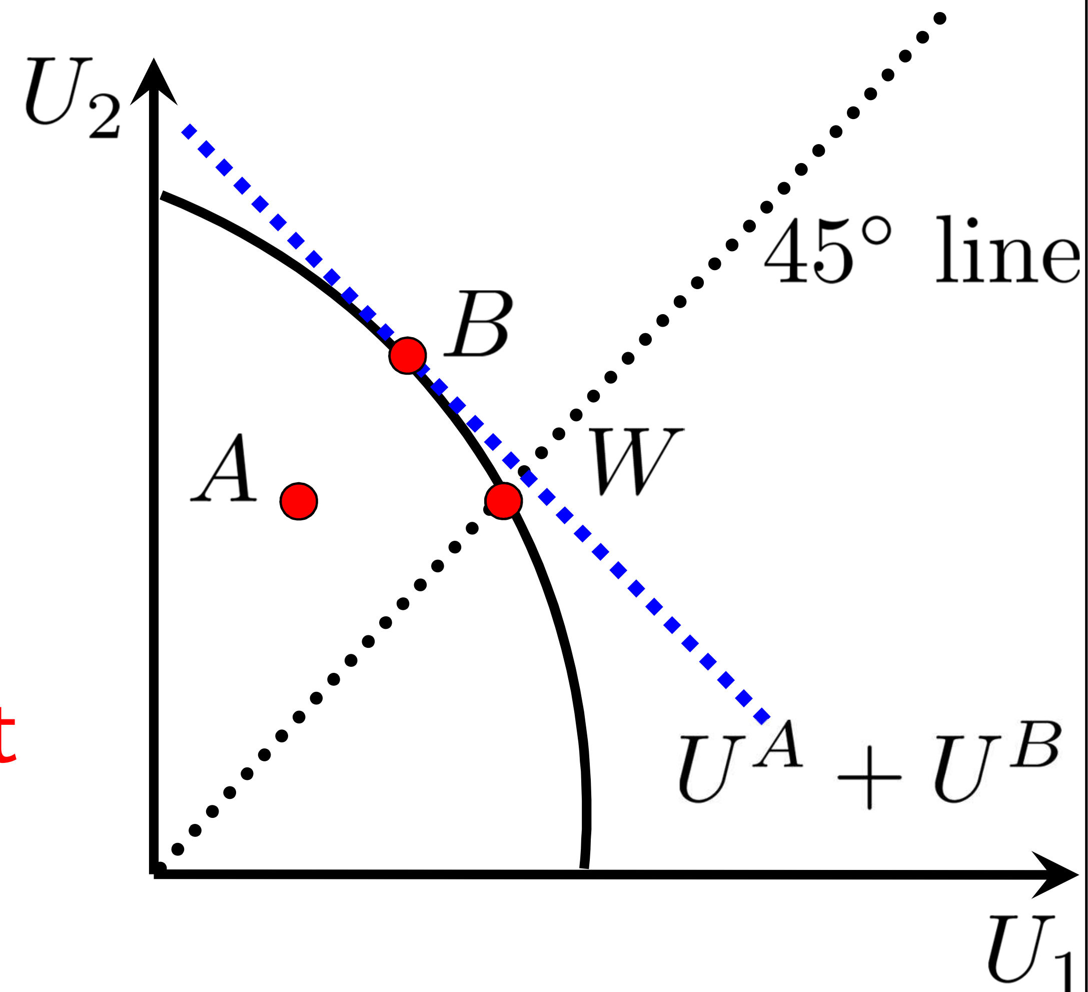
- Benthamite:
  - Behind Veil of Ignorance
  - Assign Prob. 50-50

$$\max \tfrac{1}{2}U^A + \tfrac{1}{2}U^B$$

- Rawlsian:
  - Infinitely Risk Averse

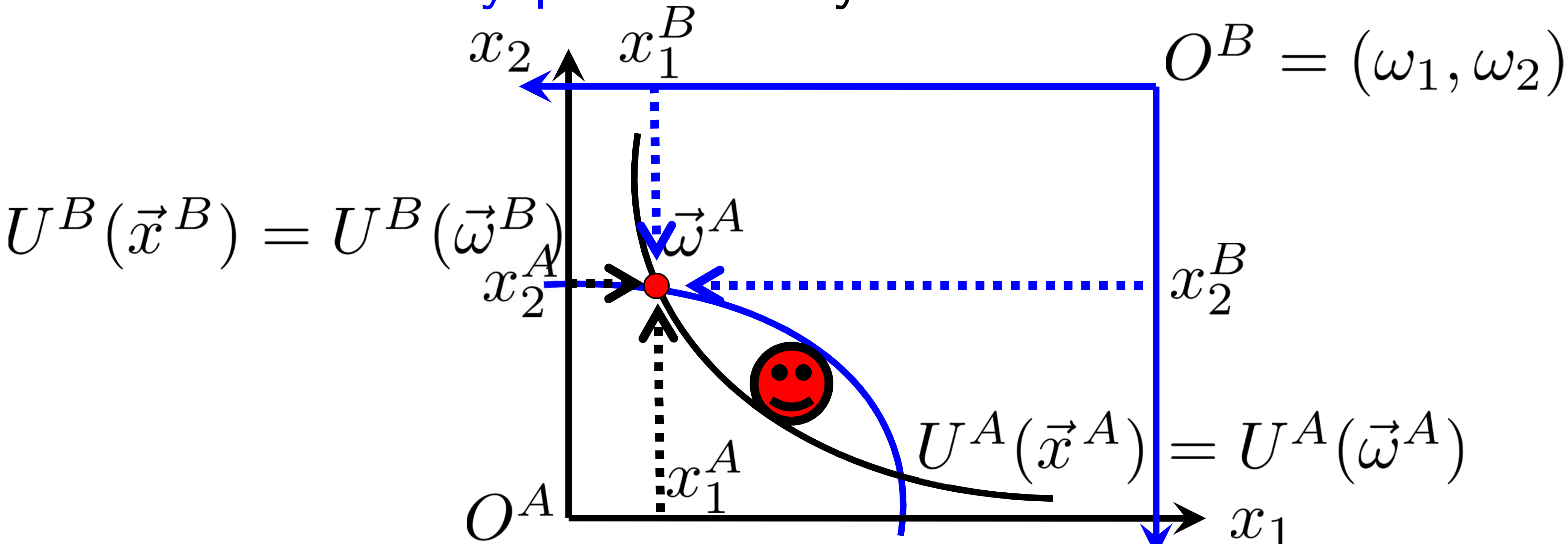
$$\max \min\{U^A, U^B\}$$

- Both are Pareto Efficient
  - But A is not



Pareto Efficiency

- A feasible allocation is Pareto efficient if
- there is no other feasible allocation that is
- strictly preferred by at least one consumer
- and is weakly preferred by all consumers.

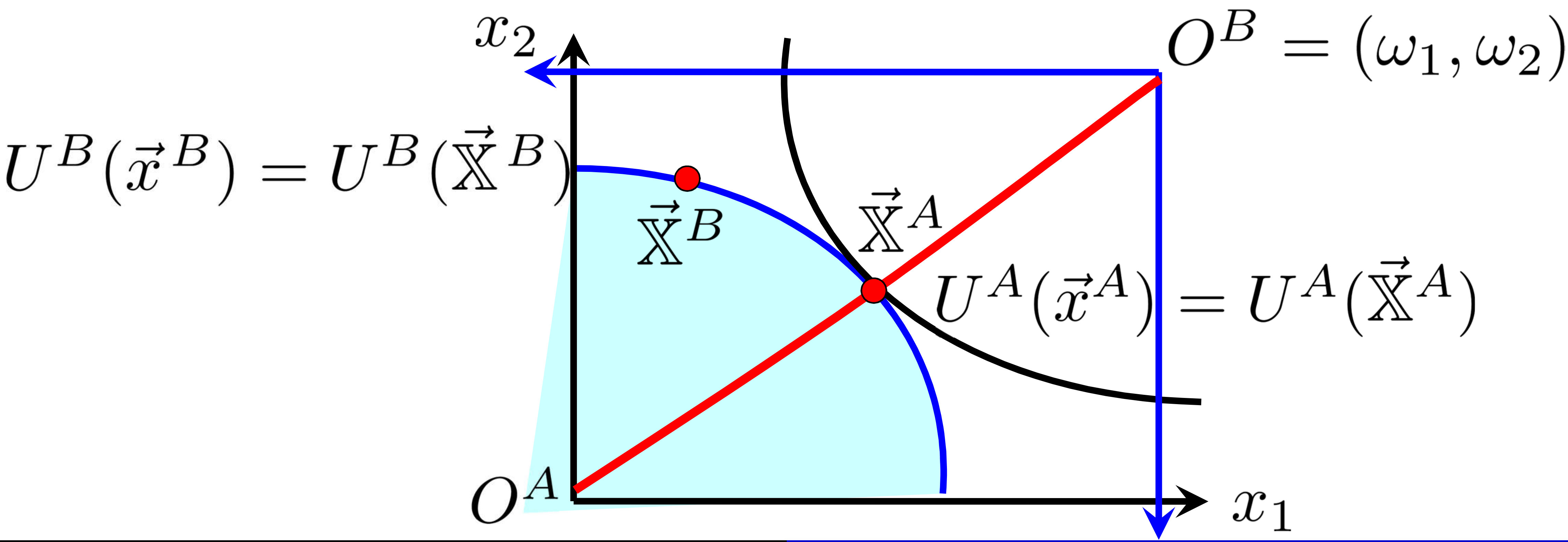


Pareto Efficient Allocations

For $\vec{\omega} = (\omega_1, \omega_2)$, consider

$$\max_{\vec{x}^A, \vec{x}^B} \left\{ U^A(\vec{x}^A) | U^B(\vec{x}^B) \geq U^B(\vec{\mathbb{X}}^B), \vec{x}^A + \vec{x}^B \leq \vec{\omega} \right\}$$

Need $MRS^A(\vec{\mathbb{X}}^A) = MRS^B(\vec{\mathbb{X}}^A)$ (interior solution)



Example: CES Preferences

- CES: $$U(x) = \left(\alpha_1 x_1^{1-\frac{1}{\theta}} + \alpha_2 x_2^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}}$$
- MRS: $$MRS^h(\vec{x}^h) = k\left(\frac{x_2^h}{x_1^h}\right)^{1/\theta}, h = A, B$$
- Equal MRS for PEA in interior of Edgeworth box
$$\Rightarrow \frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{x_2^A + x_2^B}{x_1^A + x_1^B} = \frac{\omega_2}{\omega_1}$$
- Thus, $$MRS^h(\vec{x}^h) = k\left(\frac{\omega_2}{\omega_1}\right)^{1/\theta}, h = A, B$$

Walrasian Equilibrium - 2x2 Exchange Economy

- All Price-takers: Price vector $\vec{p} \geq 0$
- 2 Consumers: Alex and Bev - $h \in \mathcal{H} = \{A, B\}$
  - Endowment: $\vec{\omega}^h = (\omega_1^h, \omega_2^h)$, $\omega_i = \omega_i^A + \omega_i^B$
  - Consumption Set: $\vec{x}^h = (x_1^h, x_2^h) \in \mathbb{R}_+^2$
  - Wealth: $W^h = \vec{p} \cdot \vec{\omega}^h$
- Market Demand: $\vec{x}(\vec{p}) = \sum_h \vec{x}^h(\vec{p}, \vec{p} \cdot \vec{\omega}^h)$

  (Solution to consumer problem)
- Vector of Excess Demand: $\vec{z}(\vec{p}) = \vec{x}(\vec{p}) - \vec{\omega}$
  - Where vector of total Endowment: $\vec{\omega} = \sum_h \vec{\omega}^h$

Definition: Market Clearing Prices

- Let Excess Demand for Commodity j be $z_j(\vec{p})$
- The Market for Commodity j Clears if
  - Excess Demand = 0 or Price = 0 (and ED < 0)
    - Excess demand = shortage; negative ED means surplus

$$z_j(\vec{p}) \leq 0 \text{ and } p_j \cdot z_j(\vec{p}) = 0$$

- Why is this important?

1. Walras Law
   - The last market clears if all other markets clear
2. Market clearing defines Walrasian Equilibrium

Local Non-Satiation Axiom (LNS)

For any consumption bundle $\vec{x} \in C \subset \mathbb{R}^n$
and any δ-neighborhood $N(\vec{x}, \delta)$ of $\vec{x}$,
there is some bundle $\vec{y} \in N(\vec{x}, \delta)$ s.t. $\vec{y} \succ_h \vec{x}$

- LNS implies consumer must spend all income
- If not, we have $\vec{p} \cdot \vec{x}^h < \vec{p} \cdot \vec{\omega}^h$ for optimal $\vec{x}^h$
- But then there exist $\delta\text{-neighborhood } N(\vec{x}^h, \delta)$
- In the budget set $\text{for sufficiently small } \delta > 0$
- LNS $\Rightarrow \vec{y} \in N(\vec{x}^h, \delta), \vec{y} \succ_h \vec{x}^h$, $\vec{x}^h$ is not optimal!

Walras Law

- For any price vector $\vec{p}$, the market value of excess demands must be zero, because:

$$\vec{p} \cdot \vec{z}(\vec{p}) = \vec{p} \cdot (\vec{x} - \vec{\omega}) = \vec{p} \cdot \left(\sum_h (\vec{x}^{\,h} - \vec{\omega}^{\,h}) \right)$$

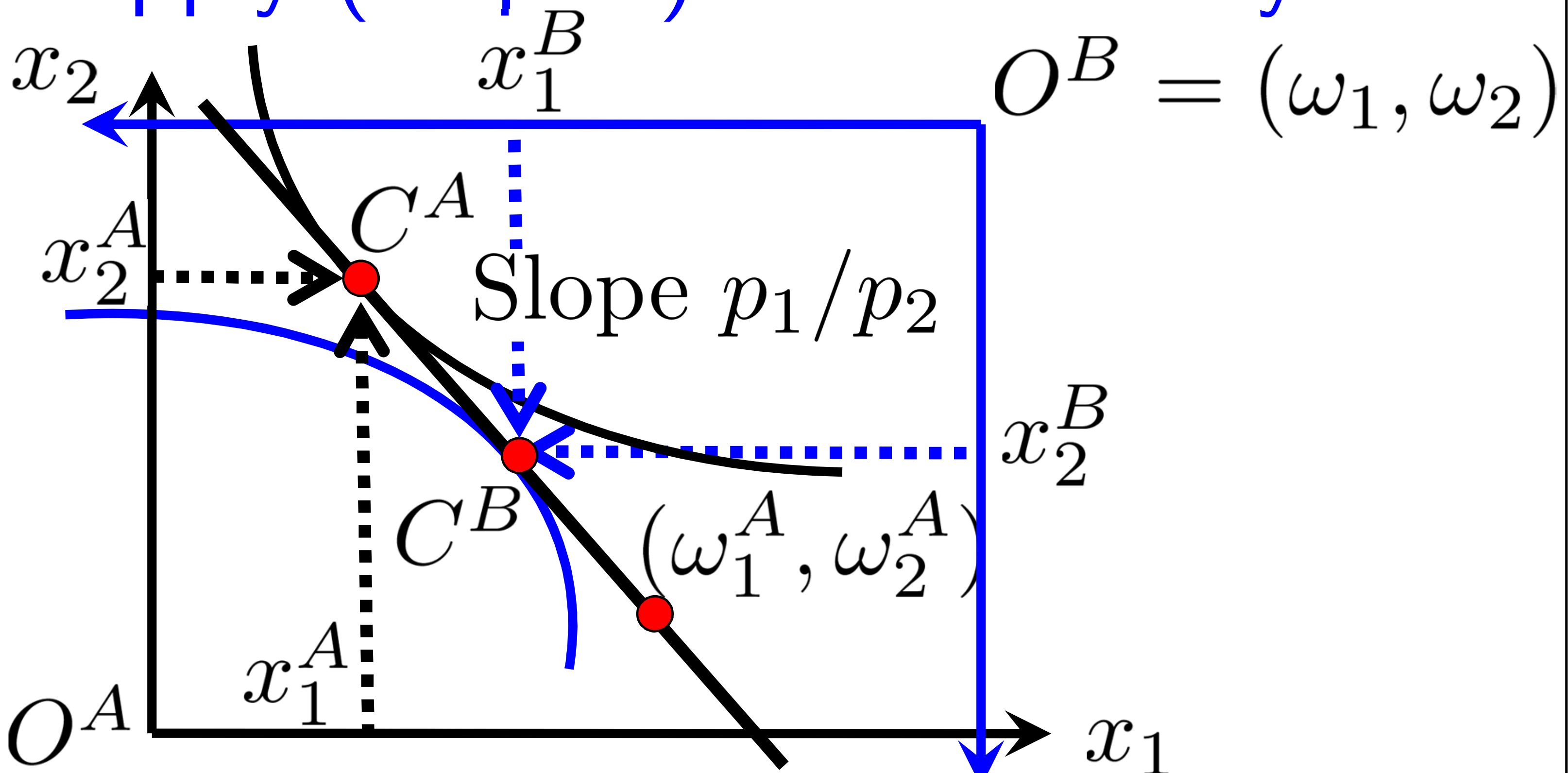
$$= \sum_h (\vec{p} \cdot \vec{x}^{\,h} - \vec{p} \cdot \vec{\omega}^{\,h}) = 0 \text{ by LNS}$$

$$p_1 z_1(\vec{p}) + p_2 z_2(\vec{p}) = 0$$

- If one market clears, so must the other.

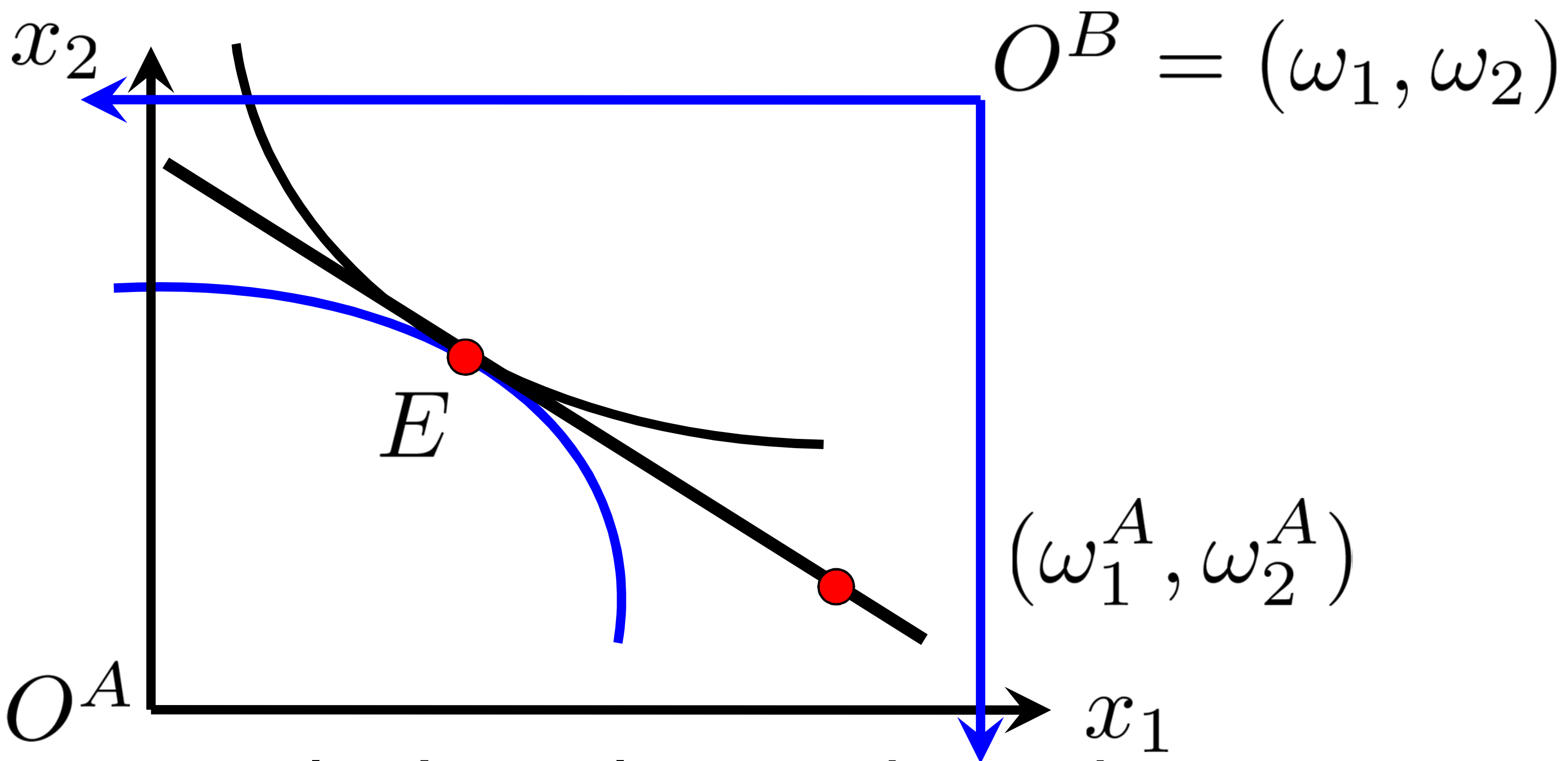
Definition: Walrasian Equilibrium

- The price vector $\vec{p} \geq \vec{0}$ is a Walrasian Equilibrium price vector if all markets clear.
  - WE = price vector!!!
- EX: Excess supply (surplus) of commodity 1…



Definition: Walrasian Equilibrium

- Lower price for commodity 1 if excess supply
  - Until Markets Clear



- Cannot raise Alex's utility without hurting Bev
  - Hence, we have FWT…

First Welfare Theorem: WE → PEA

- If preferences satisfy LNS, then a Walrasian Equilibrium allocation $(\vec{\mathbb{x}}^A, \vec{\mathbb{x}}^B)$ (in an exchange economy) is Pareto efficient.
- Sketch of Proof:

1. Any weakly (strictly) preferred bundle must cost at least as much (strictly more) as WE
2. Markets clear

   → Pareto preferred allocation not feasible

First Welfare Theorem: WE → PEA

1. Since WE allocation $\vec{\mathrm{x}}^h$ maximizes utility, so

$$U^h(\vec{x}^h) > U(\vec{\mathrm{x}}^h) \Rightarrow \vec{p}\cdot\vec{x}^h > \vec{p}\cdot\vec{\mathrm{x}}^h$$

Now need to show: (Duality Lemma 2.2-3!)

$$U^h(\vec{x}^h) \geq U(\vec{\mathrm{x}}^h) \Rightarrow \vec{p}\cdot\vec{x}^h \geq \vec{p}\cdot\vec{\mathrm{x}}^h$$

- Recall Proof: If not, we have $\vec{p}\cdot\vec{x}^h < \vec{p}\cdot\vec{\mathrm{x}}^h$
- But then LNS yields a δ-neighborhood $N(\vec{x}^h, \delta)$
- In the budget set for sufficiently small $\delta > 0$
- In which there exists a point $\vec{\chi}^h$ such that

$$U^h(\vec{\chi}^h) > U^h(\vec{x}^h) \geq U(\vec{\mathrm{x}}^h)$$ Contradiction!

First Welfare Theorem: WE → PEA

1. $U^h(\vec{x}^h) > U(\vec{\mathbb{x}}^h) \Rightarrow \vec{p} \cdot \vec{x}^h > \vec{p} \cdot \vec{\mathbb{x}}^h$

   $U^h(\vec{x}^h) \geq U(\vec{\mathbb{x}}^h) \Rightarrow \vec{p} \cdot \vec{x}^h \geq \vec{p} \cdot \vec{\mathbb{x}}^h$

- Satisfied by Pareto preferred allocation $(\vec{x}^A, \vec{x}^B)$

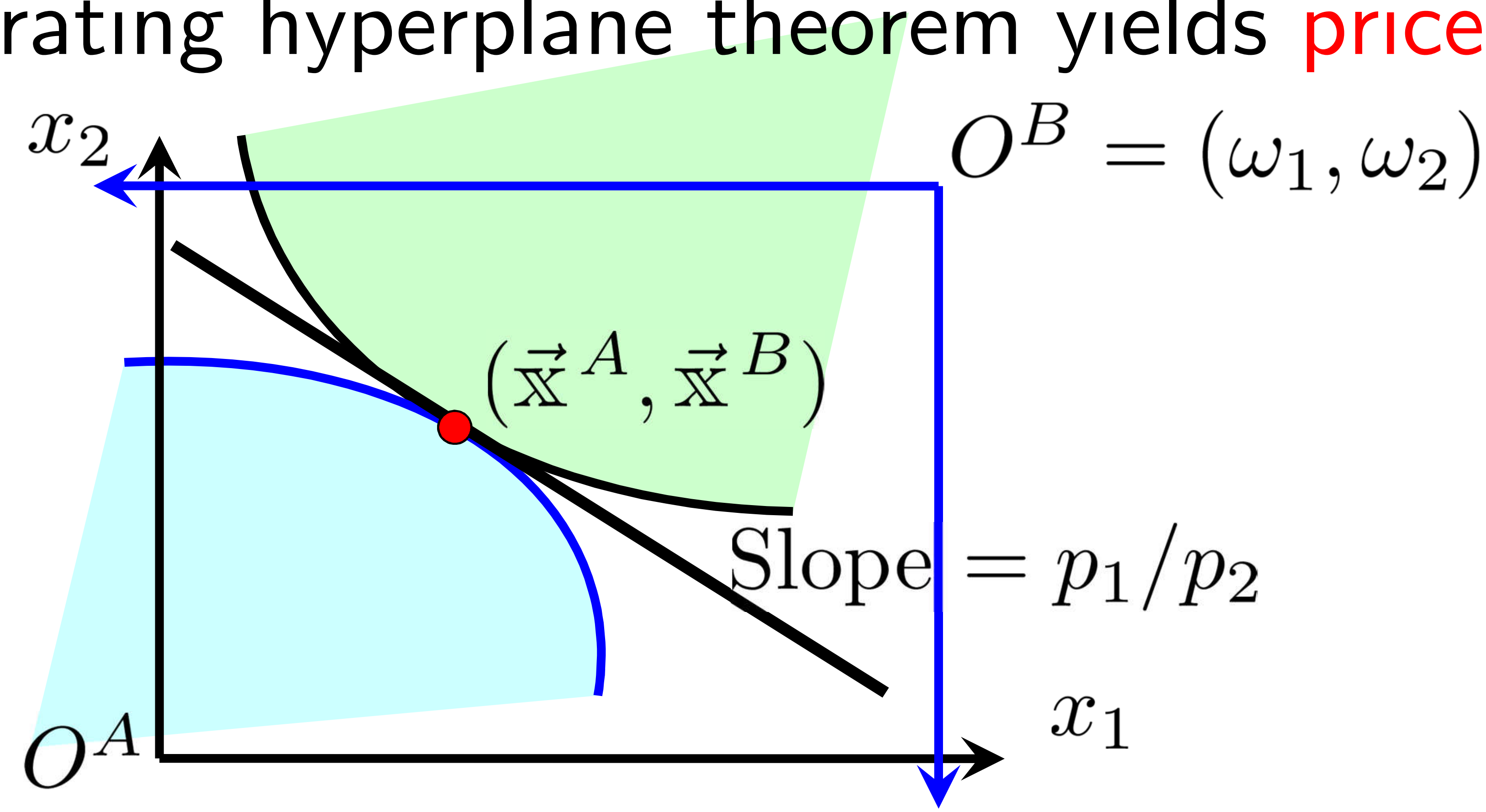
2. Hence, $\vec{p} \cdot \vec{x}^h > \vec{p} \cdot \vec{\mathbb{x}}^h$ for at least one, and

- $\vec{p} \cdot \vec{x}^h \geq \vec{p} \cdot \vec{\mathbb{x}}^h$ for all others (preferred)
- Thus, $\vec{p} \cdot \sum_h \vec{x}^h > \vec{p} \cdot \sum_h \vec{\mathbb{x}}^h = \vec{p} \cdot \sum_h \vec{\omega}^h$
- Since $\vec{p} \geq \vec{0}$, at least one $j \Rightarrow \sum_h x_j^h > \sum_h \omega_j^h$
  - Not feasible, so can't improve!

Second Welfare Theorem: PEA → WE

- (2-commodity) For PE allocation $(\vec{\mathbb{x}}^A, \vec{\mathbb{x}}^B)$

1. Convex preferences imply convex regions
2. Separating hyperplane theorem yields prices



Second Welfare Theorem: PEA → WE

3. Alex and Bev are both optimizing

- For interior Pareto efficient allocation $(\vec{\mathbb{x}}^A, \vec{\mathbb{x}}^B)$

$$\frac{\frac{\partial U^A}{\partial x_1}(\vec{\mathbb{x}}^A)}{\frac{\partial U^A}{\partial x_2}(\vec{\mathbb{x}}^A)} = \frac{\frac{\partial U^B}{\partial x_1}(\vec{\mathbb{x}}^B)}{\frac{\partial U^B}{\partial x_2}(\vec{\mathbb{x}}^B)} \Rightarrow \frac{\partial U^A}{\partial \vec{x}}(\vec{\mathbb{x}}^A) = \theta \cdot \frac{\partial U^B}{\partial \vec{x}}(\vec{\mathbb{x}}^B)$$

- Since we have convex upper contour set

$$X^A = \{\vec{x}^A | U^A(\vec{x}^A) \geq U^A(\vec{\mathbb{x}}^A)\}$$

- Lemma 1.1-2 yields:

$$U^A(\vec{x}^A) \geq U^A(\vec{\mathbb{x}}^A) \Rightarrow \frac{\partial U^A}{\partial \vec{x}}(\vec{\mathbb{x}}^A) \cdot (\vec{x}^A - \vec{\mathbb{x}}^A) \geq 0$$

Second Welfare Theorem: PEA → WE

$$U^B(\vec{x}^B) \geq U^B(\vec{\mathbb{x}}^B) \Rightarrow \frac{\partial U^B}{\partial \vec{x}}(\vec{\mathbb{x}}^B) \cdot (\vec{x}^B - \vec{\mathbb{x}}^B) \geq 0$$

- Choose $\vec{p} = \frac{\partial U^B}{\partial \vec{x}}(\vec{\mathbb{x}}^B)$, then $\frac{\partial U^A}{\partial \vec{x}}(\vec{\mathbb{x}}^A) = \theta \vec{p}$
- And we have:

$$U^A(\vec{x}^A) \geq U^A(\vec{\mathbb{x}}^A) \Rightarrow \vec{p} \cdot \vec{x}^A \geq \vec{p} \cdot \vec{\mathbb{x}}^A$$

$$U^B(\vec{x}^B) \geq U^B(\vec{\mathbb{x}}^B) \Rightarrow \vec{p} \cdot \vec{x}^B \geq \vec{p} \cdot \vec{\mathbb{x}}^B$$

- In words, weakly "better" allocations are at least as expensive (under this price vector)
  - For $\vec{\mathbb{x}}^A, \vec{\mathbb{x}}^B$ optimal, need them not affordable…

Second Welfare Theorem: PEA → WE

- Suppose a strictly "better" allocation is feasible
- i.e. $U^A(\vec{x}^A) > U^A(\vec{\mathbb{x}}^A)$ and $\vec{p} \cdot \vec{x}^A = \vec{p} \cdot \vec{\mathbb{x}}^A$
- Since U is strictly increasing and continuous,
- Exists $\vec{\delta} \gg \vec{0}$ such that

$$U^A(\vec{x}^A - \vec{\delta}) > U^A(\vec{\mathbb{x}}^A) \text{ and } \vec{p} \cdot (\vec{x}^A - \vec{\delta}) < \vec{p} \cdot \vec{\mathbb{x}}^A$$

- Contradicting:

$$U^A(\vec{x}^A) \geq U^A(\vec{\mathbb{x}}^A) \Rightarrow \vec{p} \cdot \vec{x}^A \geq \vec{p} \cdot \vec{\mathbb{x}}^A$$

  – Strictly "better" allocations are not affordable!

Second Welfare Theorem: PEA → WE

– Strictly "better" allocations are not affordable:

- i.e. $U^h(\vec{x}^h) > U^h(\vec{\mathbb{x}}^h) \Rightarrow \vec{p} \cdot \vec{x}^h > \vec{p} \cdot \vec{\mathbb{x}}^h, h \in \mathcal{H}$
- So both Alex and Bev are optimizing under $\vec{p}$
- Since markets clear at $\vec{\mathbb{x}}^A, \vec{\mathbb{x}}^B$, it is a WE!
- In fact, to achieve this WE, only need transfers

$$T^h = \vec{p} \cdot (\vec{\mathbb{x}}^h - \vec{\omega}^h), h \in \mathcal{H}$$

– Add up to zero (feasible transfer payment), so:

- Budget Constraint is $\vec{p} \cdot \vec{x}^h \leq \vec{p} \cdot \vec{\omega}^h + T^h, h \in \mathcal{H}$

Proposition 3.1-3: Second Welfare Theorem

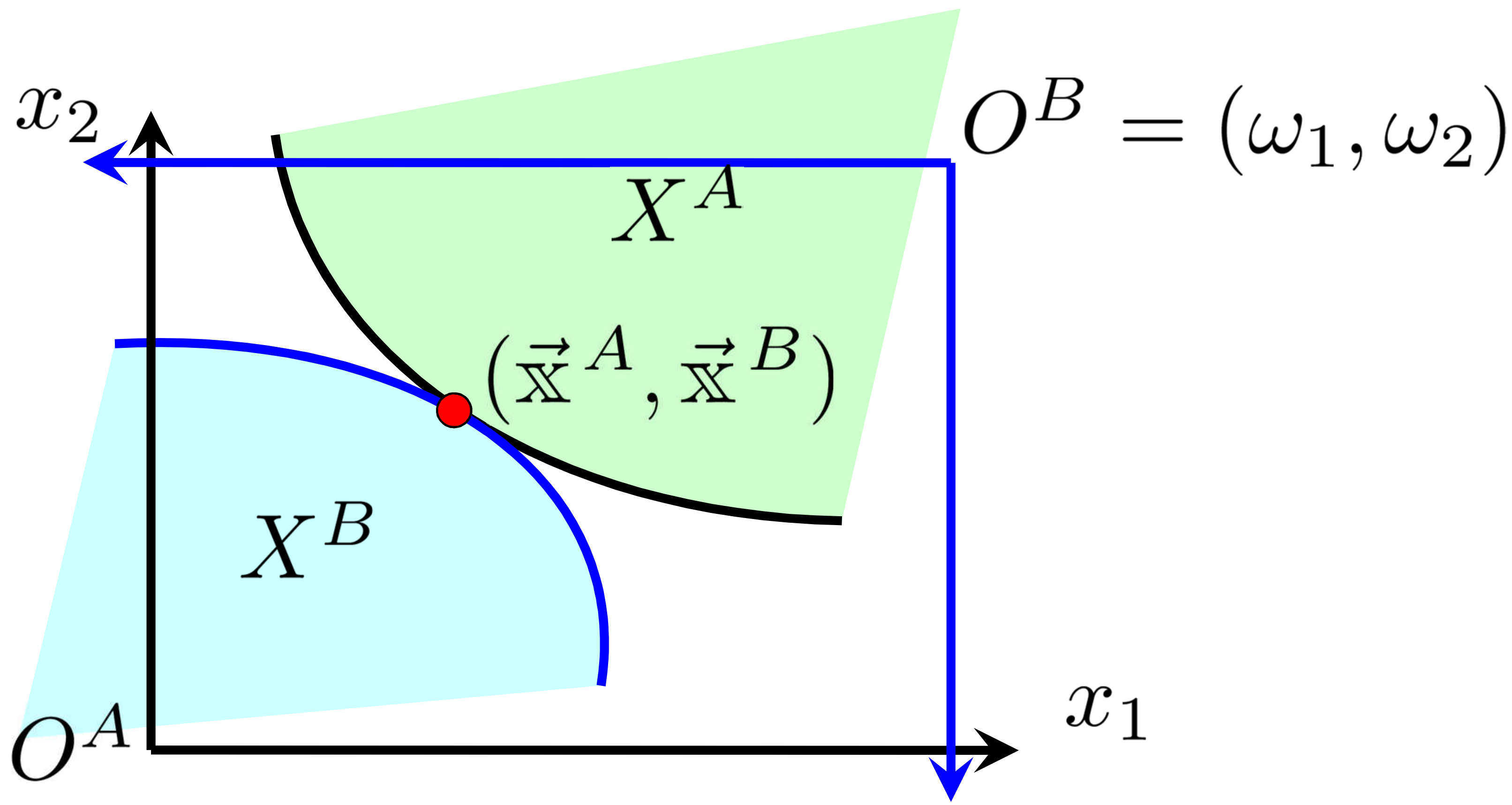
- In an exchange economy with endowment $\{\vec{\omega}^h\}_{h\in\mathcal{H}}$
- Suppose $U^h(\vec{x})$ is continuously differentiable,
- quasi-concave on $\mathbb{R}^n_+$ and $\frac{\partial U^h}{\partial \vec{x}}(\vec{x}^h) \gg \vec{0}, h \in \mathcal{H}$
- Then any PE allocation $\{\vec{\mathbb{x}}^h\}_{h\in\mathcal{H}}$ where $\vec{\mathbb{x}}^h \neq \vec{0}$
- can be supported by a price vector $\vec{p} \geq \vec{0}$ (as WE)
- Sketch of Proof: (Need not be interior as above!)

1. Constraint Qualification of the PE problem ok
2. Kuhn-Tucker conditions give us (shadow) prices
3. Alex and Bev both maximizing under these prices

Proof of Second Welfare Theorem

- (Proof for 2-player case) PEA $\Rightarrow \vec{\mathbb{x}}^A$ solves:

$$\max_{\vec{x}^A,\ \vec{x}^B} \{U^A(\vec{x}^A) | \vec{x}^A + \vec{x}^B \leq \vec{\omega}, U^B(\vec{x}^B) \geq U^B(\vec{\mathbb{x}}^B)\}$$



Proof of Second Welfare Theorem

$$\max_{\vec{x}^A,\ \vec{x}^B} \{U^A(\vec{x}^A) | \vec{x}^A + \vec{x}^B \le \vec{\omega}, U^B(\vec{x}^B) \ge U^B(\vec{\mathrm{x}}^B)\}$$

- Consider the feasible set of this problem:

1. The feasible set has a non-empty interior

- Since $U^B(\vec{x})$ is strictly increasing, for small $\vec{\delta}$,

$$\vec{0} < \vec{\mathrm{x}}^B < \vec{\omega} \Rightarrow U^B(\vec{\mathrm{x}}^B) < U^B(\vec{\omega} - \vec{\delta}) < U^B(\vec{\omega})$$

2. The feasible set is convex $(U^B(\cdot)$ quasi-concave$)$
3. Constraint function have non-zero gradient

➢ Constraint Qualifications ok, use Kuhn-Tucker

Proof of Second Welfare Theorem

$$\mathfrak{L} = U^A(\vec{x}^A) + \nu_1(\omega_1 - x_1^A - x_1^B) + \nu_2(\omega_2 - x_2^A - x_2^B)$$

$$+ \mu\left[U^B(\vec{x}^B) - U^B(\vec{\mathbb{x}}^B)\right]$$ – Kuhn-Tucker (Inequalities!)

$$\frac{\partial \mathfrak{L}}{\partial x_i^A} = \frac{\partial U^A}{\partial x_i^A}(\vec{\mathbb{x}}^A) - \nu_i \le 0, \quad \mathbb{x}_i^A \left[\frac{\partial U^A}{\partial x_i^A}(\vec{\mathbb{x}}^A) - \nu_i\right] = 0$$

$$\frac{\partial \mathfrak{L}}{\partial x_i^B} = \mu\frac{\partial U^B}{\partial x_i^B}(\vec{\mathbb{x}}^B) - \nu_i \le 0, \mathbb{x}_i^B \left[\mu\frac{\partial U^B}{\partial x_i^B}(\vec{\mathbb{x}}^B) - \nu_i\right] = 0$$

$$\frac{\partial \mathfrak{L}}{\partial \nu_i} = \omega_i - \mathbb{x}_i^A - \mathbb{x}_i^B \ge 0, \quad \nu_i\left[\omega_i - \mathbb{x}_i^A - \mathbb{x}_i^B\right] = 0$$

$$\frac{\partial \mathfrak{L}}{\partial \mu} = U^B(\vec{x}^B) - U^B(\vec{\mathbb{x}}^B) \ge 0, \mu\left[U^B(\vec{x}^B) - U^B(\vec{\mathbb{x}}^B)\right] = 0$$

Proof of Second Welfare Theorem

- For positive MU: $\dfrac{\partial U^h}{\partial \vec{x}^h}(\vec{\mathbb{x}}^h) \gg \vec{0} \Rightarrow \dfrac{\partial U^A}{\partial x_i^A}(\vec{\mathbb{x}}^A) > 0$

1. $\dfrac{\partial \mathfrak{L}}{\partial x_i^A} = \dfrac{\partial U^A}{\partial x_i^A}(\vec{\mathbb{x}}^A) - \nu_i \leq 0 \quad \Rightarrow \nu_i \geq \dfrac{\partial U^A}{\partial x_i^A}(\vec{\mathbb{x}}^A) > 0$
2. $\nu_i \left[\omega_i - \mathbb{x}_i^A - \mathbb{x}_i^B\right] = 0 \quad \Rightarrow \omega_i - \mathbb{x}_i^A - \mathbb{x}_i^B = 0$
3. $\dfrac{\partial \mathfrak{L}}{\partial x_i^B} \leq 0, \quad \mathbb{x}_i^B \left[\mu \dfrac{\partial U^B}{\partial x_i^B}(\vec{\mathbb{x}}^B) - \nu_i\right] = 0$

For $\omega_i \neq \mathbb{x}_i^A$, $\vec{\mathbb{x}}^B \gg 0$, $\dfrac{\partial U^B}{\partial x_i^B}(\vec{\mathbb{x}}^B) \gg 0$, $\Rightarrow \mu > 0$

Proof of Second Welfare Theorem

- Consider Alex's consumer problem with $\vec{p} = \vec{\nu} \gg \vec{0}$
$$\max_{\vec{x}^A} \{U^A(\vec{x}^A) | \vec{\nu} \cdot \vec{x}^A \leq \vec{\nu} \cdot \vec{\mathrm{x}}^A\}$$
- FOC: (sufficient since $U^h(\cdot)$ is quasi-concave)

$$\frac{\partial \mathfrak{L}}{\partial x_i^A} = \frac{\partial U^A}{\partial x_i^A}(\vec{x}^A) - \lambda^A \nu_i \leq 0,$$

$$x_i^A \left[\frac{\partial U^A}{\partial x_i^A}(\vec{x}^A) - \lambda^A \nu_i \right] = 0$$

- Same for Bev's consumer problem...

Proof of Second Welfare Theorem

- FOC: (sufficient for $U^h(\cdot)$ is quasi-concave)

$$\frac{\partial U^A}{\partial x_i^A}(\vec{x}^A) - \lambda^A \nu_i \leq 0, x_i^A \left[\frac{\partial U^A}{\partial x_i^A}(\vec{x}^A) - \lambda^A \nu_i\right] = 0$$

$$\frac{\partial U^B}{\partial x_i^B}(\vec{x}^B) - \lambda^B \nu_i \leq 0, x_i^B \left[\frac{\partial U^B}{\partial x_i^B}(\vec{x}^B) - \lambda^B \nu_i\right] = 0$$

- Set, $\lambda^A = 1, \lambda^B = 1/\mu$,
- Then, FOCs are satisfied at $\vec{x}^A = \vec{\mathbb{x}}^A, \vec{x}^B = \vec{\mathbb{x}}^B$
- At price $\vec{p} = \vec{\nu} \gg \vec{0}$, neither Alex nor Bev want to trade, so this PE allocation is indeed a WE!

Proof of Second Welfare Theorem

- Define transfers $$T^A = \vec{\nu} \cdot (\vec{\mathbb{x}}^A - \vec{\omega}^A)$$ $$T^B = \vec{\nu} \cdot (\vec{\mathbb{x}}^B - \vec{\omega}^B)$$
- With $\vec{\omega} - \vec{\mathbb{x}}^A - \vec{\mathbb{x}}^B = \vec{\omega}^A + \vec{\omega}^B - \vec{\mathbb{x}}^A - \vec{\mathbb{x}}^B = \vec{0}$
- Alex and Bev's new budget constraints with these transfers are: $$\vec{\nu} \cdot \vec{x}^A \leq \vec{\nu} \cdot \vec{\omega}^A + T^A = \vec{\nu} \cdot \vec{\mathbb{x}}^A$$ $$\vec{\nu} \cdot \vec{x}^B \leq \vec{\nu} \cdot \vec{\omega}^B + T^B = \vec{\nu} \cdot \vec{\mathbb{x}}^B$$
- Thus, PE allocation can be support as WE with these transfers. Q.E.D.

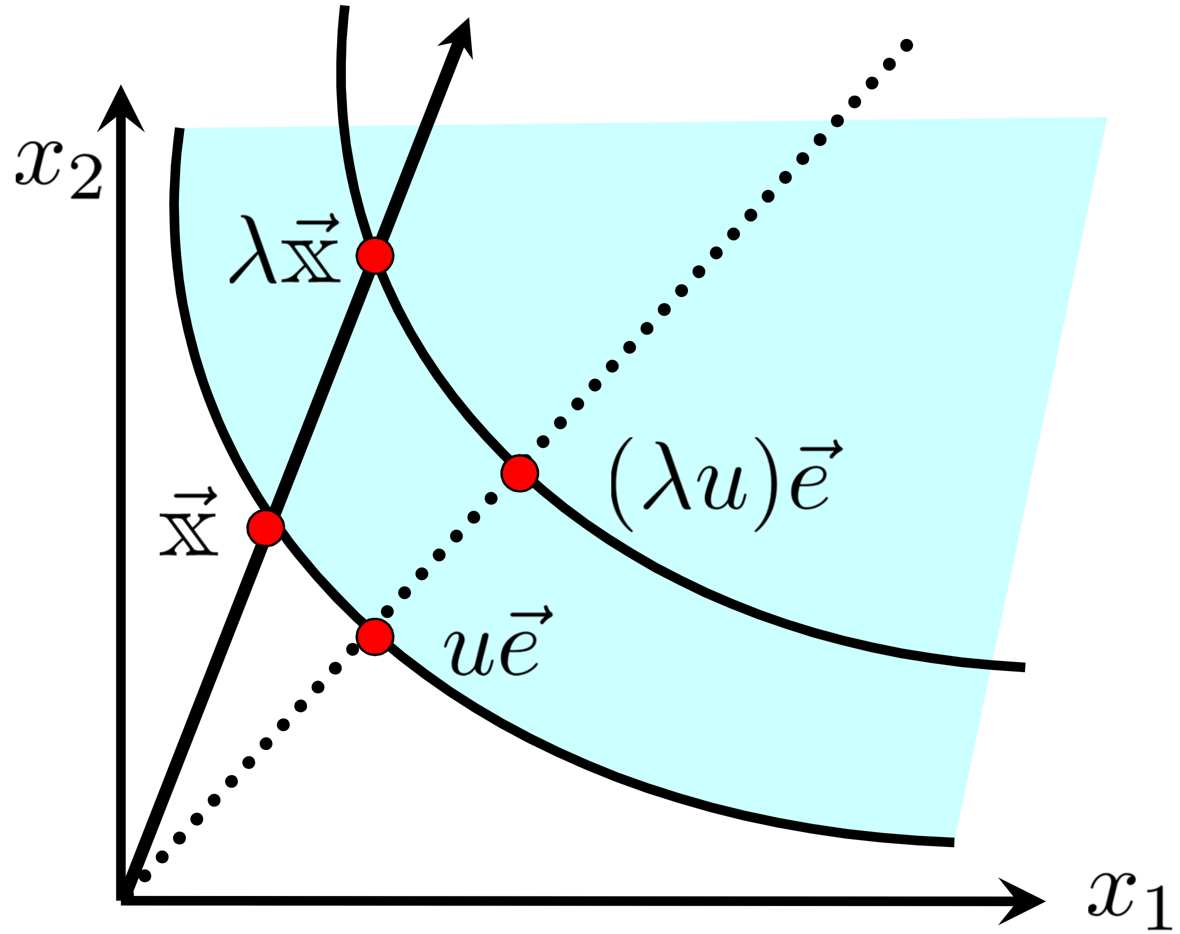
Example: Quasi-Linear Preferences

- Alex has utility function $U^A(\vec{x}^A) = x_1^A + \ln x_2^A$
- Bev has utility function $U^B(\vec{x}^B) = x_1^B + 2\ln x_2^B$

- Draw the Edgeworth box and find:
- All PE allocations
- Can they be supported as WE?
- What are the supporting price ratios?

Homothetic Preferences: Radial Parallel Pref.

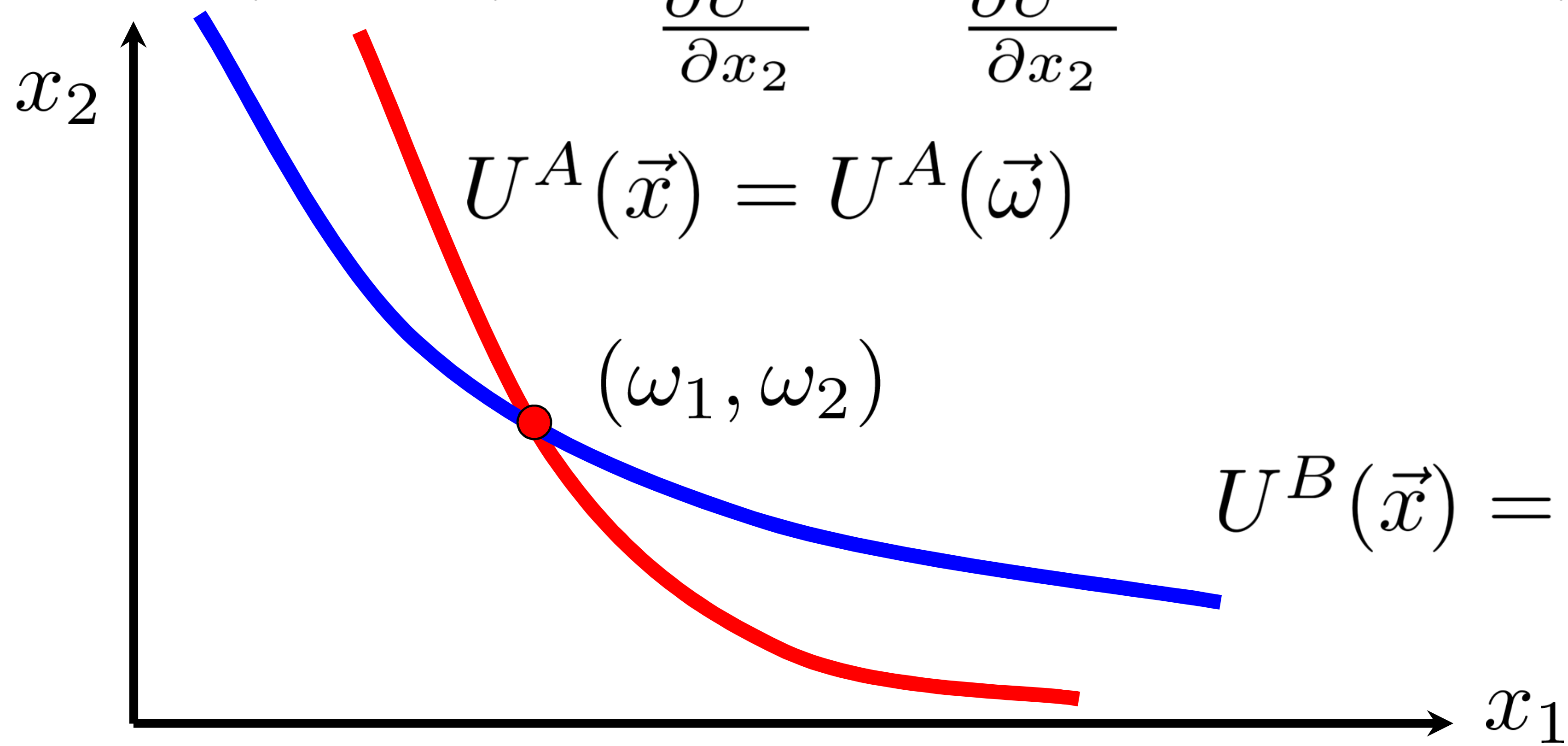
- Consumers have homothetic preferences (CRS)
  - MRS same on each ray, increases as slope of the ray increase



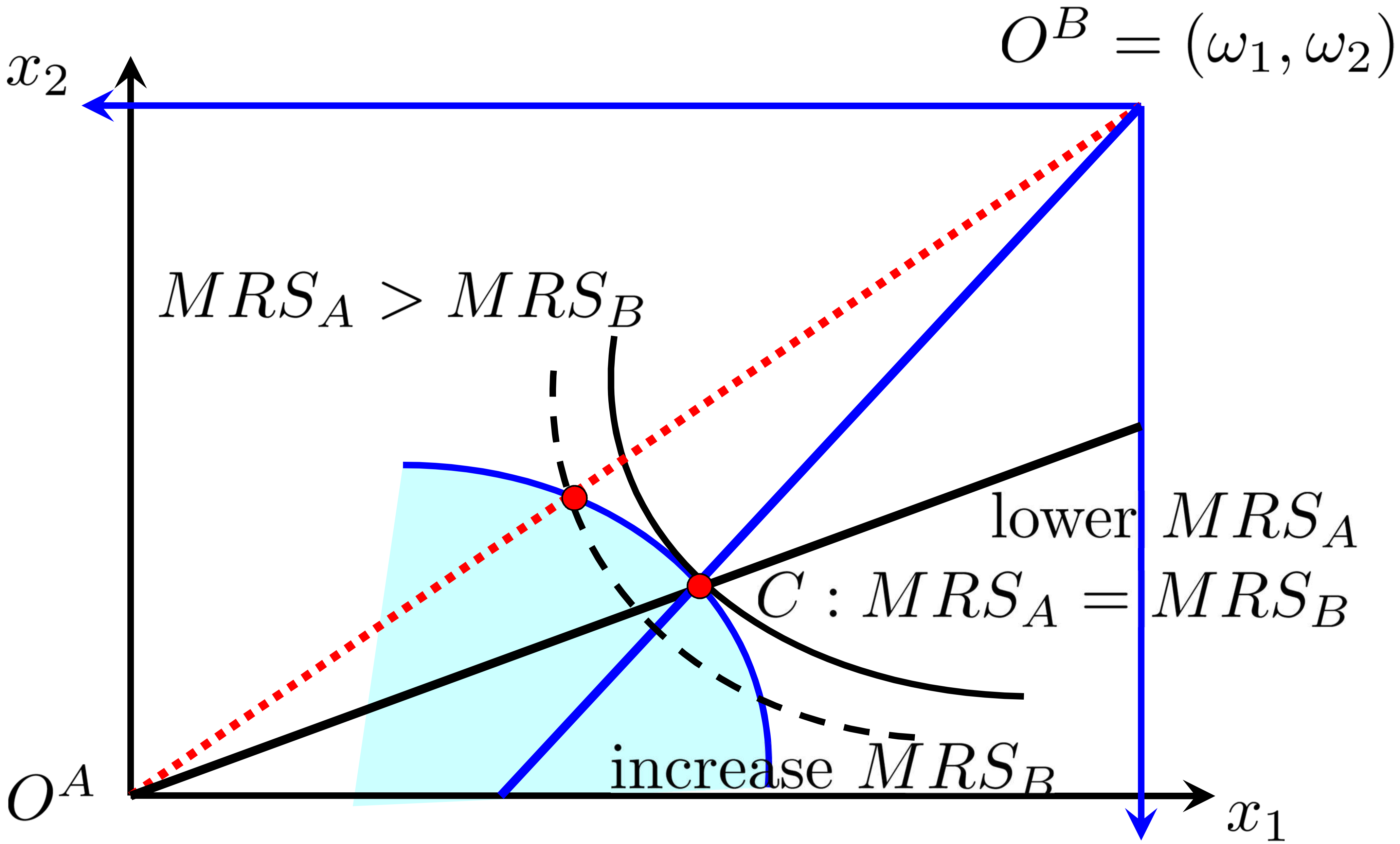
Assumption: Intensity of Preferences

- At aggregate endowment, Alex has a stronger preference for commodity 1 than Bev.

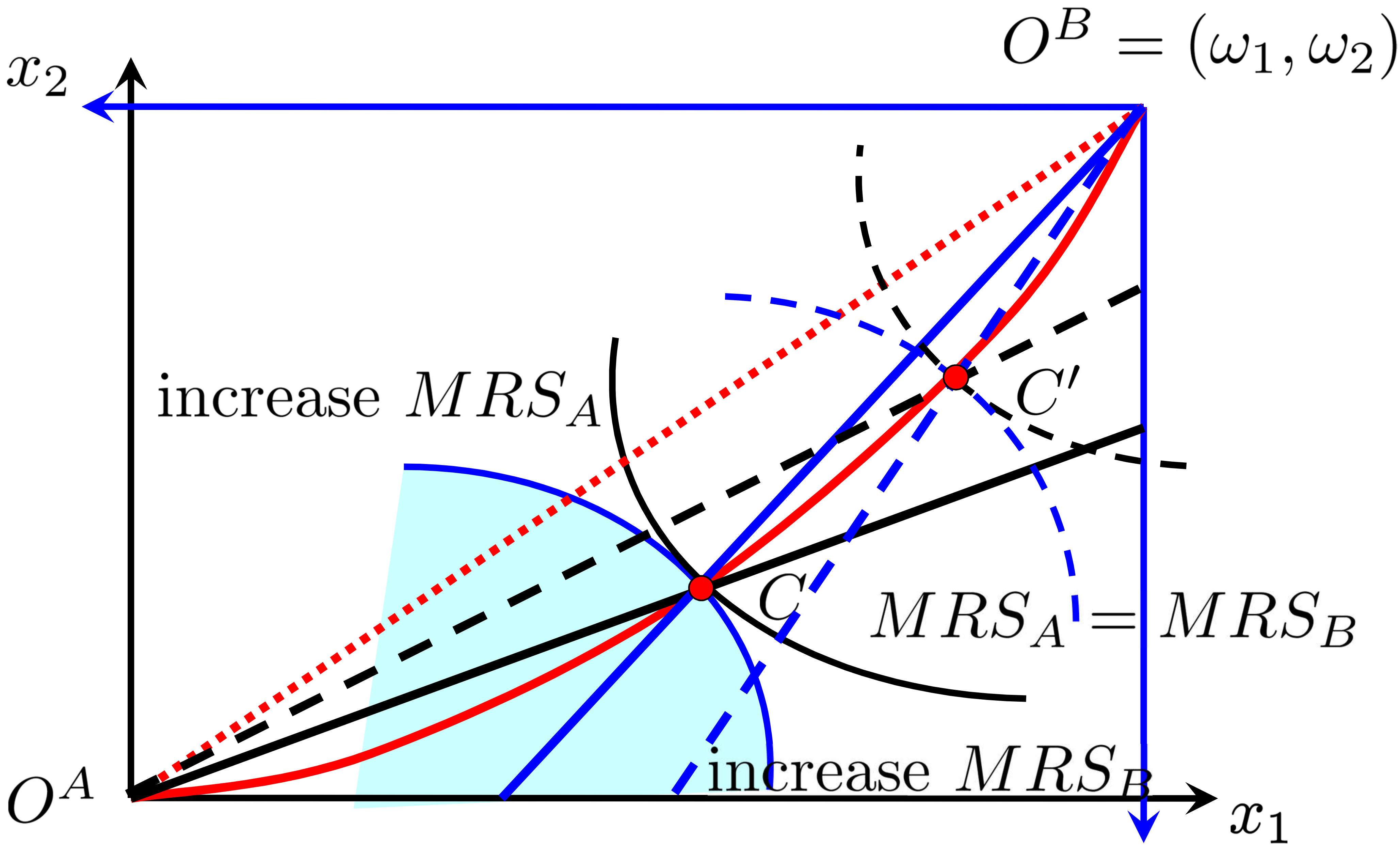
$$MRS_A(\omega_1, \omega_2) = \frac{\frac{\partial U^A}{\partial x_1}}{\frac{\partial U^A}{\partial x_2}} > \frac{\frac{\partial U^B}{\partial x_1}}{\frac{\partial U^B}{\partial x_2}} = MRS_B(\omega_1, \omega_2)$$



PE Allocations with Homothetic Preferences



PE Allocations with Homothetic Preferences



PE Allocations with Homothetic Preferences

- 2x2 Exchange Economy: Alex and Bev have convex and homothetic preferences
- At aggregate endowment, Alex has a stronger preference for commodity 1 than Bev.
- Then, at any interior PE allocation, we have:

$$\frac{x_2^A}{x_1^A} < \frac{\omega_2}{\omega_1} < \frac{x_2^B}{x_1^B}$$

- And, as $U^A(\vec{x}^A)$ rises, consumption ratio $\frac{x_2^A}{x_1^A}$ and MRS both rise.

Summary of 3.1

- Pareto Efficiency:
  - Can't make one better off without hurting others
- Walrasian Equilibrium: market clearing prices
- First Welfare Theorem: WE is PE
- Second Welfare Theorem: PE allocations can be supported as WE (with transfers)
- Homework: 2008 midterm-Question 3
  - (Optional: 2009 midterm-Part A and Part B)

In-Class Exercise 3.1-4: Linear Preferences

- Alex has utility function $U^A(\vec{x}^A) = 2x_1^A + x_2^A$
- Bev has utility function $U^B(\vec{x}^B) = x_1^B + 2x_2^B$
  - Total endowment is (30, 20)

a) Depict PE allocations in an Edgeworth box

- Show that if Alex has sufficiently large fraction of total endowment, equilibrium price ratio is

$$p_1/p_2 = 2$$

- What if Bev has a large fraction of the total endowment?

In-Class Exercise 3.1-4: Linear Preferences

- Alex has utility function $U^A(\vec{x}^A) = 2x_1^A + x_2^A$
- Bev has utility function $U^B(\vec{x}^B) = x_1^B + 2x_2^B$
  - Total endowment is (30, 20)

b) For what endowment will the price ratio lie between these two extremes? Find the WE.

c) Show that for some endowments a transfer of wealth from Alex to Bev has no effect on prices, and for other endowment there is no effect on WE allocation.

In-Class Exercise: Quasi-Linear Preferences

- Alex has utility function $U^A(\vec{x}^A) = x_1^A + \ln x_2^A$
- Bev has utility function $U^B(\vec{x}^B) = x_1^B + 2\ln x_2^B$

- Draw the Edgeworth box and find:
- All PE allocations
- Can they be supported as WE?
- What are the supporting price ratios?

In-Class Homework: Exercise 3.1-1

- Consider a two-person economy in which the aggregate endowment is $(\omega_1, \omega_2) = (100, 200)$
- Both have same quasi-linear utility function

$$U(\vec{x}^h) = x_1^h + \sqrt{x_2^h}$$

a) Solve for the Walrasian equilibrium price ratio assuming equilibrium consumption of good 1 is positive for both individuals.

b) What is the range of possible equilibrium price ratios in this economy?

In-Class Homework: Exercise 3.1-2

a) If U^A and U^B are strictly increasing, explain why the allocation $\{\vec{x}^A, \vec{x}^B\} = \{\vec{\omega}^A + \vec{\omega}^B, \vec{0}\}$ is a PE and WE allocation.

- Suppose that $U^A(\vec{x}^A) = x_1^A + 10 \ln x_2^A$ and
$$U^B(\vec{x}^B) = \ln x_1^B + x_2^B$$
- Aggregate endowment is $(\omega_1, \omega_2) = (20, 10)$

In-Class Homework: Exercise 3.1-2

- Let $U^A = x_1^A + 10 \ln x_2^A$ and $U^B = \ln x_1^B + x_2^B$
- Aggregate endowment is $(\omega_1, \omega_2) = (20, 10)$

b) Show that PEA in the interior of the Edgeworth box can be written as $\mathbb{x}_2^A = f(\mathbb{x}_1^A)$

c) Suppose that $\omega_2^A = f(\omega_1^A)$. How does the equilibrium price ratio change as ω_1^A increases along the curve?

d) Which allocations on the boundary of the Edgeworth box are PE allocations?