Microeconomic Theory I - Quiz Fall 2014

Exam Time: 10/17 2:20pm-4:20pm. You have 2 hours; allocate your time wisely.

Part A (35%): The CES Utility Function

Consider the utility function $U(x_1, x_2) = \left(\alpha_1 x_1^{1-\frac{1}{\theta}} + \alpha_2 x_2^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}}, \alpha_1, \alpha_2, \theta > 0, \theta \neq 1.$

1. (5%) Is the preference represented by this utility function homothetic or not? Why?
2. (15%) For a consumer with this utility function facing price vector p and income I, solve for the demand $x_j(p, I)$. (Note that you can either solve the consumer problem or derive the indirect utility function and appeal to Roy's identity. Either case, you have to state all assumptions required and verify that they indeed hold.)
3. (5%) Show that when $\theta = 1$, the demand $x_j(p, I)$ of this utility function coincides with that of Cobb-Douglas preference $U(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}, \alpha_1, \alpha_2 > 0$.
4. (5%) What is $\mathcal{E}\left(\frac{x_2^c}{x_1^c}, \frac{p_1}{p_2}\right)$ for this utility function?
5. (5%) Depict the income expansion path for a consumer with this utility function.

Part B (25%): The 2x2 Exchange Economy

Consider two consumers, A and B, having utility functions $\begin{cases} u^A(x_b, x_g) = 2x_b + x_g \\ u^B(x_b, x_g) = \sqrt{x_b x_g} \end{cases}$

1. (10%) Draw the Edgeworth box for this 2-person economy and carefully depict these Pareto efficient allocations. (Note: You will have to justify your answers, say, by appealing to the Kuhn-Tucker conditions.)
2. (10%) Suppose endowments are $(\omega_b^A, \omega_g^A) = (689, 689)$ and $(\omega_b^B, \omega_g^B) = (300, 300)$. What is the Walrasian equilibrium for these two consumers?
3. (5%) Are all Pareto efficient allocations implementable as Walrasian Equilibrium? Why or why not?

   (Hint: You should use what you have learned in the previous part.)

Part C (40%): Edgeworth Box Bargaining of 689 and the Zhan-Zhong Trio

Consider an exchange economy with 692 consumers. Three "big" consumers each have endowment (100, 100) and utility function $u(x_b, x_g) = \frac{1}{2}\ln x_b + \frac{1}{2}\ln x_g$, while the remaining consumers have endowment (1,1) and utility function $u(x_b, x_g) = x_b + 10x_g$.

1. (15%) What is the Walrasian equilibrium for these 692 consumers? Is the equilibrium outcome Pareto efficient? Why or why not?

2. (15%) Consider the exchange economy replacing the three big consumers with one "huge" consumer with endowment (300, 300) and utility function $u(x_b, x_g) = 100x_b$ What is the Walrasian equilibrium for these 690 consumers? Is the equilibrium outcome Pareto efficient? Why or why not?
3. (5%) Are small consumers better-off facing one huge consumer than three big consumers? Why or why not?
4. (5%) What assumptions do you need to take this model to real world bargaining situations? Are they likely to hold? Explain.

(Hint: You should use what you have learned in the previous parts.)