

Exam Time: 10/25 2:20pm-5:20pm. You have 3 hours; allocate your time wisely.

**Part A (30+bonus 10%): Who Should Pay for the (4th Nuclear) Power Plant?**

1. (5%) Daiwan's electricity market has two sectors ( $s = 1, 2$ ), the industrial sector and non-industrial sector (households and services), and two periods ( $t = 1, 2$ ), peak and off-peak. Demand for each sector and period is  $P_{st}(q_{st})$ . What is the total revenue (as a function of quantities from each sector/period) for Dai-Power Corporation?
2. (5%) Dai-Power needs to build power plants to provide electricity. In particular, the current capacity of electricity is  $Q_0$ . Assume the marginal cost of producing electricity is  $c$  and the marginal cost to build power plants is  $C_0$ . What is the total cost (as a function of quantities from each sector/period) for Dai-Power?
3. (10%) Suppose Dai-Power is an unregulated monopoly. Solve for profit-maximizing prices for each sector and each period. What is the capacity choice? According to the shadow price, which sector bears the cost of building power plants?
4. (10%) Suppose the government requires Dai-Power to set prices at  $P_{st}^*$ . What are profit-maximizing quantities of electricity for each sector/period? What is the capacity choice? Who is bearing the cost of building power plants now?
5. (bonus 10%) What are the policy implications of your answers to the above questions? Are there assumptions that are likely to fail in the real world?

**Part B (25%): Laws of Economics**

1. (21%) Is revealed preference sufficient to prove the following statements? If yes, prove them; if not, provide a counter-example.
  - a. Let  $x^0$  be utility maximizing when the price vector is  $p^0$  and  $x^1$  be utility maximizing when the price vector is  $p^1$ , both under a fixed income  $I$ . Then,  $(p^0 - p^1) \cdot (x^0 - x^1) \leq 0$ .
  - b. Let  $x^0$  be expenditure minimizing when the price vector is  $p^0$  and  $x^1$  be expenditure minimizing when the price vector is  $p^1$ , both weakly preferred to a default consumption  $x^*$ . Then,  $(p^0 - p^1) \cdot (x^0 - x^1) \leq 0$ .
  - c. Let  $p^0$  and  $p^1$  be two different price vectors and let  $y^0$  and  $y^1$  be profit maximization production plans at these prices. Then,  $(p^0 - p^1) \cdot (y^0 - y^1) \geq 0$ .
2. (4%) Which of the following requires the least assumptions? Law of demand, law of supply and law of compensated demand. Explain.

**Part C (45+bonus 10%): The Edgeworth Cube of Quasi-Linear Utility**

Alex and Bev share total endowment  $(\omega_1, \omega_2, \omega_3) = (10, 20, 30)$  and have utility functions:

$$\begin{cases} u_A(x_1, x_2, x_3) = x_1 + 2x_2 + \ln(x_3) \\ u_B(x_1, x_2, x_3) = \sqrt{x_1} + x_2 + 2x_3 \end{cases}$$

1. (15%) Find all Pareto efficient allocations. (Note: You will have to justify your answers, say, by appealing to the Kuhn-Tucker conditions.)
2. (5%) Draw the Edgeworth cube and carefully depict these Pareto efficient allocations. (Hint: Draw several 2D projections of the cube if 3D drawing is too complicated.)
3. (15%) Which of the Pareto efficient allocations inside the Edgeworth Cube can be supported as a Walrasian equilibrium outcome? Solve for the Walrasian equilibrium for those that can, and show why there is no market-clearing price vector for the rest.
4. (10%) Suppose initial endowment for Alex is  $(\omega_1^A, \omega_2^A, \omega_3^A) = (10, 20, 0)$ . Solve for the Walrasian equilibrium and the consumption bundles for Alex and Bev.
5. (bonus 10%) Can the Pareto efficient allocations on the boundary of the Edgeworth Cube be supported as Walrasian equilibrium? Why or why not?