Microeconomic Theory I - Midterm Fall 2013

Exam Time: 11/8 2:20pm-5:20pm. You have 3 hours; allocate your time wisely.

Part A (50%): Prediction Markets for Lan and Lu

Consider three states of the world, DL, HZ, TY, which leads to different total endowment $(\omega_1, \omega_2, \omega_3) = (10, 20, 30)$ in Daiwan, say, due to whether Dailiok helps boost Daiwan's economy. The Daiwanese people can be classified into two types, represented by either Zi Lu or Gee Lan, each believing the three states would occur with probability (0.25, 0.5, 0.25) and (0.25, 0.25, 0.5).

1. (2%) Suppose Lu and Lan are both risk neutral with Von Neumann-Morgenstern utility functions. Write down utility functions that would represent each of them.
2. (2%) Now suppose Lu feels extremely depressed in state TY, which transforms his utility at that state by logarithm, say, $v(y) = \sqrt{y}$, while Lan would be extremely nervous in state DL, transforming her utility by taking square root (say $u(x) = \ln(x)$). Write down the new utility functions.

You can ask for utility functions and proceed, but would have to forfeit the above scores.

3. (4%) Which of the axiom(s) satisfied by expected utility theory is violated after the above transformations?
4. (12%) Consider the negotiation between Lu and Lan. Assume that the two would negotiate until there is no further room for improvement (of one's welfare without hurting the other party). Find all the possible contracts they could reach.
5. (4%) Draw all possible allocations and carefully depict contracts the two would reach (if nothing is left on the table). (Hint: Draw 2D projections if 3D is too complicated.)
6. (7%) Consider zFuture in Daiwan that offers event futures for each state. If Lu and Lan both trade in the future markets, but only for states HZ and TY, Solve the consumer problem and identify which person buys/sells state claims of HZ.
7. (12%) Assume future markets are competitive so the Walrasian equilibrium is a good prediction. Which of the contracts where both parties get something in all states (in Question 4) can be a competitive outcome of these markets? Why?
8. (7%) Suppose the initial endowment for Lu is $(\omega_1^{Lu}, \omega_2^{Lu}, \omega_3^{Lu}) = (10, 20, 0)$. What bundle would Lu and Lan consume if they can trade in zFuture? Are they better off?

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Part B (50 + bonus 10%): The Professor-Student Problem

Consider the relationship between Professor Joseph and Student Yu. Professor Joseph has expected utility function satisfying $u'(x) = x^{-R}$ where $R > 0$, while Student Yu has expected utility function satisfying $v'(x) = x^{-r}$ with $r < R$. Consider the ten lottery choices of Holt and Laury (2002) listed below:

You will roll a ten-sided die and get paid according to your decision (choice A or B):

Decision	Lottery A	Lottery B	Your choice (A or B)
Question 1	1：Gain NT\$200 2～10：Gain NT\$160	1：Gain NT\$385 2～10：Gain NT\$10	
Question 2	1～2 ：Gain NT\$200 3～10：Gain NT\$160	1～2 ：Gain NT\$385 3～10：Gain NT\$10	
Question 3	1～3 ：Gain NT\$200 4～10：Gain NT\$160	1～3 ：Gain NT\$385 4～10：Gain NT\$10	
Question 4	1～4 ：Gain NT\$200 5～10：Gain NT\$160	1～4 ：Gain NT\$385 5～10：Gain NT\$10	
Question 5	1～5 ：Gain NT\$200 6～10：Gain NT\$160	1～5 ：Gain NT\$385 6～10：Gain NT\$10	
Question 6	1～6 ：Gain NT\$200 7～10：Gain NT\$160	1～6 ：Gain NT\$385 7～10：Gain NT\$10	
Question 7	1～7 ：Gain NT\$200 8～10：Gain NT\$160	1～7 ：Gain NT\$385 8～10：Gain NT\$10	
Question 8	1～8 ：Gain NT\$200 9～10：Gain NT\$160	1～8 ：Gain NT\$385 9～10：Gain NT\$10	
Question 9	1～9 ：Gain NT\$200 10：Gain NT\$160	1～9 ：Gain NT\$385 10：Gain NT\$ 10	
Question 10	1～10：Gain NT\$200	1～10：Gain NT\$385	

1. (3%) Show that both Professor Joseph and Student Yu exhibit constant relative risk aversion. Hence or otherwise, solve for their Von Neumann-Morgenstern utility functions $u(.)$, $v(.)$, and corresponding degree of relative risk aversion $R(x)$.
2. (3%) Show that a risk neutral person would choose lottery A for Questions 1~4 and lottery B otherwise.
3. (4%) Would Professor Joseph choose more or less lottery A's than a risk neutral person? Why or why not? What about Student Yu (compared to a risk neutral person and/or to Professor Joseph)?

4. (4%) Show that if Professor Joseph chooses lottery B in Question k, he would also choose lottery B in Question (k+1).
5. (6%) Show that if a person follows expected utility theory and chooses lottery B in Question k, he would also choose lottery B in Question (k+1).
6. (bonus 10%) What is the critical assumption required for the above statement to be true? Is expected utility theory really required? Why or why not?

Student Yu decides how much to study for the midterm exam ($e \in [0,1]$); his midterm exam outcome can be either satisfactory ($s = 1$) or disappointing ($s = 0$). Suppose the probability of having a satisfactory/disappointing outcome, $\pi_s(e)$, depends on effort. In particular, $\pi_1(e) = e$, so $\pi_1(e) = 1 - e$. Professor Joseph cares about the exam outcome y_s of Student Yu, and decides to design a scoring scheme to induce Student Yu to put more effort into micro theory. Assume the two parties have expected utility:

$$U_P = \sum_{s=0}^{1} \pi_s(e) u(y_s - w_s) \qquad U_A = \sum_{s=0}^{1} \pi_s(e) v(w_s) - C(e)$$

where $u'(x) = x^{-R}$; $v'(x) = x^{-r}$, $C(e) = \dfrac{e^2}{4}$, and Student Yu's outside option (retake the course next year) yields reservation utility $\overline{U}_A = 0$. First assume Professor Joseph can somehow observe the effort Student Yu puts into studying microeconomic theory because he has access to espionage technology and abuses it.

7. (2%) Show that the increasing likelihood ratio assumption holds. In other words, show that for $\tilde{e} > e$, $\dfrac{\pi_1(\tilde{e})}{\pi_1(e)} > \dfrac{\pi_0(\tilde{e})}{\pi_0(e)}$
8. (6%) Write down the Professor Joseph's optimization problem and derive the Kuhn-Tucker conditions for its solution.
9. (4%) Suppose $R = 0$. Show that the optimal scoring scheme involves Professor Joseph giving the same score regardless of the midterm outcome (i.e. $w_1 = w_0$).
10. (4%) Suppose $r = 0$. What is the optimal scoring scheme for Professor Joseph?

Now suppose more realistically that Professor Joseph cannot observe Student Yu's effort.

11. (2%) If $r = 0$, what is the optimal scoring scheme for Professor Joseph?
12. (10%) If $R = 0$ and $r = 2$, show that optimal $e = 1$ and solve for the optimal scoring scheme for Professor Joseph. (Note that you might get negative w!)
13. (2%) Did your micro theory professor adopt such an optimal scheme? Why or why not? (What assumptions could fail in real life?)