Theory of Choice

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2009/9/25

(Lecture 3, Micro Theory I)

Preferences, Utility and Choice

- Empirically, we see people make choices
- Can we come up with a theory about "why" people made these choices?
- Preferences: People choose certain things instead of others because they "prefer" them
  - As an individual, preferences are primitive; my choices are made based on my preferences
- Can we do some reverse engineering?

Preferences, Utility and Choice

- Revealed Preferences: Inferring someone's preferences by his/her choices
  - As an econometrician, choices are primitive; preferences are "revealed" by observing them
- Not formally discussed in our textbook, but the idea of revealed preferences is everywhere…
- Also, MWG show WARP "=" total + transitive
  - See my presentation on Predictably Irrational…
- Can we do further reverse engineering?

Preferences, Utility and Choice

Choices ←→ Preferences ←→ Utility

- Can we describe preferences as a function?
- Utility: A function that "describes" preferences
  - Someone's true utility may not be the same as what economists assume, but they behave "as if"
- What are the axioms needed for a preference to be described by a utility function?

Why do we care about this?

- Objective function required for constrained max.
- We cannot observe one's real utility (objective)
  - Neuroeconomics is trying this, but "not there yet" (Except places that don't care about human rights)…
- However, we can observe one's choices
  - We can try to elicit preferences by asking people to make a lot of choices (= revealed preference!)
- Can we find a utility function (an economic model) that describes these preferences?
- If yes, we can use it as our objective function…

Preferences: How alternatives are ordered?

- A binary relation for household h: $\succsim_h$

  $x^1 \succsim_h x^2$ (x^1 is ordered as least as high as x^2)

  - But order may not be defined for all bundles…
- Weak inequality order:

  $$x^1 \succsim_h x^2 \text{ if and only if } x^1 \geq x^2$$

  - Cannot define order between (1,2) and (2,1)…

Preferences: Completeness and Transitivity

- To represent preferences with utility function, consumer needs to be compare all bundles…
- Complete Axiom: (Total Order)

For any consumption bundle $x^1, x^2 \in X$,

either $x^1 \succsim_h x^2$ or $x^2 \succsim_h x^1$.

- Also need consistency across pair-wise rankings…
- Transitive Axiom:

For any consumption bundle $x^1, x^2, x^3 \in X$,

if $x^1 \succsim_h x^2$ and $x^2 \succsim_h x^3$, then $x^1 \succsim_h x^3$.

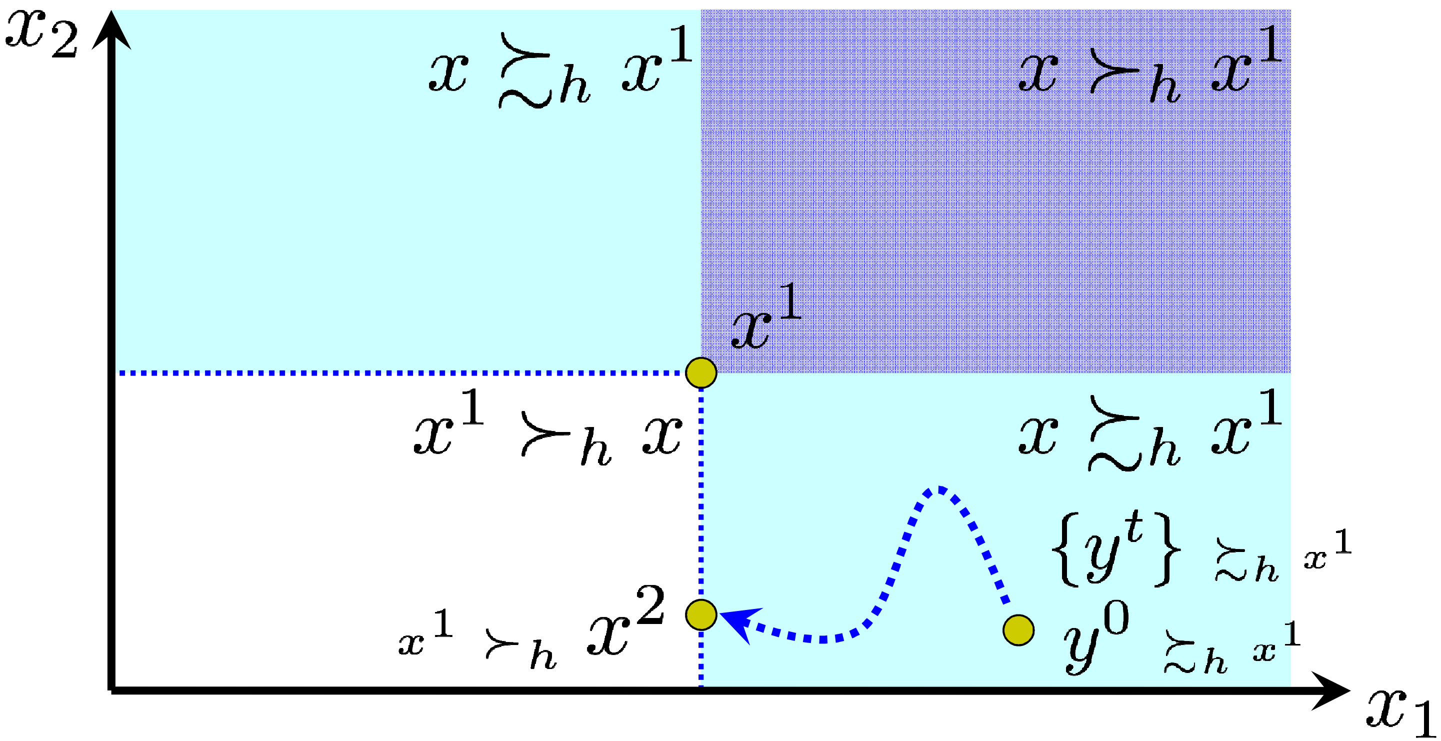
Preferences: Indifference; Strictly Preferred

- Indifference:
  $x^1 \sim_h x^2$ if and only if $x^1 \succsim_h x^2$ and $x^2 \succsim_h x^1$
- Strictly Preferred:
  $x^1 \succ_h x^2$ if and only if $x^1 \succsim_h x^2$, but $x^2 \not\succsim_h x^1$
  $x^2 \succ_h x^1$ if and only if $x^2 \succsim_h x^1$, but $x^1 \not\succsim_h x^2$
- Indifference order and strict preference order are both transitive, but not complete (total)
- The two axioms above are not enough…

Example: “Not-Less-Than” Order

- “Not-less-than” order: (Complete & Transitive)

$$x^1 \succsim_h x^2 \text{ if and only if } x^1 \nless x^2$$



Continuous Preferences

- Why is non-continuous order a problem?

  $$y^t (\sim_h x^1) \to x^2, \text{ but } x^1 \succ_h x^2$$

- Corresponding utility also not continuous!

  $$U(y^t) = U(x^1) \nrightarrow U(x^2) > U(x^1)$$

- Continuous Order:

  Suppose $\{x^t\}_{t=1,2,\cdots} \to x^0$. For any bundle y,

  If for all t, $x^t \succsim_i y$ then $x^0 \succsim_i y$.

  If for all t, $y \succsim_i x^t$ then $y \succsim_i x^0$.

Preferences: Where Do These Postulates Apply?

- More applicable to daily shopping (familiar…)
  - Can you rank things at open-air markets in Turkey?
- What if today's choice depends on past history or future plans? Consider: $x_t = (x_{1t}, x_{2t}, \cdots, x_{nt})$

  Then use $x = (x_1, x_2, \cdots, x_t, \cdots, x_T)$
- What if there is uncertainty about the complete bundle? Consider: $(x_1, x_2^g, x_2^b; \pi^g, \pi^b)$
- Would adding time and uncertainty make the commodities less "familiar"?

Preferences: LNS (rules out "total indifference")

- Back to full information, static (1 period) case:
- A "everything-is-as-good-as-everything" order satisfies all other postulates so far
  - But this isn't really useful for explaining choices…
- Local non-satiation (LNS):

  For any consumption bundle $x \in C \subset \mathbb{R}^n$

  and any δ-neighborhood $N(x, \delta)$ of x,

  there is some bundle $y \in N(x, \delta)$ s. t. $y \succ_h x$

Preferences: Strict Monotonicity

- Another strong assumption is "More is always strictly preferred."
  - Natural for analyzing consumption of commodity groups (food, clothing, housing…)
- Strict Monotonicity:

  If $y > x$, then $y \succ_h x$.

Preferences: Convexity

- Final postulate: "Individuals prefer variety."
- Convexity:

  Let C be a convex subset of $\mathbb{R}^n$

  For any $x^0, x^1 \in C$, if $x^0 \succsim_h y$ and $x^1 \succsim_h y$,

  then $x^\lambda = (1-\lambda)x^0 + \lambda x^1 \succsim_h y$, $0 < \lambda < 1$.
- Strict Convexity:

  For any $x^0, x^1, y \in C$, if $x^0 \succsim_h y$ and $x^1 \succsim_h y$,

  then $x^\lambda \succ_h y$, $0 < \lambda < 1$.

Proposition 2.1-1: When is the Utility Function Continuous?

- Utility Function Representation of Preferences

  If preferences are complete, (reflective? $x \succsim_h x$),

  transitive and continuous on $C \subset \mathbb{R}^n$,

  they can be represented by a function $U(x)$

  which is continuous over X.
- This means we can use a utility function to represent preferences, and use it as the objective function in constraint maximization
- Special Case: Strict Monotonicity

Special Case: Strict Monotonicity

Consider $x^0, x^1 \in X$, $x^1 > x^0 \Rightarrow x^1 \succ_h x^0$

For $T = \{x \in S | x^1 \succsim_h x \succsim_h x^0\}$,

Claim:

For any $y \in T$, there exists some weight $\lambda \in [0, 1]$

such that $y \sim_h x^\lambda$ where $x^\lambda = (1 - \lambda)x^0 + \lambda x^1$

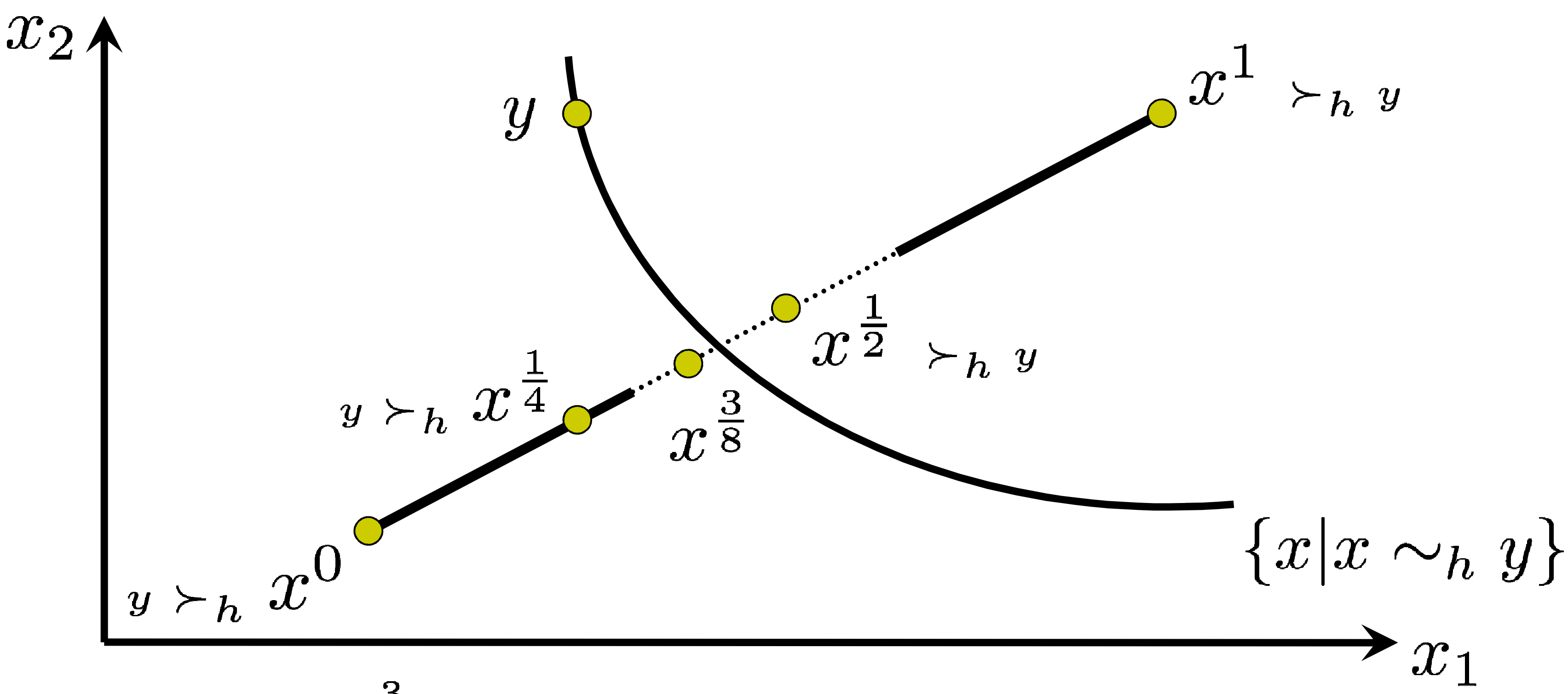
Moreover, $\lambda(y) : T \to [0, 1]$ is continuous.

Proof:

Consider the sequence of intervals $\{x^{\nu_t}, x^{\mu_t}\}$,

Appeal to the completeness of real numbers…

Special Case: Strict Monotonicity



Either $x^{\frac{3}{8}} \sim_h y$ (done),

$x^{\frac{3}{8}} \succ_h y$(consider $x^{\frac{3}{16}}$), or $y \succ_h x^{\frac{3}{8}}$ (consider $x^{\frac{7}{16}}$).

Special Case: Strict Monotonicity

Goal: Find $x^{\hat{\lambda}} \sim_h y$ as the limiting point of

Sequences $x^{\nu_t} (\succsim_h y)$ and $(y \succsim_h) x^{\mu_t}$

Start with $\nu_0 = 1,\ \mu_0 = 0$. Let $\lambda_{t+1} = \frac{1}{2}(\nu_t + \mu_t)$

If $y \sim_h x^{\lambda_t}$, we are done.

If $y \succ_h x^{\lambda_t},\ \nu_{t+1} = \nu_t,\ \mu_{t+1} = \lambda_{t+1}$

If $x^{\lambda_t} \succ_h y,\ \nu_{t+1} = \lambda_{t+1},\ \mu_{t+1} = \mu_t$

$$x^0 = x^{\nu_0} \succ_h \cdots \succ_h x^{\nu_n} \succ_h y$$

$$y \succ_h x^{\mu_n} \succ_h \cdots \succ_h x^{\mu_0} = x^1$$

Completeness of real numbers ➔ $\hat{\lambda}(y)$ exists.

Summary of 2.1

- Preference Axioms
  - Complete
  - Transitive
  - Continuous
    - Monotonic
    - Convex / Strictly Convex
- Utility Function Representation
- Homework: TBA