Decision-Making by Price-Taking Firms

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(Lecture 11, Micro Theory I)

A Price Taking Firm

- Maximize Profit vs. Minimize Cost
- Cost Function (the Minimized Cost):
  - Input Price Change (Revealed Preference)
  - Normal Input (Input Price Effect on MC)
  - Convex Cost Function (Revealed Preference)
- Profit Function (The Maximized Profit):
  - First Laws of Supply (Revealed Preference)
  - First Laws of Input Demand (Revealed Preference)
  - Convex Profit Function (Revealed Preference)
- LR vs. SR: Le Chatelier's Principle (RP too!)

Producer vs. Consumer

- Profit
- Profit Maximation
- Cost
- Cost Function
- Profit Function
- Input Price Change
- First Laws of Supply and Input Demand

- Utility
- Utility Maximation
- Expenditure
- Expenditure Function
- Indirect Utility Function
- SE and IE
- Compensated Law of Demand

Why do we care about this?

- Suppose you decide to run a small business…
- You face a changing environment
- And make various business choices everyday
- Aren't you just another "consumer" in the economy maximizing "utility"?
  - Profit maximization similar to utility maximization?
- What will your actions tell us about your choices?
  - How general can revealed preference be?
- Are these convincing?

Dual of Maximizing Profit: Minimizing Cost

- Production Plan $(z,q) \in \gamma^f$ $\quad q = F(z)$
- Input z, Input Prices r
- Cost Function $C(r,q) = \min_z \{r \cdot z | (z,q) \in \gamma^f\}$
  - Single output: $C(r,q) = \min_z \{r \cdot z | F(z) - q \geq 0\}$
- Lemma: Gradient of the Cost Function

If cost minimizing $z(q,r)$ is continuous over r,

Then, $\frac{\partial C}{\partial r_i}(r,q) = z_i(r,q)$ for $i = 1, \cdots, n$.

Lemma: Input Price Change (Gradient of the Cost Function)

Proof: $$C(r^0, q) = r^0 \cdot z^0 \leq r^0 \cdot z^1,$$
$$C(r^1, q) = r^1 \cdot z^1 \leq r^1 \cdot z^0$$

Since input vector z^0 is optimal for input price r^0

input vector z^1 is optimal for input price r^1

$$C(r^1, q) - C(r^0, q) \leq (r^1 - r^0) \cdot z^0,$$
$$C(r^1, q) - C(r^0, q) \geq (r^1 - r^0) \cdot z^1$$

Suppose $r^1 - r^0 = (0, \cdots, r_i^1 - r_i^0, \cdots, 0)$

$$\Rightarrow z_i(r^1, q) \leq \frac{C(r^1, q) - C(r^0, q)}{r_i^1 - r_i^0} \leq z_i(r^0, q)$$

Lemma: Input Price Change (Gradient of the Cost Function)

- Hence we have $$\frac{\partial C}{\partial r_i}(r,q) = z_i(r,q)$$
- Note: Only Revealed Preferences + continuity
- Recall Substitution Effect for Compensated Demand: $$\frac{\partial M}{\partial p_j} = x^c(p,U^0)$$
- Producer ~ Consumer

Proposition 4.2-1: Effect of Input Price Change on MC

- Consider the effect on MC:

$$\frac{\partial}{\partial r_j} MC_i = \frac{\partial^2 C}{\partial r_j \partial q_i} = \frac{\partial}{\partial q_i}\frac{\partial C}{\partial r_j} = \frac{\partial z_j}{\partial q_i}$$

- Hence, a rise in price of input j raises MC of output i iff input j is a normal input
- Recall (from Section 2.3): $\frac{\partial^2 M}{\partial p_i \partial p_j} = \frac{\partial x_j^c}{\partial p_i}$
- (See also Income Effect)
- Example: Quasi-linear Production
  - (Quasi-linear utility with vertical IEP…)

Proposition 4.2-2
Convex Cost Function

- If the production set is convex, then the cost function is a convex function of outputs.

i.e. For any q^0, q^1,

$$C(q^\lambda, r) \leq (1-\lambda)C(q^0, r) + \lambda C(q^1, r)$$

- (Compare: Concave Expenditure Function, but slightly different – there we fixed utility level and changed prices; here we fix input prices and change quantity produced.)

Proposition 4.2-2
Convex Cost Function

- Proof: Since $z_0 \sim q^0,\ z_1 \sim q^1,$

$$C(q^0, r) = r \cdot z^0,\ C(q^1, r) = r \cdot z^1$$

$$z^\lambda = (1-\lambda)z^0 + \lambda z^1,\ q^\lambda = (1-\lambda)\cdot q^0 + \lambda \cdot q^1$$

- is feasible, since production set is convex.
- Hence, $C(q^\lambda, r) \le r \cdot z^\lambda$
- Since $C(q,r)$ minimizes cost.
- Thus, $(1-\lambda)C(q^0, r) + \lambda C(q^1, r)$

$$= (1-\lambda)r \cdot z^0 + \lambda r \cdot z^1 = r \cdot z^\lambda \ge C(q^\lambda, r)$$

Profit Function

- Production Plan: $y^f = (y_1^f, \cdots, y_n^f)$
- Net output: $y_i^f > 0$ Net input: $y_j^f < 0$
- Profit: $p \cdot y = \underbrace{\sum_{i, y_i > 0} p_i \cdot y_i}_{revenue} - \underbrace{\sum_{j, y_j < 0} p_j \cdot (-y_j)}_{cost}$
- Profit Function (Maximized Profit):

$$\Pi(p) = \max_y \left\{ p \cdot y | y \in \gamma^f \right\}$$

- (Compare: Indirect Utility Function)

Proposition 4.2-3: Price Change Effect on Inputs and Outputs

- Consider the producer problem

$$\Pi(p) = \max_{y} \{ p \cdot y | y \in \gamma^f \}$$

$$\text{Let } y^0 \text{ be profit maximizing for prices } p^0$$

$$y^1 \text{ be profit maximizing for prices } p^1$$

$$\Rightarrow \Delta p \cdot \Delta y = (p^1 - p^0) \cdot (y^1 - y^0) \geq 0$$

- (Compare: Compensated Price Change)
  - Proposition 2.3-1

Proposition 4.2-3: Price Change Effect on Inputs and Outputs

Proof:

$$p^0 \cdot y^0 \geq p^0 \cdot y^1, \quad p^1 \cdot y^1 \geq p^1 \cdot y^0$$

Since y^0 is profit maximizing for prices p^0

y^1 is profit maximizing for prices p^1

$$-p^0 \cdot (y^1 - y^0) \geq 0, \quad p^1 \cdot (y^1 - y^0) \geq 0$$

$$\Rightarrow \Delta p \cdot \Delta y = (p^1 - p^0) \cdot (y^1 - y^0) \geq 0$$

Corollary: First Laws of Supply and Input Demand

- This is true for any pair of price vectors
- So, if only the price of commodity *j* changes,

$$\Delta p_j \cdot \Delta y_j \geq 0$$

- First Law of Supply:

$$\text{For output } y_j > 0, \text{ we have } \frac{\Delta y_j}{\Delta p_j} \geq 0$$

- First Law of Input Demand:

$$\text{For input } y_j < 0, \text{ we have } \frac{-\Delta y_j}{\Delta p_j} \leq 0$$

- (Compare: Compensated law of demand)

Proposition 4.2-4
Convex Profit Function

- The profit function is convex.

i.e. For any p^0, p^1,

$$\Pi(p^\lambda) \leq (1-\lambda)\Pi(p^0) + \lambda\Pi(p^1)$$

- (Compare: Concave Expenditure Function.)
- This is stronger than Prop. 4.2-3…
- Note similar relation between 2.3-1 & 2.3-2
- Is the Indirect Utility Function (quasi-)convex?
- Yes! See Jehle & Reny (2001), p.28,Thm 1.6

Proposition 4.2-4
Convex Profit Function

Proof: y^λ profit maximizing at p_λ,

$$\Pi(p^0) = p^0 \cdot y^0 \geq p^0 \cdot y^\lambda,$$

$$\Pi(p^1) = p^1 \cdot y^1 \geq p^1 \cdot y^\lambda$$

Since $\Pi(p)$ maximizes profit.

Hence,

$$(1-\lambda)\Pi(p^0) + \lambda\Pi(p^1)$$

$$\geq \left[(1-\lambda)(p^0 \cdot y^\lambda)\right] + \left[\lambda(p^1 \cdot y^\lambda)\right]$$

$$= p^\lambda \cdot y^\lambda = \Pi(p^\lambda)$$

Application: SR vs. LR Adjustment to Price Change

- Firm face price p^0, choose production plan y^0
- One (input or output) price changes $p^0 \Rightarrow p^1$
- Assume firm's feasible set more limited in SR
  - Set of feasible LR plans: γ
  - Set of feasible SR plans: $\gamma^S(y^0) \subset \gamma$
- Le Chatelier Principle: Own price effects are larger in the LR than in the SR. i.e.

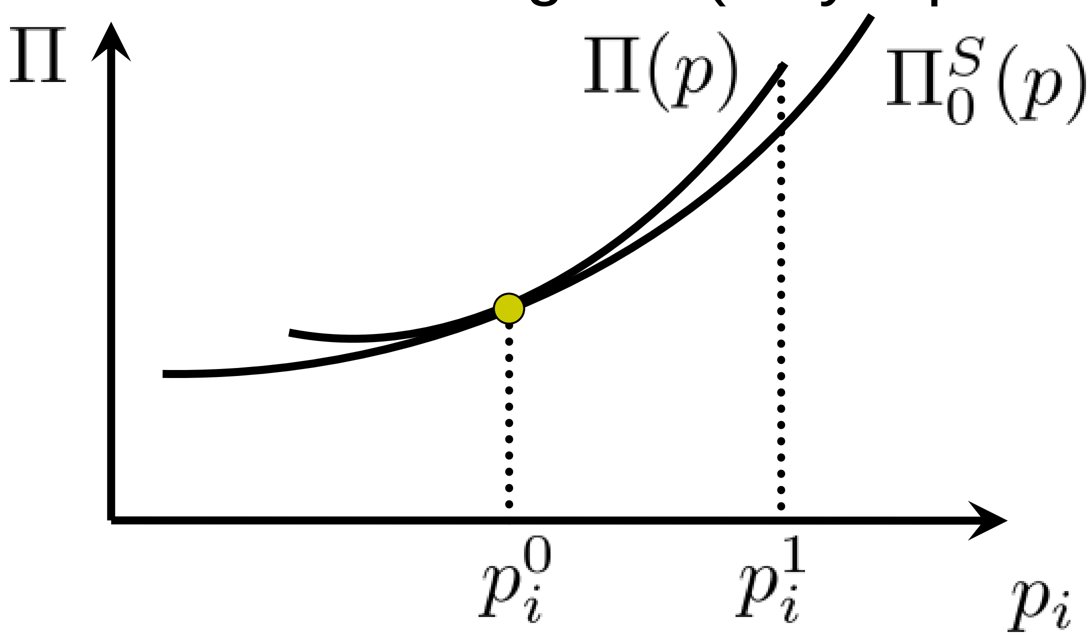
$$\frac{\partial y_i}{\partial p_i} \geq \frac{\partial y_i^S}{\partial p_i}$$

Proposition 4.2-5: Le Chatelier Principle

- LR Profit Function: $\Pi(p)$
- SR Profit Function: $\Pi_0^S(p) < \Pi(p)$ for $p \neq p^0$

$$\text{But } \Pi_0^S(p^0) = \Pi(p^0)$$

  - SR constraints bind tighter (only if plan changes)



Proposition 4.2-5:
Le Chatelier Principle

Proof: $\Pi(p^0) = p^0 \cdot y(p^0) \geq p^0 \cdot y(p^1),$

$$\Pi(p^1) = p^1 \cdot y(p^1) \geq p^1 \cdot y(p^0),$$

Since $y(p^0)$ is most profitable at price vector p^0

$y(p^1)$ is most profitable at price vector p^1

$$\Pi(p^1) - \Pi(p^0) \leq (p^1 - p^0) \cdot y(p^1),$$

$$\Pi(p^1) - \Pi(p^0) \geq (p^1 - p^0) \cdot y(p^0)$$

Suppose $p^1 - p^0 = (0, \cdots, p_i^1 - p_i^0, \cdots, 0)$

$$\Rightarrow y_i(p^1) \geq \frac{\Pi(p^1) - \Pi(p^0)}{p_i^1 - p_i^0} \geq y_i(p^0)$$

Proposition 4.2-5: Le Chatelier Principle

- Hence, $$\frac{\partial \Pi}{\partial p_i} = y_i(p), \quad \frac{\partial^2 \Pi}{\partial p_i^2} = \frac{\partial y_i}{\partial p_i}$$
- Similarly, $$\frac{\partial \Pi_0^S}{\partial p_i} = y_i^S(p), \quad \frac{\partial^2 \Pi_0^S}{\partial p_i^2} = \frac{\partial y_i^S}{\partial p_i}$$
- Since, $\frac{\partial \Pi}{\partial p_i} = \frac{\partial \Pi_0^S}{\partial p_i}$ at p^0 and $\Pi(p) \geq \Pi_0^S(p)$
- Hence, $$\frac{\partial y_i}{\partial p_i} = \frac{\partial^2 \Pi}{\partial p_i^2} \geq \frac{\partial^2 \Pi_0^S}{\partial p_i^2} = \frac{\partial y_i^S}{\partial p_i}$$
  - Note how similar this is to the first Lemma

What Have We Learned?

- Cost Function (the Minimized Cost):
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  - Convex Profit Function (Revealed Preference)
- LR vs. SR: Le Chatelier's Principle (RP too!)
- Homework: Riley - 4.2-3, 4, 6, 7, J/R - TBA

What Have We Learned?

- Cost Function vs. Profit Function
- Method of “Revealed Preferences” used in:

1. Input Price Change
2. First Laws of Supply
3. First Laws of Input Demand
4. Cost and Profit Functions are Convex
5. Le Chatelier Principle

Producer vs. Consumer

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- Compensated Law of Demand