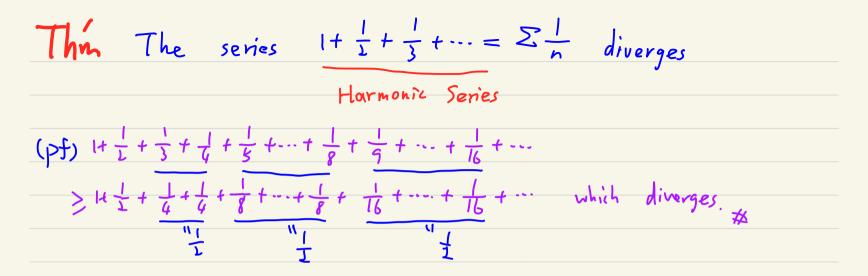
Online Math Camp (235) TA Session Note (5/8)



Comparison Test
Let
$$n_n \ge b_n \ge 0$$
. \Rightarrow $\begin{cases} D & If \ge a_n \text{ converges, then } \ge b_n \text{ converges.} \\ \hline @ If \ge b_n \text{ diverges, then } \ge a_n \text{ diverges.} \end{cases}$

Root Test
Let
$$a_n \neq 0$$
. $\Rightarrow \int \mathbb{D}$ If lineng $\exists a_n < 1$, then Ξa_n converges.
(\Im If lineng $\exists a_n > 1$, then Ξa_n diverges.
($\operatorname{Sketch} \operatorname{of} pf$)
 \mathbb{D} linesup $\exists a < 1 \Rightarrow \exists \beta < 1$, $N \in \mathbb{N}$ such that $\exists a_n < \beta$ (e.g. $\beta = \frac{\operatorname{lin} p \exists \overline{n} + 1}{2}$)
 $\Rightarrow a_n < \beta^n$. By companison test $\Xi a_n < \Xi \beta^n$ converges.
 $\Rightarrow \forall N \in \mathbb{N}$, $\exists n \in \mathbb{N}$ such that $n > N$, $a_n > 1 \Rightarrow a_n \Rightarrow 0$

Ratio Test
Let
$$\Sigma a_n$$
 be a series, and $a_n \neq v$ for large enough n .
 \mathbb{O} If $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} < 1$, then Σa_n converges.
 \Im If $\left|\frac{a_{n+1}}{a_n}\right| \ge 1$ for n large enough, then Σa_n diverges.

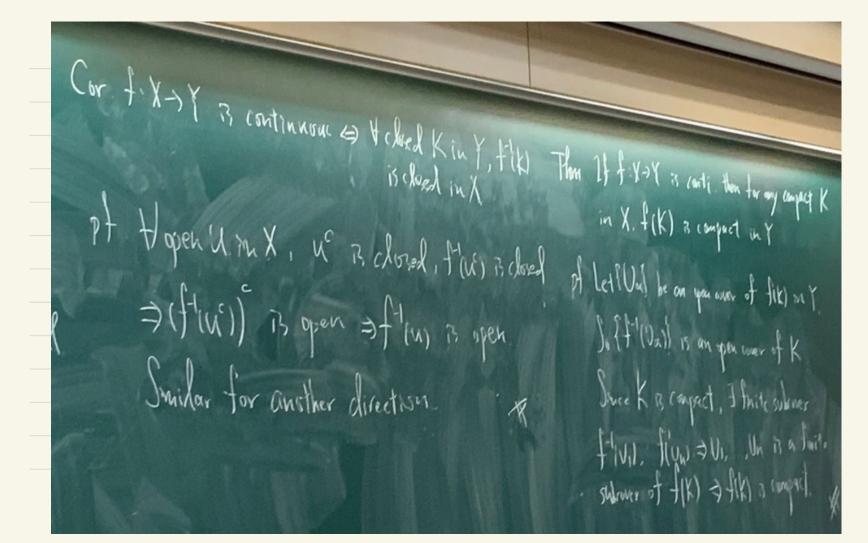
Abel's Thin

$$A_n = \sum_{i=0}^{n} a_i$$
 is bounded, by is monotonic decreasing, and $b_n \rightarrow 0$.
Then, Σ and b_n converges.
 $E_X: \sum \frac{1}{b_n} \frac{\sin\left(\frac{h\pi}{4}\right)}{a_n}$ converges by Ahel's Thin.



$$\begin{array}{c} Re \operatorname{qrrangement} \\ \Sigma \operatorname{qi} \ converges \ absolutely, \ so \exists N \in \mathbb{N} \ \operatorname{such} \ that \ n > \mathbb{N} \Rightarrow \widetilde{\Sigma} |\operatorname{qi}| < \xi. \\ \Sigma \operatorname{q'}(i) \ is \ a \ rearrangement \ ef \ \Sigma \ \operatorname{qi} \Rightarrow \left| \widetilde{\Sigma} \operatorname{q'}_{j} \right| < \xi \ for \ n \ large \ enough \\ \operatorname{(pl)} \ Let \ S = \left\{ \sigma(i) \mid i \geq 1, \cdots, N \right\}. \ Take \ M = \max \ S \ t \ 1 \\ Then \ n > \mathbb{M} \Rightarrow \widetilde{\Sigma} \atop{\sigma(i) \geq M} \left| \operatorname{q'}_{\Gamma(i)} \right| < \widetilde{\Sigma} \ |\operatorname{qi}| < \xi \\ \widetilde{\Sigma} \ |\operatorname{qi}| < \xi \\ \mathcal{T} \ duct \\ \Sigma \ a_n, \ \Sigma \ b_n \ converges \ absolutely, \ C_n = \sum_{i \geq 0}^{n} \operatorname{qi}_{i} \ b_{n-i}, \Rightarrow \Sigma \ c_n \ converges. \\ \operatorname{(pl)} \ a_1 \ b_1 \ a_2 \ b_1 \ a_3 \ b_1 \ a_4 \ b_2 \\ a_1 \ b_2 \ a_2 \ b_3 \ a_3 \ b_3 \ a_4 \ b_2 \\ a_1 \ b_1 \ a_2 \ b_3 \ a_3 \ b_4 \ a_4 \ b_4 \\ a_1 \ b_4 \ a_3 \ b_4 \ a_4 \ b_4 \end{array}$$

Limit of Function



Homeomorphism than If f:X->X is conti bijective on a compact set X, then I is a homeomorphism, i.e. f-1 is continuous. pf. ft is continuous (=> H dosed K in X, f(K) is dosed in Y V closed K in X, K is compact. She f is conti, f(k) is compact ? f(k) is closed. Cor. IO, 17 is not homeomorphic to R'.

Example in Economics: Consumer Theory
Suppose in a narbet, there are a goods,
$$\kappa_1, \dots, \kappa_n$$
 with price $p_1, \dots, p_n > 0$.
Every individual maximizes its utility under the budget constraint:
Max $U(x)$ s.t. $\Sigma p_1 \kappa_1 \leq y_1$
If $u(\cdot)$ is continuous, then the maximization point exists, and the best
consumption bundle is called the Mashallian demand.
(pf) Let $A = \{(\kappa_1, \dots, \kappa_n)\} o \leq \Sigma p_1 \kappa_1 \leq y, \kappa_1 \neq 0\}$.
A is bounded: A is bounded by $[0, \frac{y}{p_1}] \times \dots \times [0, \frac{y}{p_n}]$.
A is closed: Since $f = \Sigma P_1 \kappa_1$ is continuous, $f^{-1}([0, y]) = A$ is closed.
Hence, by Heien-Borel Thin, A is compared.
By Weierstrass Thin, M achieves its maximum.