Dominance-Solvable Games

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Dominance

- Dominance
  - Strategy A gives you better payoffs than Strategy B regardless of opponent strategy
- Dominance Solvable
  - A game that can be solved by iteratively deleting dominated strategy

Dominance

- Do people obey dominance?
  - Looking both sides to cross a 1-way street
  - "If you can see this, I can't see you."
  - p-Beauty Contest behavior (guess above 67)
- Will you bet on others obeying dominance?
  - Workers respond to incentives rationally
  - Companies don't use "optimal" contracts
- SOPH: Knowing other's steps of reasoning

Hierarchy of Iterated Reasoning

- D0: Strict dominance,
- D1: Belief that others obey dominance,
- D2: Belief that others believe you'll obey dominance, …
- Vince Crawford (AER co-editor):
  - on Level-k: I'll treat any student for dinner at a conference if s/he can show an example of "4 levels of reasoning" in history or literature

Outline

- A Simple Test:
  - Beard & Beil (MS 94')
- Other Games:
  - Centipede:
    - McKelvey & Palfrey (Econometrica 92')
  - Mechanism Design:
    - Sefton and Yavas (GEB 96')
  - Dirty Face:
    - Weber (EE 01')
- Learning IEDS:
  - Traveler's dilemma
  - Price competition
  - Capra et al (AER 99', IER 02')
- *p*-Beauty Contest
  - Nagel + CHW (AER 95', 98')
- Theory:
  - Stahl and Wilson (GEB 95')
  - CGCB (Econometrica 01')
  - Cognitive Hierarchy (QJE 04')
  - CGC (AER 06')

A Simple Test

- Beard and Beil (Management Science 1994)

Iterated dominance game

Player 1 move	Player 2 move: l	Player 2 move: r
L	9.75, 3	
R	3, 4.75	10, 5

A Simple Test

Payoff treatments and results in Beard and Beil

Treatment	Payoffs from (L, 1)	Payoffs from (R, 1)	Payoffs from (R, r)	Frequency of L	Frequency of r/R	Number Of pairs	Threshold P(rR)
1 (baseline)	(9.75, 3)	(3, 4.75)	(10.5)	0.66	0.83	35	0.97
2 (less risk)	(9, ·)	(· , ·)	(· , ·)	0.65	1.00	31	0.85
3 (even less risk)	(7, ·)	(· , ·)	(· , ·)	0.20	1.00	25	0.57
4 (more assurance)	(· , ·)	(· , 3)	(· , ·)	0.47	1.00	32	0.97
5 (more resentment)	(· , 6)	(· , ·)	(· , ·)	0.86	1.00	21	0.97
6 (less risk, more reciprocity)	(· , 5)	(5, 9.75)	(· , 10)	0.31	1.00	26	0.95
7 (1/6 payoff)	(58.5, 18)	(18, 28.5)	(60, 30)	0.67	1.00	30	0.97

Note: (· , ·) indicates the payoffs are the same as those in the baseline case.

A Simple Test

- Player 2 mostly DO obey dominance
- Player 1 is inclined to believe this
  - Though they can be convinced if incentives are strong for the other side to comply
- Follow-up studies show similar results:
  - Goeree and Holt (PNAS 1999)
  - Schotter, Weigelt and Wilson (GEB 1994)

A Simple Test: Follow-up 1

- Jacob Goeree and Charles Holt (PNAS 1999)

Table 5.3 Goeree and Holt's credible threat games

Condition	Number of pairs	Threshold p (r/R)	Payoffs (L)	Payoffs (R, I)	Payoffs (R, r)	Frequency of L	Frequency of r/R
Baseline 1	25	0.33	(70,60)	(60,10)	(90,50)	0.12	1.00
Lower assurance	25	0.33	(70,60)	(60,48)	(90,50)	0.32	0.53
Baseline 2	15	0.85	(80,50)	(20,10)	(90,70)	0.13	1.00
Lower assurance	25	0.85	(80,50)	(20,68)	(90,70)	0.52	0.75
Very low assurance	25	0.85	(400,250)	(100, 348)	(450,350)	0.80	0.80

A Simple Test: Follow-up 2

- Strategic Form vs. Sequential Form

Games 1M and 1S of Schotter et al.

Player 1	Player 2: 1	Player 2: r	Actual frequency
Normal form (1M)			
L	4, 4	4, 4	(0.57)
R	0, 1	6, 3	(0.43)
Frequency	(0.20)	(0.80)	
Sequential form			
L	4, 4		(0.08)
	1	r	
R	0, 1	6, 3	(0.92)
Frequency	(0.02)	(0.98)	

A Simple Test: Follow-up 2

- Observing Elimination or not…

Games 3M and 3S of Schotter et al.

Player 1 move	Player 2 move: T	Player 2 move: M	Player 2 move: B	Frequency
Normal form 3M				
T	4, 4	4, 4	4, 4	(0.82)
M	0, 1	6, 3	0, 0	(0.16)
B	0, 1	0, 0	3, 6	(0.02)
Frequency	(0.70)	(0.26)	(0.04)	

Sequential form 3S

						Conditional frequency
T	4, 4					(0.70)
		T				
		0, 1				
				M	B	
			M	6, 3	0, 0	(1.00)
			B	0, 0	3, 6	(0.00)
Frequency		(0.13)		(0.31)	(0.69)	

A Simple Test: Follow-up 2

- Schotter et al. (1994)'s conclusion:
- Limited evidence of iteration of dominance (beyond 1-step), or SPE, forward induction
  - Can more experience fix this?
- Not for forward induction in 8 periods…
  - Brandts and Holt (1995)
- Yes for 3-step iteration in 160 periods
  - Rapoport and Amaldoss (1997): Patent Race

Learn to Play Iterated Dominance

- Rapoport & Amaldoss (1997): Patent Race
- “Strong” invests 0~$5; “weak” invests 0~$4
  - – Highest investor earns $10; both earn $0 if tie
- s=0 is dominated by s=5
  - – Deleting s=0, w=1 is dominated by w=0
- Deleting w=1, s=2 is dominated by s=1, etc.
  - – I.D. kills s=0, 2, 4 and w=1, 3
- MSE: s=1,3,5- (.2, .2, .6); w=0,2,4- (.6, .2,.2)

Centipede Game

- McKelvey and Palfrey (Econometrica 1992)

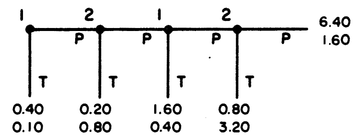


FIGURE 1.—The four move centipede game.

Centipede Game

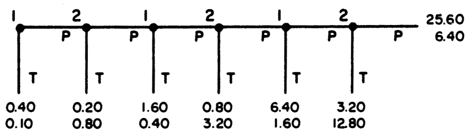


FIGURE 2.—The six move centipede game.

Centipede Game

TABLE I

EXPERIMENTAL DESIGN

Session #	Subject pool	# subjects	Games/ subject	Total # games	# moves	High Payoffs
1	PCC	20	10	100	4	No
2	PCC	18	9	81	4	No
3	CIT	20	10	100	4	No
4	CIT	20	10	100	4	Yes
5	CIT	20	10	100	6	No
6	PCC	18	9	81	6	No
7	PCC	20	10	100	6	No

Centipede Game

TABLE IIA

PROPORTION OF OBSERVATIONS AT EACH TERMINAL NODE

	Session		N	f_1	f_2	f_3	f_4	f_5	f_6	f_7
	1	(PCC)	100	.06	.26	.44	.20	.04		
Four	2	(PCC)	81	.10	.38	.40	.11	.01		
Move	3	(CIT)	100	.06	.43	.28	.14	.09		
	Total	1–3	281	.071	.356	.370	.153	.049		
High Payoff	4	(High-CIT)	100	.150	.370	.320	.110	.050		
	5	(CIT)	100	.02	.09	.39	.28	.20	.01	.01
Six	6	(PCC)	81	.00	.02	.04	.46	.35	.11	.02
Move	7	(PCC)	100	.00	.07	.14	.43	.23	.12	.01
	Total	5–7	281	.007	.064	.199	.384	.253	.078	.014

Centipede Game

TABLE IIB[a]

IMPLIED TAKE PROBABILITIES FOR THE CENTIPEDE GAME

	Session	p_1	p_2	p_3	p_4	p_5	p_6
	1 (PCC)	.06	.28	.65	.83		
		(100)	(94)	(68)	(24)		
Four	2 (PCC)	.10	.42	.76	.90		
Move		(81)	(73)	(42)	(10)		
	3 (CIT)	.06	.46	.55	.61		
		(100)	(94)	(51)	(23)		
	Total 1–3	.07	.38	.65	.75		
		(281)	(261)	(161)	(57)		
High	4 (CIT)	.15	.44	.67	.69		
Payoff		(100)	(85)	(48)	(16)		
	5 (CIT)	.02	.09	.44	.56	.91	.50
		(100)	(98)	(89)	(50)	(22)	(2)
Six	6 (PCC)	.00	.02	.04	.49	.72	.82
Move		(81)	(81)	(79)	(76)	(39)	(11)
	7 (PCC)	.00	.07	.15	.54	.64	.92
		(100)	(100)	(93)	(79)	(36)	(13)
	Total 5–7	.01	.06	.21	.53	.73	.85
		(281)	(279)	(261)	(205)	(97)	(26)

[a]The number in parentheses is the number of observations in the game at that node.

Centipede Game

TABLE IIIA

CUMULATIVE OUTCOME FREQUENCIES

$(F_j = \Sigma_{i=1}^{j} f_i)$

Treatment	Game	N	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Four	1–5	145	.062	.365	.724	.924	1.00		
Move	6–10	136	.081	.493	.875	.978	1.00		
Six	1–5	145	.000	.055	.227	.558	.889	.979	1.000
Move	6–10	136	.015	.089	.317	.758	.927	.993	1.000

Centipede Game

TABLE IIIB

IMPLIED TAKE PROBABILITIES

COMPARISON OF EARLY VERSUS LATE PLAYS IN THE LOW PAYOFF CENTIPEDE GAMES

Treatment	Game	p_1	p_2	p_3	p_4	p_5	p_6
Four	1–5	.06	.32	.57	.75		
Move		(145)	(136)	(92)	(40)		
	6–10	.08	.49	.75	.82		
		(136)	(125)	(69)	(17)		
Four	1–5	.00	.06	.18	.43	.75	.81
Move		(145)	(145)	(137)	(112)	(64)	(16)
	6–10	.01	.07	.25	.65	.70	.90
		(136)	(134)	(124)	(93)	(33)	(10)

Centipede Game

- What theory can explain this?
- Altruistic Types (7%): Prefer to Pass
- Normal Types:
  - Mimic altruistic types up to a point (gain more)
- Unraveling: error rate shrinks over time

Centipede Game

- Selfish players sometimes pass, to mimic an altruist. By imitating an altruist one might lure an opponent into passing at the next move, thereby raising one's final payoff in the game.
- The amount of imitation in equilibrium depends directly on the beliefs about the likelihood *(1-q)* of a randomly selected player being an altruist. The more likely players believe there are altruists in the population, the more imitation there is.

Centipede Game

Property 1:
For any q, Blue chooses TAKE with probability 1 on its last move.

Property 2:
If $1 - q > \frac{1}{7}$, both Red and Blue always choose PASS, except on the last move, when Blue chooses TAKE.

Property 3:
If $1 - q \in (0, \frac{1}{7})$ the equilibrium involves mixed strategies.

Property 4:
If $q = 1$, then both Red and Blue always choose TAKE.

Centipede Game

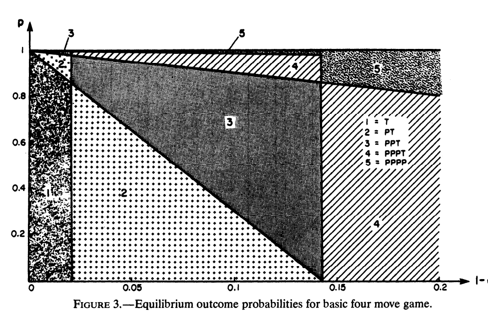


FIGURE 3.—Equilibrium outcome probabilities for basic four move game.

Centipede Game

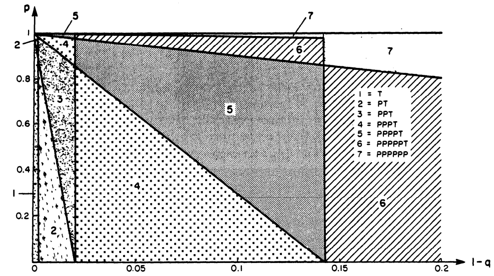


FIGURE 4.—Equilibrium outcome probabilities for basic six move game.

Centipede Game

- We model noisy play in the following way. In game *t,* at node *s*, if *p** is the equilibrium probability of TAKE that the player at that node attempts to implement, we assume that the player actually chooses TAKE with probability $(1-\varepsilon_t)p^*$, and makes a random move (i.e. TAKE or PASS with probability 0.5) with probability ε_t.
- $\varepsilon_t = \varepsilon e^{-\delta(t-1)}$

Centipede Game: Follow-ups

- Fey, McKelvey and Palfrey (IJGT 1996)
  - Use constant-sum to kill social preferences
  - Take 50% at 1st, 80% at 2nd
- Nagel and Tang (JMathPsych 1998)
  - Don't know other's choice if you took first
  - Take about half way
- Rapoport et al. (GEB 2003)
  - 3-person & high stakes: Many take immediately
  - CH can explain this (but not QRE) – see theory

Mechanism Design

- Pure coordination game with $1.20 & $0.60
- How can you implement a Pareto-inferior equilibrium in a pure coordination games?
- Abreu & Matsushima (Econometrica 1992)
  - Slice the game into "T periods"
  - F: Fine paid by first subject to deviate
  - Won't deviate if F > $1.20/T
  - Can set T=1, F=$1.20; more credible if T large

Mechanism Design

- Glazer and Rosenthal (Economtrica 1992)
  - Comment: AM mechanism requires more steps of iterated deletion of dominated strategies
- Abreu & Matsushima (Econometrica 1992)
  - Respond: "[Our] gut instinct is that our mechanism will not fare poorly in terms of the essential feature of its construction, that is, the significant multiplicative effect of 'fines.'"
- This invites an experiment!

Mechanism Design

- Sefton and Yavas (GEB 1996)
- F=$0.225
- T=4, 8, or 12
  - Theory: Play inferior NE at T=8 or 12, not T=4
- Results: Opposite, and diverge…
- Why? Choose only 1 switchpoint in middle
  - Goal: switch soon, but 1 period after opponent

Mechanism Design

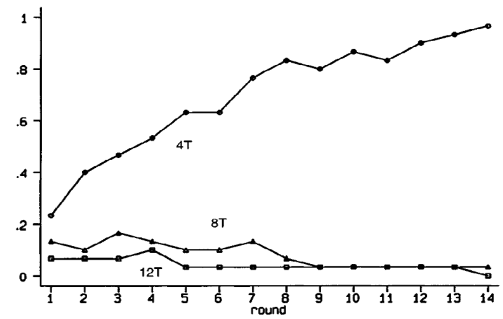


FIG. 3. All-Blue Play in Experiment I.

Mechanism Design

- Glazer and Perry (GEB 1996)
  - Implemental can work in sequential game via backward induction
- Katok, Sefton and Yavas (JET 2002)
  - Doesn't work either
- Can any "approximately rational explanation" get this result?
  - Maybe "Limited steps of IDDS + Learning"

Dirty Face Game

- Three ladies, A, B, C, in a railway carriage all have dirty faces and are all laughing. It sudden flashes on A: why doesn't B realize C is laughing at her? Heavens! *I* must be laughable.
  - Littlewood (1953), "*A Mathematician's Miscellany*
- Requires A to think that B is rational enough to draw inference from C

Dirty Face Game

- Weber (EE 2001)
- Independent types X (Prob=.8) or O (Prob=.2)
  - X is like "dirty face"
- Commonly told "At least one player is type X."
- Observe other's type
- Choose Up or Down (figure out one is type X)
- If nobody chooses Down, reveal other's choice and play again

Dirty Face Game

		Type	
		X	O
Probability		0.8	0.2
Action	Up	$0	$0
	Down	$1	-$5

Dirty Face Game

- Case XO: Players play (Up, Down)
- Type X player thinks…
  - I know that "at least one person is type X"
  - I see the other person is type O
- So, I must be type X → Chooses Down
- Type O player thinks…
  - I know that "at least one person is type X"
  - I see the other person is type X
- No inference → Chooses Up

Dirty Face Game

- Case XX - First round:
- No inference (since at least one is type X, but the other guy is type X) → Both choose Up
- Case XX - Second round:
- Seeing UU in first
  - the other is not sure about his type
  - He must see me being type X
- I must be Type X → Both choose Down

Dirty Face Game

		Trial 1		Trial 2	
		XO	XX	XO	XX
Round 1	UU	0	7*	1	7*
	DU	3*	3	4*	1
	DD	0	0	0	0
Round 2 (after UU)	UU	-	1	-	2
	DU	-	5	-	2
	DD	-	1*	-	3*
	Other	-	-	1	-

Dirty Face Game

- Results: 87% rational in XO, but only 53% in 2nd round of XX
- Significance:
- Upper bound of iterative reasoning
  - Caltech students still don't do 2 steps
- Choices reveal limited reasoning, not pure cooperativeness
  - More iteration is better here…

Initial Response and Equilibration

- Price Competition
  - Capra, Goeree, Gomez and Holt (IER 2002)
- Traveler's Dilemma
  - Capra, Goeree, Gomez and Holt (AER 1999)
- *p*-Beauty Contest
  - Nagel (AER 1995)
  - Camerer, Ho, Weigelt (AER 1998)

Price Competition

- Capra, Goeree, Gomez & Holt (IER 2002)
  - Two firms pick prices p_1 & p_2 from $0,60~$1.60
  - Both get $(1+a)*p_1 / 2$ if tied; but if $p_1 < p_2$
  - Low-price firm gets p_1; other firm gets $a* p_1$
- *a* = responsiveness to "best price" (=0.2/0.8)
  - *a=1:* Meet-or-release
  - *a<1:* Bertrand competition predicts lowest price

Price Competition

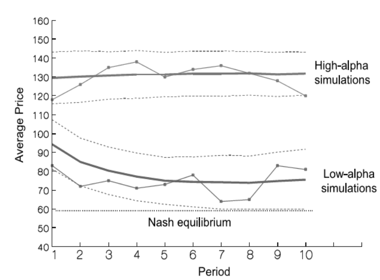


Figure 4

Simulated average prices obtained from 1000 simulations (dark lines) ±2 standard deviations (dotted lines) and a typical run (lines connecting squares)

Traveler's Dilemma

- Capra, Goeree, Gomez & Holt (AER 1999)
  - Two travelers state claim p_1 and p_2 : 80~200
  - Airline awards both the minimum claim, but
  - reward R to the one who stated the lower claim
  - penalize the other by R
- Unique NE: race to the bottom → lowest claim
  - Like price competition game or beauty contest

Traveler's Dilemma

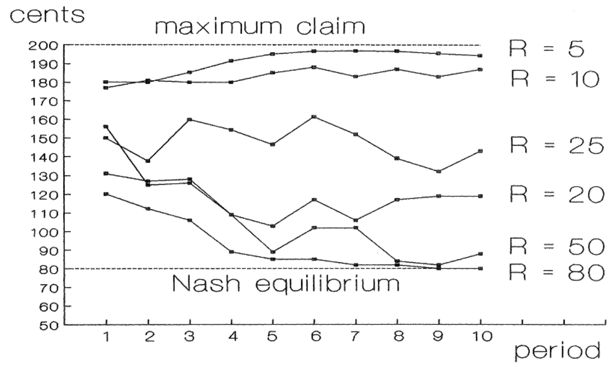


FIGURE 1. DATA FOR PART A FOR VARIOUS VALUES OF THE REWARD/PENALTY PARAMETER

p-Beauty Contest

- Each of *N* players choose x_i from [0,100]
- Target is *p*(average of* x_i*)*
- Closest x_i wins fixed prize
- (67,100] violates 1st order dominance
- (45, 67] obeys 1 step (not 2) of dominance
- Nagel (AER 1995): BGT, Figure 5.1b
- Ho, Camerer and Weigelt (AER 1998)
  - BGT, Figure 1.3, 5.1

p-Beauty Contest

- Ho, Camerer, and Weigelt (AER 1998)
- Keynes (1936 pp. 155-56) said,
  - "…professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole.…"

p-Beauty Contest

- Keynes (1936 pp. 155-56) said,
  - "It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees."

p-Beauty Contest

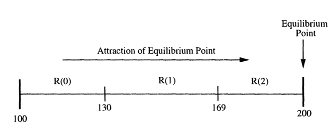


FIGURE 1A. A FINITE-THRESHOLD GAME, $FT(n) = ([100, 200], 1.3, n)$

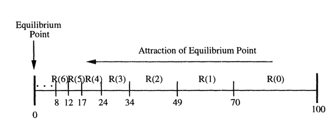


FIGURE 1B. AN INFINITE-THRESHOLD GAME, $IT(n) = ([0, 100], 0.7, n)$

p-Beauty Contest

TABLE 1—THE EXPERIMENTAL DESIGN

Group size: 3	Group size: 7
Finite → Infinite	
FT(1.3, 3) → *IT*(0.7, 3) (7 groups)	*FT*(1.3, 7) → *IT*(0.7, 7) (7 groups)
FT(1.1, 3) → *IT*(0.9, 3) (7 groups)	*FT*(1.1, 7) → *IT*(0.9, 7) (7 groups)
Infinite → Finite	
IT(0.7, 3) → *FT*(1.3, 3) (7 groups)	*IT*(0.7, 7) → *FT*(1.3, 7) (7 groups)
IT(0.9, 3) → *FT*(1.1, 3) (6 groups)	*IT*(0.9, 7) → *FT*(1.1, 7) (7 groups)

p-Beauty Contest

- **RESULT 1:**
  First-period choices are far from equilibrium, and centered near the interval midpoint. Choices converge toward the equilibrium point over time.

- **RESULT 2:**
  On average, choices are closer to the equilibrium point for games with finite thresholds, and for games with p further from 1.

p-Beauty Contest

- **RESULT 3:**
  Choices are closer to equilibrium for large (7-person) groups than for small (3-person) groups.

- **RESULT 4:**
  Choices by [cross-game] experienced subjects are no different than choices by inexperienced subjects in the first round, but converge faster to equilibrium.

p-Beauty Contest

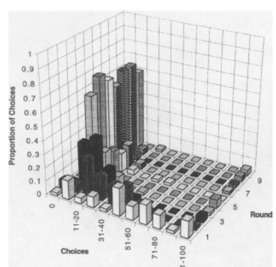


FIGURE 2B. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 7)

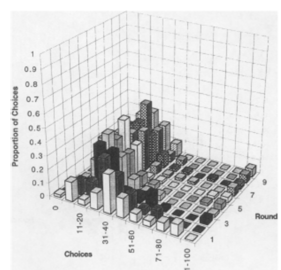


FIGURE 2A. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 7)

p-Beauty Contest

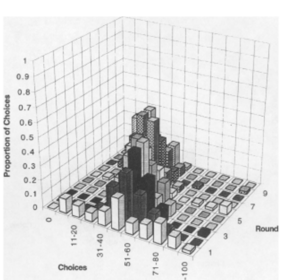


FIGURE 2C. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 7)

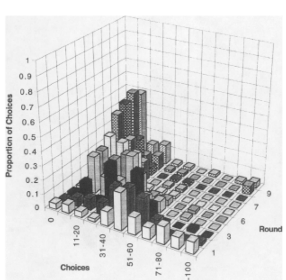


FIGURE 2D. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 7)

p-Beauty Contest

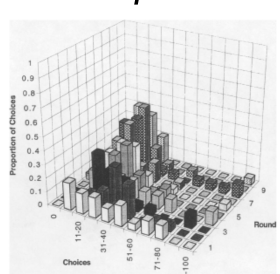


FIGURE 2E. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

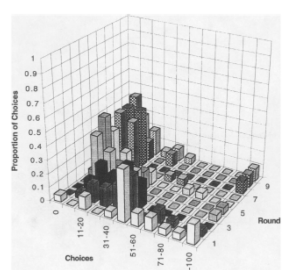


FIGURE 2F. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.7, 3)

p-Beauty Contest

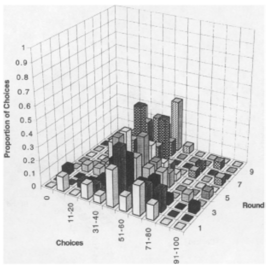
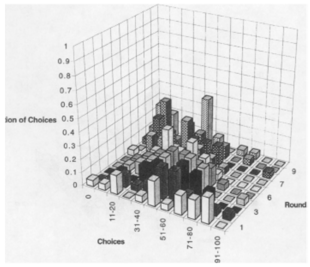


FIGURE 2G. INEXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 3)



IGURE 2H. EXPERIENCED SUBJECTS' CHOICES OVER ROUND IN *IT*(0.9, 3)

p-Beauty Contest

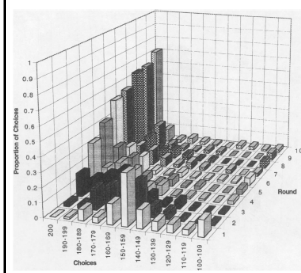


FIGURE 3A. CHOICES OVER ROUND IN *FT* GAMES PLAYED BY 3-PERSON GROUPS

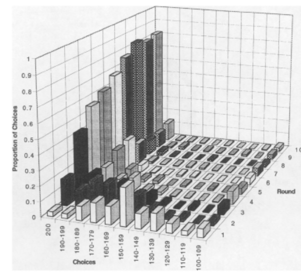


FIGURE 3B. CHOICES OVER ROUND IN *FT* GAMES PLAYED BY 7-PERSON GROUPS

p-Beauty Contest

TABLE 2—FREQUENCIES OF LEVELS OF ITERATED DOMINANCE OVER ROUND IN *FT* AND *IT* GAMES WITH VARYING *p*-VALUES

Games/Round	1–2	3–4	5–6	7–8	9–10	Total
FT(1.3, *n*)						
R(0)	44	27	14	14	11	110
R(1)	102	18	12	10	4	146
R(2)	101	70	49	22	7	249
Equilibrium Play	33	165	205	234	258	895
FT(1.1, *n*)						
R(0)	12	9	10	7	13	51
R(1)	9	2	4	2	3	20
R(2)	14	4	2	1	1	22
R(3)	27	7	5	4	2	45
R(4)	96	24	1	6	4	131
R(5)	65	59	13	7	11	155
R(6)–*R*(10)	42	103	118	76	72	411
Equilibrium Play	9	66	121	171	168	535
IT(0.7, *n*)						
R(0)	42	11	13	16	15	97
R(1)	65	21	5	7	3	101
R(2)	53	30	14	8	12	117
R(3)	35	53	37	21	21	167
R(4)	39	50	44	47	41	221
R(5)	13	43	35	36	32	159
R(6)–*R*(10)	25	71	108	102	91	397
>*R*(11)	2	1	12	18	25	58
Equilibrium Play	6	0	12	25	40	83
IT(0.9, *n*)						
R(0)	12	3	4	2	7	28
R(1)	7	2	1	0	1	11
R(2)	23	4	3	2	1	33
R(3)	17	12	1	0	2	32
R(4)	33	18	10	5	3	69
R(5)	14	21	12	6	3	56
R(6)–*R*(10)	117	142	100	80	60	499
>*R*(11)	47	69	136	162	175	589
Equilibrium Play	4	3	7	17	22	53

p-Beauty Contest

- Classification of Types
  - Follow Stahl and Wilson (GEB 1995)
- Level-0: pick randomly from N(mu, sigma)
- Level-1: BR to level-0 with noise
- Level-2: BR to level-1 with noise
- Level-3: BR to level-2 with noise
- Estimate type, error using MLE

p-Beauty Contest

(1) $$B_L = \frac{p}{n} \cdot \left(B_L + \sum_{k=2}^{n} B_{L-1}^k \right),$$

(2) $$B_L = \frac{p}{n-p} \cdot \sum_{k=2}^{n} B_{L-1}^k.$$

(3) $$E(B_L) = \frac{p \cdot (n-1)}{n-p} \cdot E(B_{L-1}),$$

(4) $$\mathrm{Var}(B_L) = \frac{p^2}{(n-p)^2} \cdot \left[(n-1) + 2\rho \cdot \frac{(n-1)\cdot(n-2)}{2} \right] \cdot \mathrm{Var}(B_{L-1}).$$

p-Beauty Contest

(5) $$B(x) = \sum_{L=0}^{b} \omega_L^b \cdot B_L(x).$$

(6) $$LL_1(\mu_1, \sigma_1, \rho, \omega_L^b; b = 0, \ldots, L_m;\ L = 0, \ldots, L_m) = \sum_{i=1}^{N} \mathrm{Log}(B(x_i)).$$

(7) $$B_L(t) = p \cdot \frac{B_L(t) + \sum_{j=2}^{n} G_L^j(t)}{n}.$$

Or

(8) $$B_L(t) = \frac{p}{n-p} \cdot \sum_{j=2}^{n} G_L^j(t).$$

p-Beauty Contest

$$G^j_L(t) = B_{L-1}(t). \quad (9)$$

$$\mu(t) = \sum_{s=1}^{R} \beta_s \cdot w(t-s). \quad (10)$$

$$\sigma(t) = \sigma \cdot e^{\gamma \cdot t}. \quad (11)$$

$$B(x) = \sum_{L=0}^{L_m} \alpha_L \cdot B_L(x), \quad (12)$$

$$LL_2 = \sum_{i=1}^{N} \cdot \sum_{s=1}^{10} \text{Log}(B(x_i(s))). \quad (13)$$

p-Beauty Contest

TABLE 3—MAXIMUM-LIKELIHOOD ESTIMATES AND LOG-LIKELIHOODS FOR LEVELS OF ITERATED DOMINANCE (FIRST-ROUND DATA ONLY)

Parameter estimates	Out data (groups of 3 or 7)		Nagel's data (groups of 16–18)	
	$IT(p, n)$	$FT(p, n)$	$IT(0.5, n)$	$IT(2/3, n)$
ω_0	15.93	21.72	45.83 (23.94)	28.36 (13.11)
ω_1	20.74	31.46	37.50 (29.58)	34.33 (44.26)
ω_2	13.53	12.73	16.67 (40.84)	37.31 (39.34)
ω_3	49.50	34.08	0.00 (5.63)	0.00 (3.28)
μ	70.13	100.50	35.53 (50.00)	52.23 (50.00)
σ	28.28	26.89	22.70	14.72
ρ	1.00	1.00	0.24	1.00
$-LL$	1128.29	1057.28	168.48	243.95

p-Beauty Contest

- Robustness checks:
  - High stakes (Fig.1.3 - small effect lowering numbers)
  - Median vs. Mean (Nagel 99' - same)
  - *p* (Median +18)*: equilibrium inside
- Subject Pool Variation:
  - Portfolio managers
  - Econ PhD, Caltech undergrads
  - Caltech Board of Trustees (CEOs)
  - Readers of Financial Times and Expansion
- Experience vs. Inexperience (for the same game)
  - Slonim (EE 2005) – Experience good only for 1st round

p-Beauty Contest

TABLE 4—MAXIMUM-LIKELIHOOD ESTIMATES AND LOG-LIKELIHOODS FOR THE ITERATED BEST-RESPONSE LEARNING MODELS

Game parameter estimates	Infinite-threshold (N = 2711); Finite-threshold (N = 2668) — Recall period		
	$R = 1$	$R = 2$	$R = 3$
$IT(p, n)$			
α_0	0.2878	0.3132	0.2850
α_1	0.7122	0.6868	0.7150
α_2	0.0000	0.0000	0.0000
α_3	0.0000	0.0000	0.0000
β_1	0.962	1.464	1.414
β_2	—	−0.464	0.197
β_3	—	—	−0.573
$w(0)$	50.97	45.27	44.87
$w(-1)$	—	37.03	48.61
$w(-2)$	—	—	41.85
σ	38.66	30.122	41.08
γ	−0.118	−0.133	−0.125
ρ	0.000	0.000	0.000
LL	−2317.94	−2242.49	−2098.70
χ^2	—	150.90	287.58

p-Beauty Contest

TABLE 4—MAXIMUM-LIKELIHOOD ESTIMATES AND LOG-LIKELIHOODS FOR THE ITERATED BEST-RESPONSE LEARNING MODELS

Game parameter estimates	Infinite-threshold (N = 2711); Finite-threshold (N = 2668) — Recall period		
	$R = 1$	$R = 2$	$R = 3$
$FT(p, n)$			
α_0	0.1185	0.1195	0.1135
α_1	0.6771	0.6801	0.6771
α_2	0.2044	0.2004	0.2094
α_3	0.0000	0.0000	0.0000
β_1	1.027	0.970	0.913
β_2	—	0.060	0.059
β_3	—	—	0.060
$w(0)$	149.08	148.13	143.657
$w(-1)$	—	154.64	222.159
$w(-2)$	—	—	224.626
σ	30.52	29.73	29.954
γ	−0.012	−0.008	−0.008
ρ	0.000	0.000	0.000
LL	−442.75	−437.80	−435.02
χ^2	—	9.90	5.62

Level-k Theory

- Theory for Initial Response (BGT, Ch. 5)
  vs. Theory for Equilibration (BGT, Ch. 6)
- First: Stahl and Wilson (GEB 1995)
- Better: Costa-Gomes, Crawford & Broseta (Econometrica 2001)
- New: Camerer, Ho and Chong (QJE 2004)
- New: Costa-Gomes & Crawford (AER 2006)
  - See Student Presentation

Level-k Theory

- Stahl and Wilson (GEB 1995)

$$q_{ij}(\mu, \varepsilon) \equiv \varepsilon \frac{\exp(\mu U_{ij} P_0)}{\sum_k \exp(\mu U_{ik} P_0)} + (1-\varepsilon) p_{ij}^{\mathrm{NE}}, \tag{1}$$

$$y_{ij}(\mu, \varepsilon) \equiv U_{ij} q_i(\mu, \varepsilon). \tag{2}$$

$$P_{ij}(\gamma, \mu, \varepsilon) \equiv \frac{\exp[\gamma y_{ij}(\mu, \varepsilon)]}{\sum_k \exp[\gamma y_{ik}(\mu, \varepsilon)]}, \tag{3}$$

Level-k Theory

(0) If $\gamma = 0$, then we have a level-0 type—uniform play.

(1) If $\gamma > 0$, $\mu = 0$, and $\varepsilon = 1$, then we have a level-1 type—a (perhaps imprecise) best response to the uniform distribution.

(2) If $\gamma > 0$, $\mu > 0$, and $\varepsilon = 1$, then we have a level-2 type—a (perhaps imprecise) best response to a (perhaps imprecise) best response to the uniform distribution.

(3) If $\gamma > 0$ and $\varepsilon = 0$, then we have a naive Nash-type—a (perhaps imprecise) best response to the Nash equilibrium prior.

(4) If $\gamma > 0.1$ and $\varepsilon \in (0, 1)$, then we have a "worldly" player who chooses a best response (perhaps imprecisely) to a prior based on a belief that some players are level-0, level-1, and naive Nash types.

Level-k Theory

$$P_l^h(\gamma_l, \mu_l, \varepsilon_l) \equiv \prod_i P_{is(h,i)}(\gamma_l, \mu_l, \varepsilon_l), \qquad \text{for } l = 1, 2, 3, 4,$$

and (7)

$$P_5^h(\gamma_5) \equiv \prod_i R_{is(h,i)}(\gamma_5).$$

$$L(s^h|\beta) \equiv \alpha_0 P_0 + \sum_{l=1}^{4} \alpha_l P_l^h(\gamma_l, \mu_l, \varepsilon_l) + \alpha_5 P_5^h(\gamma_5), \tag{8}$$

$$\mathcal{L} \equiv \sum_h \log[L(s^h|\beta)]. \tag{9}$$

Level-k Theory

TABLE IV

PARAMETER ESTIMATES AND CONFIDENCE INTERVALS FOR MIXTURE MODEL WITHOUT RE TYPES

	Estimate	Std. Dev.	95 percent conf. int.	
γ_1	0.2177	0.0425	0.1621	0.3055
μ_2	0.4611	0.0616	0.2014	0.8567
			[0.2360	0.8567]
γ_2	3.0785	0.5743	1.9029	4.9672
			[2.5631	5.0000]
γ_3	4.9933	0.9357	1.9964	5.0000
μ_4	0.0624	0.0063	0.0527	0.0774
ϵ_4	0.4411	0.0773	0.2983	0.5882
γ_4	0.3326	0.0549	0.2433	0.4591
α_0	0.1749	0.0587	0.0675	0.3047
α_1	0.2072	0.0575	0.1041	0.3298
α_2	0.0207	0.0202	0.0000	0.0625
α_3	0.1666	0.0602	0.0600	0.2957
α_4	0.4306	0.0782	0.2810	0.5723
$\mathcal{L}$	-442.727			

Other Level-k Theory

- Choices + Lookups
  - Costa-Gomes, Crawford and Broseta (Econometrica 2001)
- Poisson Cognitive Hierarchy:
  - Camerer, Ho and Chong (QJE 2004)
- Level-k Model (State-of-the-art)
  - Costa-Gomes & Crawford (AER 2006)
  - See Student Presentation

Level-k Theory

- Costa-Gomes, Crawford and Broseta (Econometrica 2001)
- 18 2-player NF games designed to separate:
- Naïve (L1), Altruistic (max sum)
- Optimistic (maximax), Pesimistic (maximin)
- L2 (BR to L1)
- D1/D2 (1/2 round of DS deletion)
- Sophisticated (BR to empirical)
- Equilibrium (play Nash)

Level-k Theory

- Three treatments (all no feedback):
- Baseline (B)
  - Mouse click to open payoff boxes
- Open Box (OB)
  - Payoff boxes always open
- Training (TS)
  - Rewarded to choose equilibrium strategies

Level-k Theory

- Results 1: Consistency of Strategies with Iterated Dominance
- B, OB: 90%, 65%, 15% equilibrium play
  - For Equilibria requiring 1, 2, 3 levels of ID
- TS: 90-100% equilibrium play
  - For all levels
- Game-theoretic reasoning is not computationally difficult, but unnatural.

Level-k Theory

- Result 2: Estimate Subject Decision Rule

Rule	E(u)	Choice (%)	Choice+Lookup (%)
Altruistic	17.11	8.9	2.2
Pessimistic	20.93	0	4.5
Naïve	21.38	22.7	44.8
Optimistic	21.38	0	2.2
L2	24.87	44.2	44.1
D1	24.13	19.5	0
D2	23.95	0	0
Equilibrium	24.19	5.2	0
Sophisticated	24.93	0	2.2

Level-k Theory

- Result 3: Information Search Patterns

Subject / Rule	↕ own payoff		↔ other payoff	
	Predicted	Actual	Predicted	Actual
TS (Equil.)	>31	63.3	>31	69.3
Equilibrium	>31	21.5	>31	79.0
Naïve/Opt.	<31	21.1	-	48.3
Altruistic	<31	21.1	-	60.0
L2	>31	39.4	=31	30.3
D1	>31	28.3	>31	61.7

Level-k Theory

- Result 3: Information Search Patterns
- Occurrence (weak requirement)
  - All necessary lookups exist somewhere
- Adjacency (strong requirement)
  - Payoffs compared by rule occur next to each other
- H-M-L: % of Adjacency | 100% occurrence

Level-k Theory

- Result 3: Information Search Patterns

TABLE V

AGGREGATE RATES OF COMPLIANCE WITH TYPES' OCCURRENCE AND ADJACENCY FOR TS AND BASELINE SUBJECTS, AND FOR BASELINE SUBJECTS BY MOST LIKELY TYPE ESTIMATED FROM DECISIONS ALONE, IN PERCENTAGES (— VACUOUS)

Treatment (# subjects)	*Altruistic* $J = H,M,L,0$	*Pessimistic* $j = H,M,L,0$	*Naïve* $j = H,M,L,0$	*Optimistic* $j = A,0$	*L2* $j = H,M,L,0$	*D1* $j = H,M,L,0$	*D2* $j = H,M,L,0$	*Equilibrium* $j = H,M,L,0$	*Sophisticated* $j = H,M,L,0$
TS (12)	3,10,50,27	44,7,36,13	83,2,0,15	86,14	76,2,0,22	92,3,1,5	92,3,1,5	96,1,1,3	75,1,1,24
Baseline (45)	14,11,51,24	74,2,11,14	78,4,4,14	85,15	67,14,5,14	52,19,15,14	50,19,15,14	42,23,19,16	39,21,20,21
Altruistic (2)	78,6,11,6	56,8,33,3	53,3,42,3	97,3	47,8,39,6	36,6,56,3	33,8,56,3	31,11,56,3	28,14,56,3
Pessimistic (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Naïve/Optim. (11)	9,5,53,33	85,1,9,5	89,5,3,4	96,4	42,24,3,31	45,22,20,13	43,18,23,16	26,24,28,23	23,23,27,27
L2 (23)	8,12,58,22	72,2,9,17	78,3,0,18	80,20	85,6,3,6	57,20,9,15	54,21,10,15	49,24,12,15	46,22,12,20
D1 (7)	23,21,26,29	59,3,16,23	63,7,6,23	77,23	53,21,6,21	48,17,14,20	45,19,15,21	42,20,17,21	38,14,21,27
D2 (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—
Equilibrium (2)	6,8,86,0	100,0,0,0	97,3,0,0	100,0	64,36,0,0	69,17,14,0	67,19,14,0	56,25,19,0	53,19,28,0
Sophisticated (0)	—,—,—,—	—,—,—,—	—,—,—,—	—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—	—,—,—,—

(Poisson) Cognitive Hierarchy

- Camerer, Ho and Chong (QJE 2004)
- Frequency of level-k thinkers is $f(k|\tau)$
  - *τ = mean number of thinking steps*
- Level-0: choose randomly or use heuristics
- Level-k thinkers use *k* steps of thinking BR to a mixture of lower-step thinkers
  - Belief about others is Truncated Poisson
- Easy to compute; Explains many data

Conclusion

- Do you obey dominance?
- Would you count on others obeying dominance?
- Limit of Strategic Thinking: 2-3 steps
- Theory (for initial responses)
  - Level-k Types: Stahl-Wilson95, CGCB01
  - Cognitive Hierarchy: CHC04