Dominance-Solvable Games

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4/10/2008

Dominance

- Strategy A gives you better payoffs than Strategy B regardless of opponent strategy
- Dominance Solvable
  - A game that can be solved by iteratively deleting dominated strategy

- Do people obey dominance?
  - Looking both sides to cross a 1-way street
  - “If you can see this, I can’t see you.”
  - p-Beauty Contest behavior (guess above 67)
- Will you bet on others obeying dominance?
  - Workers respond to incentives rationally
  - Companies don’t use “optimal” contracts
- SOPH: Knowing other’s steps of reasoning

Hierarchy of Iterated Reasoning

- D0: Strict dominance,
- D1: Belief that others obey dominance,
- D2: Belief that others believe you’ll obey dominance, ...
- Vince Crawford (AER co-editor):
  - on Level-k: I’ll treat any student for dinner at a conference if s/he can show an example of “4 levels of reasoning” in history or literature

Outline

- A Simple Test:
  - Beard & Beil (MS 94’)
- Other Games:
  - Centipede:
    - McKelvey & Palfrey (Econometrica 92’)
    - Mechanism Design:
      - Sefton and Yavas (GEB 96’)
      - Dirty Face:
        - Weber (EE 01’)
  - Learning IEDS:
    - Traveler’s dilemma
    - Price competition
    - Capra et al (AER 99’, IER 02’)
  - p-Beauty Contest
    - Nagel + CHW (AER 95’, 98’)
  - Theory:
    - Stahl and Wilson (GEB 95’)
    - CGCB (Econometrica 01’)
    - Cognitive Hierarchy (QJE 04’)
    - CGC (AER 06’)

A Simple Test

<table>
<thead>
<tr>
<th>Iterated dominance game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1 move</td>
</tr>
<tr>
<td>L</td>
</tr>
<tr>
<td>R</td>
</tr>
</tbody>
</table>
A Simple Test

Payoff treatments and results in Beard and Bell

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Payoffs from (L, 1)</th>
<th>Frequency of (R, 1)</th>
<th>Number of pairs</th>
<th>Threshold P(rR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (baseline)</td>
<td>(9.75, 3)</td>
<td>(10.5)</td>
<td>35</td>
<td>0.97</td>
</tr>
<tr>
<td>2 (less risk)</td>
<td>(9, ..)</td>
<td>(.. ..)</td>
<td>31</td>
<td>0.83</td>
</tr>
<tr>
<td>3 (even less risk)</td>
<td>(7, ..)</td>
<td>(.. ..)</td>
<td>26</td>
<td>0.97</td>
</tr>
<tr>
<td>4 (more assurance)</td>
<td>(.. ..)</td>
<td>(.. ..)</td>
<td>25</td>
<td>0.97</td>
</tr>
<tr>
<td>5 (more resentment)</td>
<td>(.. ..)</td>
<td>(10)</td>
<td>21</td>
<td>0.97</td>
</tr>
<tr>
<td>6 (less risk, more reciprocity)</td>
<td>(.. ..)</td>
<td>(5, 9.75)</td>
<td>26</td>
<td>0.97</td>
</tr>
<tr>
<td>7 (1/6 payoff)</td>
<td>(58.5, 18)</td>
<td>(18, 28.5)</td>
<td>3</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Note: (.. ..) indicates the payoffs are the same as those in the baseline case.

A Simple Test: Follow-up 1

- Jacob Goeree and Charles Holt (PNAS 1999)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of pairs</th>
<th>Threshold p(rR)</th>
<th>Payoffs (L, 1)</th>
<th>Frequency (R, 1)</th>
<th>Number of pairs</th>
<th>Threshold P(rR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>15</td>
<td>0.85</td>
<td>(80,50)</td>
<td>(20,10)</td>
<td>90,70</td>
<td>0.53</td>
</tr>
<tr>
<td>Lower assurance</td>
<td>25</td>
<td>0.85</td>
<td>(80,50)</td>
<td>(20,10)</td>
<td>90,70</td>
<td>0.53</td>
</tr>
<tr>
<td>Very low assurance</td>
<td>25</td>
<td>0.85</td>
<td>(400,250)</td>
<td>(100,348)</td>
<td>450,350</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 5.3 Goeree and Holt's credible threat games

A Simple Test: Follow-up 2

- Strategic Form vs. Sequential Form

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of pairs</th>
<th>Threshold p(rR)</th>
<th>Payoffs (L, 1)</th>
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<th>Number of pairs</th>
<th>Threshold P(rR)</th>
</tr>
</thead>
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<td>0.80</td>
</tr>
</tbody>
</table>

A Simple Test: Follow-up 2

- Schotter et al. (1994)’s conclusion:
  - Limited evidence of iteration of dominance (beyond 1-step), or SPE, forward induction
  - Can more experience fix this?
- Not for forward induction in 8 periods…
  - Brandts and Holt (1995)
- Yes for 3-step iteration in 160 periods
Learn to Play Iterated Dominance

  - “Strong” invests 0~$5; “weak” invests 0~$4
    - Highest investor earns $10; both earn $0 if tie
  - s=0 is dominated by s=5
    - Deleting s=0, w=1 is dominated by w=0
  - Deleting w=1, s=2 is dominated by s=1, etc.
    - I.D. kills s=0, 2, 4 and w=1, 3
  - MSE: s=1,3,5- (.2, .2, .6); w=0,2,4- (.6, .2, .2)

Centipede Game

- McKelvey and Palfrey (Econometrica 1992)

![Centipede Game Diagram]

<table>
<thead>
<tr>
<th>Session</th>
<th># Subjects</th>
<th>Subject</th>
<th># Games/Subject</th>
<th>Total # Games</th>
<th># Moves</th>
<th>High Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (PCC)</td>
<td>100</td>
<td>2</td>
<td>2.8</td>
<td>70</td>
<td>4</td>
<td>0.6</td>
</tr>
<tr>
<td>2 (PCC)</td>
<td>100</td>
<td>2</td>
<td>2.6</td>
<td>44</td>
<td>4</td>
<td>0.4</td>
</tr>
<tr>
<td>3 (CTT)</td>
<td>100</td>
<td>2</td>
<td>2.8</td>
<td>40</td>
<td>4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Proportion of Observations at Each Terminal Node**

<table>
<thead>
<tr>
<th>Session</th>
<th>s1</th>
<th>s3</th>
<th>t1</th>
<th>t3</th>
<th>t5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (PCC)</td>
<td>.06</td>
<td>.44</td>
<td>.20</td>
<td>.04</td>
<td></td>
</tr>
<tr>
<td>2 (PCC)</td>
<td>.10</td>
<td>.40</td>
<td>.11</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>3 (CTT)</td>
<td>.06</td>
<td>.43</td>
<td>.14</td>
<td>.09</td>
<td></td>
</tr>
</tbody>
</table>

**Experimental Design**

*The number in parentheses is the number of observations in the game at that node.*
Centipede Game

• What theory can explain this?
• Altruistic Types (7%): Prefer to Pass
• Normal Types:
  – Mimic altruistic types up to a point (gain more)
  – Unraveling: error rate shrinks over time

TABLE IIIA
Cumulative Outcome Frequencies

t_i = \sum_{j} t_{j}

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Game</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( t_3 )</th>
<th>( t_4 )</th>
<th>( t_5 )</th>
<th>( t_6 )</th>
<th>( t_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four</td>
<td>1-5</td>
<td>0.02</td>
<td>0.25</td>
<td>0.55</td>
<td>0.85</td>
<td>0.95</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>Move</td>
<td>6-10</td>
<td>0.13</td>
<td>0.20</td>
<td>0.37</td>
<td>0.58</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>Six</td>
<td>1-5</td>
<td>0.00</td>
<td>0.05</td>
<td>0.27</td>
<td>0.55</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>Move</td>
<td>6-10</td>
<td>0.13</td>
<td>0.20</td>
<td>0.37</td>
<td>0.58</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
</tr>
</tbody>
</table>

TABLE IIIB
Implied Take Probabilities
Comparison of Early Versus Late Plays in the Low Payoff Centipede Game

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Game</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_3 )</th>
<th>( p_4 )</th>
<th>( p_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four</td>
<td>1-5</td>
<td>0.06</td>
<td>0.32</td>
<td>0.57</td>
<td>0.75</td>
<td>0.84</td>
</tr>
<tr>
<td>Move</td>
<td>6-10</td>
<td>0.08</td>
<td>0.36</td>
<td>0.54</td>
<td>0.75</td>
<td>0.82</td>
</tr>
<tr>
<td>Four</td>
<td>1-5</td>
<td>0.09</td>
<td>0.31</td>
<td>0.49</td>
<td>0.75</td>
<td>0.85</td>
</tr>
<tr>
<td>Move</td>
<td>6-10</td>
<td>0.08</td>
<td>0.35</td>
<td>0.47</td>
<td>0.76</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Selfish players sometimes pass, to mimic an altruist. By imitating an altruist one might lure an opponent into passing at the next move, thereby raising one's final payoff in the game.

The amount of imitation in equilibrium depends directly on the beliefs about the likelihood \((1-q)\) of a randomly selected player being an altruist. The more likely players believe there are altruists in the population, the more imitation there is.

Property 1:
For any \( q \), Blue chooses TAKE with probability 1 on its last move.

Property 2:
If \( 1 - q > \frac{1}{2} \), both Red and Blue always choose PASS, except on the last move, when Blue chooses TAKE.

Property 3:
If \( 1 - q \in (0, \frac{1}{2}) \) the equilibrium involves mixed strategies.

Property 4:
If \( q = 1 \), then both Red and Blue always choose TAKE.
We model noisy play in the following way. In game $t$, at node $s$, if $p^*$ is the equilibrium probability of TAKE that the player at that node attempts to implement, we assume that the player actually chooses TAKE with probability $(1-\varepsilon_t)p^*$, and makes a random move (i.e. TAKE or PASS with probability 0.5) with probability $\varepsilon_t$.

Fey, McKelvey and Palfrey (IJGT 1996)
- Use constant-sum to kill social preferences
- Take 50% at 1$^{st}$, 80% at 2$^{nd}$

Nagel and Tang (JMathPsych 1998)
- Don’t know other’s choice if you took first
- Take about half way

Rapoport et al. (GEB 2003)
- 3-person & high stakes: Many take immediately
- CH can explain this (but not QRE) – see theory

Mechanism Design
- Pure coordination game with $1.20 & $0.60
- How can you implement a Pareto-inferior equilibrium in a pure coordination games?

Abreu & Matsushima (Econometrica 1992)
- Slice the game into “$T$ periods”
- F: Fine paid by first subject to deviate
- Won’t deviate if F > $1.20/T$
- Can set $T=1$, F=$1.20$; more credible if $T$ large

Glazer and Rosenthal (Economtrica 1992)
- Comment: AM mechanism requires more steps of iterated deletion of dominated strategies

Abreu & Matsushima (Econometrica 1992)
- Respond: “[Our] gut instinct is that our mechanism will not fare poorly in terms of the essential feature of its construction, that is, the significant multiplicative effect of ‘fines.’”
- This invites an experiment!

Sefton and Yavas (GEB 1996)
- F=$0.225$
- $T=4$, 8, or 12
  - Theory: Play inferior NE at $T=8$ or 12, not $T=4$
  - Results: Opposite, and diverge…
  - Why? Choose only 1 switchpoint in middle
    - Goal: switch soon, but 1 period after opponent
Mechanism Design

- Glazer and Perry (GEB 1996)
  - Implemental can work in sequential game via backward induction
- Katok, Sefton and Yavas (JET 2002)
  - Doesn’t work either
- Can any “approximately rational explanation” get this result?
  - Maybe “Limited steps of IDDS + Learning”

Dirty Face Game

- Three ladies, A, B, C, in a railway carriage all have dirty faces and are all laughing. It sudden flashes on A: why doesn’t B realize C is laughing at her? Heavens! I must be laughable.
  - Littlewood (1953), “A Mathematician’s Miscellany
- Requires A to think that B is rational enough to draw inference from C

Dirty Face Game

- Weber (EE 2001)
  - Independent types X (Prob=.8) or O (Prob=.2)
    - X is like “dirty face”
  - Commonly told “At least one player is type X.”
  - Observe other’s type
  - Choose Up or Down (figure out one is type X)
  - If nobody chooses Down, reveal other’s choice and play again

Dirty Face Game

<table>
<thead>
<tr>
<th>Probability</th>
<th>Type</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Up</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>$0</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>Down</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>$1</td>
</tr>
</tbody>
</table>

Dirty Face Game

- Case XO: Players play (Up, Down)
- Type X player thinks…
  - I know that “at least one person is type X”
  - I see the other person is type O
- So, I must be type X → Chooses Down
- Type O player thinks…
  - I know that “at least one person is type X”
  - I see the other person is type X
- No inference → Chooses Up
Dirty Face Game

• Case XX - First round:
  • No inference (since at least one is type X, but the other guy is type X) → Both choose Up
• Case XX - Second round:
  • Seeing UU in first
    – the other is not sure about his type
    – He must see me being type X
  • I must be Type X → Both choose Down

<table>
<thead>
<tr>
<th>Round</th>
<th>XO</th>
<th>XX</th>
<th>XO</th>
<th>XX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UU</td>
<td>0</td>
<td>7*</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>DU</td>
<td>3*</td>
<td>3</td>
<td>4*</td>
</tr>
<tr>
<td></td>
<td>DD</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (after UU)</td>
<td>DD</td>
<td>-</td>
<td>1*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Dirty Face Game

• Results: 87% rational in XO, but only 53% in 2nd round of XX
• Significance:
  • Upper bound of iterative reasoning
    – Caltech students still don’t do 2 steps
  • Choices reveal limited reasoning, not pure cooperativeness
    – More iteration is better here...

Price Competition

• Capra, Goeree, Gomez & Holt (IER 2002)
  – Two firms pick prices $p_1$ & $p_2$ from $0.60 \sim 1.60$
  – Both get $(1+a/p_1)/2$ if tied; but if $p_1 < p_2$
  – Low-price firm gets $p_1$; other firm gets $a*p_1$
• $a =$ responsiveness to “best price” (=0.2/0.8)
  – $a=1$: Meet-or-release
  – $a<1$: Bertrand competition predicts lowest price

Initial Response and Equilibration

• Price Competition
  – Capra, Goeree, Gomez and Holt (IER 2002)
• Traveler’s Dilemma
  – Capra, Goeree, Gomez and Holt (AER 1999)
• $p$-Beauty Contest
  – Nagel (AER 1995)
  – Camerer, Ho, Weigelt (AER 1998)

Price Competition

Simulated average prices obtained from 1000 simulations (dark lines ± 2 standard deviations (dotted lines) and a focal run (lines connecting squares))
Traveler's Dilemma

- Capra, Goeree, Gomez & Holt (AER 1999)
  - Two travelers state claim \( p_1 \) and \( p_2 \): 80~200
  - Airline awards both the minimum claim, but
  - reward \( R \) to the one who stated the lower claim
  - penalize the other by \( R \)
- Unique NE: race to the bottom → lowest claim
  - Like price competition game or beauty contest

\[ p \text{-Beauty Contest} \]

- Each of \( N \) players choose \( x_i \) from [0,100]
- Target is \( p^* \) (average of \( x_i \))
- Closest \( x_i \) wins fixed prize
- (67,100] violates 1st order dominance
- (45, 67] obeys 1 step (not 2) of dominance
- Nagel (AER 1995): BGT, Figure 5.1b
- Ho, Camerer, and Weigelt (AER 1998)
  - BGT, Figure 1.3, 5.1

\[ p \text{-Beauty Contest} \]

- Keynes (1936 pp. 155-56) said,
  - “...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole..."
RESULT 1:  
First-period choices are far from equilibrium, and centered near the interval midpoint. Choices converge toward the equilibrium point over time.

RESULT 2:  
On average, choices are closer to the equilibrium point for games with finite thresholds, and for games with p further from 1.

RESULT 3:  
Choices are closer to equilibrium for large (7-person) groups than for small (3-person) groups.

RESULT 4:  
Choices by [cross-game] experienced subjects are no different than choices by inexperienced subjects in the first round, but converge faster to equilibrium.
• Classification of Types
  – Follow Stahl and Wilson (GEB 1995)
• Level-0: pick randomly from N(\mu, \sigma)
• Level-1: BR to level-0 with noise
• Level-2: BR to level-1 with noise
• Level-3: BR to level-2 with noise
• Estimate type, error using MLE
**p-Beauty Contest**

(9) \[ G_L(t) = B_{L-1}(t). \]

(10) \[ \mu(t) = \sum_{i=1}^{N} \beta_i \omega(t-x). \]

(11) \[ \sigma(t) = \sigma \cdot \epsilon^{-\gamma t}. \]

(12) \[ B(x) = \sum_{L=0}^{L_{max}} \alpha_L B_L(x). \]

(13) \[ LL = \sum_{i=1}^{N} \sum_{x=1}^{x_{max}} \log(B(x(x_i))). \]

**Robustness checks:**
- High stakes (Fig. 1.3 - small effect lowering numbers)
- Median vs. Mean (Nagel 99' - same)
- \( p^* \) (Median +18): equilibrium inside

**Subject Pool Variation:**
- Portfolio managers
- Econ PhDs, Caltech undergrads
- Caltech Board of Trustees (CEOs)
- Readers of Financial Times and Expansion

**Experience vs. Inexperience (for the same game)**
- Slonim (EE 2005) – Experience good only for 1st round

**Theory for Initial Response (BGT, Ch. 5)**

**Level-k Theory**

- Theory for Initial Response (BGT, Ch. 5)
- First: Stahl and Wilson (GEB 1995)
- Better: Costa-Gomes, Crawford & Broseta (Econometrica 2001)
- New: Camerer, Ho and Chong (QJE 2004)
- New: Costa-Gomes & Crawford (AER 2006)

– See Student Presentation
Level-k Theory

\[ q_0(\mu, e) = e^{\frac{\mu U_0 P_0}{\sum_i \exp(\mu U_i P_0)} + (1 - e)P_0^{NE}}, \]  

\[ y_0(\mu, e) = U_0 q_0(\mu, e). \]  

\[ P_0(y, \mu, e) = \frac{\exp[y y_0(\mu, e)]}{\sum_i \exp[y y_0(\mu, e)]}, \]

Level-k Theory

\[ P_l(y_l, \mu_l, e_l) = \prod P_{\alpha_l, \beta_l}(y_l, \mu_l, e_l), \quad \text{for } l = 1, 2, 3, 4, \]  

\[ P(y_0) = \prod R_{\alpha, \beta}(y_0). \]

\[ L(x^{|y_0}) = \alpha_0 P_0 + \sum_{l=1}^4 \alpha_l P_l(y_l, \mu_l, e_l), \]  

\[ X = \sum \log L(x^{|y_0}). \]

Other Level-k Theory

• Choices + Lookups  
  - Costa-Gomes, Crawford and Broseta (Econometrica 2001)

• Poisson Cognitive Hierarchy:  
  - Camerer, Ho and Chong (QJE 2004)

• Level-k Model (State-of-the-art)  
  - Costa-Gomes & Crawford (AER 2006)  
  - See Student Presentation

Level-k Theory

0. If \( y = 0 \), then we have a level-0 type—uniform play.

1. If \( y > 0, \mu = 0, \) and \( e = 1 \), then we have a level-1 type—a (perhaps imprecise) best response to the uniform distribution.

2. If \( y > 0, \mu > 0, \) and \( e = 1 \), then we have a level-2 type—a (perhaps imprecise) best response to the uniform distribution.

3. If \( y > 0 \) and \( e = 0 \), then we have a naive Nash-type—a (perhaps imprecise) best response to the Nash equilibrium prior.

4. If \( y > 0.1 \) and \( e \in (0, 1) \), then we have a “worldly” player who chooses a best response (perhaps imprecisely) to a prior based on a belief that some players are level-0, level-1, and naive Nash types.

Other Level-k Theory

• Costa-Gomes, Crawford and Broseta (Econometrica 2001)

• 18 2-player NF games designed to separate:  
  - Naive (L1), Altruistic (max sum)
  - Optimistic (maximmax), Pesimistic (maximin)
  - L2 (BR to L1)
  - D1/D2 (1/2 round of DS deletion)
  - Sophisticated (BR to empirical)
  - Equilibrium (play Nash)

Level-k Theory

\[ \begin{array}{cccc}
\text{Parameter} & \text{Estimate} & \text{Std. Dev.} & \text{95\% conf. int.} \\
\hline
\gamma_1 & 0.3177 & 0.0428 & 0.2259 & 0.4095 \\
\gamma_2 & 0.4511 & 0.0616 & 0.3294 & 0.5728 \\
\gamma_3 & 0.3726 & 0.3743 & 0.2039 & 0.5422 \\
\gamma_4 & 3.0785 & 5.7343 & 1.9549 & 4.6672 \\
\gamma_5 & 2.5353 & 6.0000 & 1.9994 & 3.0000 \\
\gamma_6 & 0.9992 & 0.0967 & 0.9103 & 1.0884 \\
\gamma_7 & 0.0066 & 0.0063 & 0.0003 & 0.0127 \\
\gamma_8 & 0.4411 & 0.0773 & 0.3683 & 0.5142 \\
\gamma_9 & 0.3226 & 0.0349 & 0.2943 & 0.3517 \\
\gamma_{10} & 0.1749 & 0.0547 & 0.0675 & 0.2817 \\
\gamma_{11} & 0.2971 & 0.2875 & 0.1041 & 0.3928 \\
\gamma_{12} & 0.2377 & 0.2392 & 0.0840 & 0.3925 \\
\gamma_{13} & 0.1666 & 0.0902 & 0.0600 & 0.2797 \\
\gamma_{14} & 0.4236 & 0.0792 & 0.3810 & 0.5723 \\
\gamma_{15} & -4.4737 & & & \\
\end{array} \]
### Level-k Theory

- Three treatments (all no feedback):
  - Baseline (B)
    - Mouse click to open payoff boxes
  - Open Box (OB)
    - Payoff boxes always open
  - Training (TS)
    - Rewarded to choose equilibrium strategies

### Level-k Theory

- Results 1: Consistency of Strategies with Iterated Dominance
  - B, OB: 90%, 65%, 15% equilibrium play
  - For Equilibria requiring 1, 2, 3 levels of ID
  - TS: 90-100% equilibrium play
  - For all levels
- Game-theoretic reasoning is not computationally difficult, but unnatural.

### Level-k Theory

- Result 2: Estimate Subject Decision Rule

<table>
<thead>
<tr>
<th>Rule</th>
<th>E(u)</th>
<th>Choice (%)</th>
<th>Choice+Lookup (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altruistic</td>
<td>17.11</td>
<td>8.9</td>
<td>2.2</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>20.93</td>
<td>0</td>
<td>4.5</td>
</tr>
<tr>
<td>Naïve</td>
<td>21.38</td>
<td>22.7</td>
<td>44.8</td>
</tr>
<tr>
<td>Optimistic</td>
<td>21.38</td>
<td>0</td>
<td>2.2</td>
</tr>
<tr>
<td>L2</td>
<td>24.87</td>
<td>44.2</td>
<td>44.1</td>
</tr>
<tr>
<td>D1</td>
<td>24.13</td>
<td>19.5</td>
<td>0</td>
</tr>
<tr>
<td>D2</td>
<td>23.95</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>24.19</td>
<td>5.2</td>
<td>0</td>
</tr>
<tr>
<td>Sophisticated</td>
<td>24.95</td>
<td>0</td>
<td>2.2</td>
</tr>
</tbody>
</table>

### Level-k Theory

- Result 3: Information Search Patterns

**Table**

<table>
<thead>
<tr>
<th>Subject / Rule</th>
<th>1 own payoff</th>
<th>↔ other payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted</td>
<td>Actual</td>
<td>Predicted</td>
</tr>
<tr>
<td>TS (Equil.)</td>
<td>&gt;31 63.3</td>
<td>&gt;31 69.3</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>&gt;31 21.5</td>
<td>&gt;31 79.0</td>
</tr>
<tr>
<td>Altruistic</td>
<td>&lt;31 21.1</td>
<td>- 60.0</td>
</tr>
<tr>
<td>L2</td>
<td>&gt;31 39.4</td>
<td>=31 30.3</td>
</tr>
<tr>
<td>D1</td>
<td>&gt;31 28.3</td>
<td>&gt;31 61.7</td>
</tr>
</tbody>
</table>

### Level-k Theory

- Result 3: Information Search Patterns

- Occurrence (weak requirement)
  - All necessary lookups exist somewhere
- Adjacency (strong requirement)
  - Payoffs compared by rule occur next to each other
- H-M-L: % of Adjacency | 100% occurrence

### Level-k Theory

- Result 3: Information Search Patterns
(Poisson) Cognitive Hierarchy

- Camerer, Ho and Chong (QJE 2004)
- Frequency of level-k thinkers is \( f(k|\tau) \)
  \( \tau = \text{mean number of thinking steps} \)
- Level-0: choose randomly or use heuristics
- Level-k thinkers use \( k \) steps of thinking BR to a mixture of lower-step thinkers
  - Belief about others is Truncated Poisson
- Easy to compute; Explains many data

Conclusion

- Do you obey dominance?
- Would you count on others obeying dominance?
- Limit of Strategic Thinking: 2-3 steps
- Theory (for initial responses)
  - Level-k Types: Stahl-Wilson95, CGCB01
  - Cognitive Hierarchy: CHC04