Mixed Strategy Equilibrium

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Games with Mixed-Strategy Equilibrium (MSE)

- Zero-Sum Games
  - Rock-Scissor-Paper
  - Sport events
  - Military attack
- Deter Undesired Behavior
  - Searches of passengers after Sep. 11
  - Randomizing across exam questions
- But, there are interesting “folk theories” about these games…

玩家公開猜拳遊戲必勝絕招：先出剪刀

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- 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。 L0
- 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。如果你的對手預期你會 L1 出石頭，他們就會選擇出布來贏過你，因此你要出剪刀才能贏，因為剪刀贏布。 L2

玩家公開猜拳遊戲必勝絕招：先出剪刀

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- 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是他要求佳士得與蘇富比兩家公司猜拳決定。
- 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

Mixed-Strategy Equilibrium

- What would you play in Rock-paper-scissors?
- What is the MSE of this game?
  - Mix with probabilities (1/3, 1/3, 1/3)
- Would you really play the MSE in RPS?
  - What would a level-k model predict in RPS? How does the news article above match that?
  - For more, see BGT, Ch.5

Advantages of Games with MSE

- Typically have unique equilibrium
  - All games discussed have unique equilibrium
- Constant sum (no social preference)
  - Not possible to help others without hurting self
- Maximin leads to Nash in zero sum
  - Maximin is a simple decision rule
- A good places to test standard theory!

Maximin in "Matching Pennies"

	Head	Tail
Head	1	-1
Tail	-1	1

- Row player thinks
- Head: Tail (-1)
- Tail: Head (-1)
- (1/2, 1/2): (0)*
  – This is the MSE!

Challenges of Games with MSE

- Epistemic Foundation
  – Requires precise knowledge of other's strategy
- Learning Dynamics may not work
  – Gradient processes spiral away from MSE
  – No incentive to mix properly at MSE
- Randomization can be unnatural (esp. in repeated play)
- Purification
  – MSE can occur at population level but not individually

Overall Results of MSE

Source: BGT, Ch. 3.

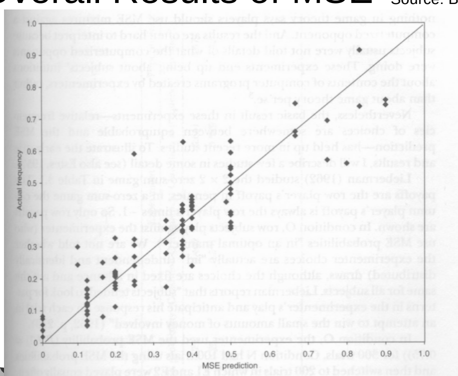

Figure 3.1. Frequencies of different strategy choices predicted by mixed-strategy equilibrium and actual frequencies.

The Joker Game: O'Neill (1987)

– Earlier studies had computerized opponents and/or low incentives (hard to interpret results)
- Modern Studies: O'Neill (1987)
- Good Design Trick:
  – Risk aversion plays no role when there are only two possible outcomes

The Joker Game: O'Neill (1987)

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

- Actual frequencies are quite close to MSE
- QRE better, but can't get "imbalances"

Quantal Response Equilibrium (QRE)

- McKelvey and Palfrey (1995)
- Better Response (not best response)
- Logit payoff response function:

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i})\right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i})\right]}}$$

Quantal Response Equilibrium (QRE)

- $\lambda = 0$: Noise (don't respond to payoffs)
- $\lambda = \infty$: Nash (perfectly respond to payoffs)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i})\right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i})\right]}}$$

Response to O'Neill (1987)

- Brown and Rosenthal (1990) criticized O'Neill:
  - Overly support MSE
  - Aggregate tests aren't good enough
- They run (temporal dependence):
- $J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1} + b_0 J^*_{t+1} + b_1 J^*_t + b_2 J^*_{t-1} + c_1 J_t J^*_t + c_2 J_{t-1} J^*_{t-1}$
- MSE implies only a_0 is nonzero

Results of Brown & Rosenthal (1990)

Effect	Coefficient	% Players s.t. p<0.05
Guessing	b_0	8%
Previous opp. choices	b_1, b_2	30%
Previous outcomes	c_1, c_2	38%
Previous choices & outcome	b_1, b_2, c_1, c_2	44%
Previous own choices	a_1, a_2	48%
All effects		62%

Source: Table 3.4, BGT.

Response to O'Neill (1987)

- Run: 2 JJJJ 1 2 33
- Too Short runs: play J twice too rarely
- Subjects react to what they had seen & done
  - But most can't use the temporal dependence outguess opponents' current action
- Equilibrium-in-beliefs is somewhat supported
  - Each player may deviate from MSE
  - But these deviations cannot be detected
- Purification interpretation of MSE
  - Equilibrium in beliefs rather than in mixtures

Response to O'Neill (1987)

- Other similar studies:
  - Rapoport and Boebel (1992) [BGT, Table 3.5]
  - Mookerjhee and Sopher (1997) [BGT, Table 3.6-3.7]
  - Tang (1996abc, 2001) [BGT, Table 3.8]
  - Binmore, Swierzbinski, and Proulx (2001) [BGT, Table 3.9]
- Stylized Facts:
  - Actual frequencies not far from MSE
  - Deviations small but significant
  - Temporal dependence at the individual level
- Can a theory explain these?

Psychology: Production Task

- Ask subjects to generate random sequences
- Subject sequences resemble the underlying statistical process <u>more closely than</u> what short random sequences actually do
  - Too balanced
  - Too many runs
  - Longest run is too short
- Children don't seem to learn this misconception until after 5th grade
  - A learned mistake

Game Play vs. Production

- Rapoport and Budescu (1992, 1994, 1997)
- Compare sequences from a production task to strategies in a constant-sum game
- Condition D: Matching pennies 150 times (1-by-1)
- Condition S: Give sequence of 150 plays at once
- Condition R: Produce the outcome of tossing an unbiased coin 150 times
- iid rejected for 40%, 65% and 80% of the subjects
  - Game playing reduce deviations from randomness
- Are subjects better motivated or are their working memory interfered and randomize "memory-lessly"?

Game Play vs. Production: Balanced

Pattern	Game Freq.	Production Freq.	iid Freq.
xx	0.269	0.272	0.333
xxx	0.073	0.063	0.111
xxy	0.196	0.209	0.222
xyy	0.196	0.210	0.222
xxxx	0.020	0.018	0.037
xxxy	0.053	0.045	0.074
yxxx	0.054	0.045	0.074
xyxx	0.056	0.035	0.074
xxyx	0.058	0.037	0.074

Game Play vs. Production: Unbalanced

Pattern	Game Freq.	Production Freq.	iid Freq.
xy	0.731	0.728	0.667
xyx	0.237	0.160	0.222
xyz	0.297	0.359	0.222
yxzx	0.096	0.078	0.074
xyxz	0.099	0.079	0.074
xyzx	0.121	0.173	0.074

Source: Table 3.10, BGT.

A Limited Memory Model

- Subjects only remember last *m* elements
- Chose the (*m+1)*st to balance the number of H and T choices in the last (*m+1)* flips
- If *m* is small, they'll alternate choices too frequently
- Experimental Data: (Should all be 0.5 if iid)
  - P(H|H)=0.42
  - P(H|HH)=0.32
  - P(H|HHH)=0.21
- Requires *m*=7 to generate this (Magic 7?)

Explicit Randomization

  - Observe the randomization subjects want to play
  - Bloomfield (1994), Ochs (1995b), Shachat (2002)
- Explicit Randomization:
  - Allocate 100 choices to either strategies
  - Choices are shuffled and computer selects one
- Deviations cannot be due to cognitive limit!
- Result: Deviations from MSE are small but significant
- About 10 percent are "purists"

Explicit Randomization

- Ex: Ochs (1995b) - Matching Pennies
  - Row player payoff of (H, H): 1→9→4
- MSE: Row MSE changes; column is same
- Allocate 10 plays of H or T
  - Becomes a 10-play sequence
- Note: Random draw without replacement
  - This is not exactly randomization of MSE…

Matching Pennies (Baseline)

	Head	Tail
Head	**1**,0	0,1
Tail	0,1	1,0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)
- Actual Frequency:
  - R: (0.500, 0.500)
  - C: (0.480, 0.520)
- QRE:
  - R: (0.500, 0.500)
  - C: (0.500, 0.500)

Matching Pennies (Game 2)

	Head	Tail
Head	**9**,0	0,1
Tail	0,1	1,0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.100, 0.900)
- Actual Frequency:
  - R: (0.600, 0.400)
  - C: (0.300, 0.700)
- QRE:
  - R: (0.649, 0.351)
  - C: (0.254, 0.746)

Matching Pennies (Game 3)

	Head	Tail
Head	**4**,0	0,1
Tail	0,1	1,0

- MSE:
  - R: (0.500, 0.500)
  - C: (0.200, 0.800)
- Actual Frequency:
  - R: (0.540, 0.460)
  - C: (0.340, 0.560)
- QRE:
  - R: (0.619, 0.381)
  - C: (0.331, 0.669)

Source: Table 3.12, BGT.

MSE in Field Context

- Rapoport and Almadoss (2000)
- Patent races games
  - Two firms with endowment *e*
  - Invest *1, 2,, …, e* (integer)
  - Win *r* if invest most
- Unique MSE: Invest *e* with prob. 1-*e/r,* invest others with prob. *1/r* (not obvious)

Patent Race Results

(Table 3.14)	Game L: *e=5,r=8*		Game H: *e=5,r=20*	
Investment	MSE	Actual	MSE	Actual
0	0.125	0.169	0.050	0.141
1	0.125	0.116	0.050	0.055
2	0.125	0.088	0.050	0.053
3	0.125	0.118	0.050	0.053
4	0.125	0.090	0.050	0.069
5	0.375	0.418	0.750	0.628

MSE in Field Context

- 3 Firm Hotelling: Collins and Sherstyuk (2000)
  - 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
  - 4-Firm: Huck, Muller and Vreiend (2002)
- Location Games (3 Firm Hotelling Model)
  - Three firms simultaneously choose [0,100]
  - Consumers go to nearest firm
  - Profits proportional to units sold
- Unique MSE: Randomize uniformly [25,75]

MSE in Field Context

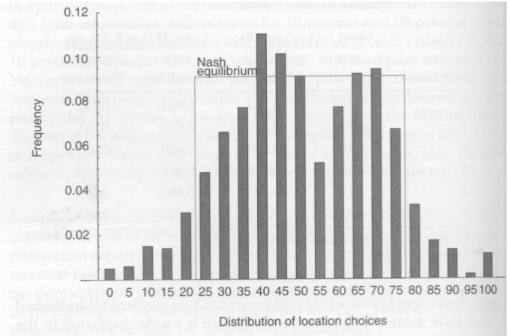

Figure 3.2. Frequency of location choices in three-person simultaneous Hotelling gam
Source: Based on Collins and Sherstyuk (2000).

Two Field Studies

- Walker and Wooders (2001)
  - serve decisions (L or R) of tennis players in 10 Grand Slam matches
- Result:
  - Win rates across two different directions are not statistically different ($p<0.10$ for only 2/40)
  - Players still exhibit some over-alteration in serve choices though temporal dependence ($p<0.10$ for 8/40) [weaker than lab subjects]

Two Field Studies

- Palacios-Huerta (2001): soccer penalty kicks
  - Code both kicker and goalie's choices
  - No selection bias (look at all games)
- Win rates are equal; no serial dependence
  - Not surprising since penalty kicks are few and are often done by different players
- Recent: Huang, Hsu, and Tang (AER 2007)
  - Chen-Ying Huang (here at NTU)

Conclusion

- Take-home Message:
- Aggregate frequencies of play are close to MSE but the deviations are statistically significant.
- QRE seems to fit behaviors well.
- Temporal dependence is frequently observed

Overall Results of QRE

Source: BGT, Ch. 3.

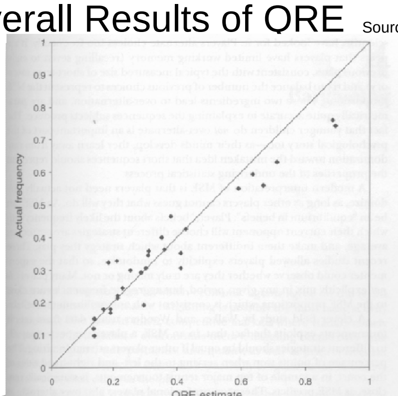

Figure 3.3. Frequencies of different strategy choices predicted by quantal response equilibrium and actual frequencies.

Conclusion

- With explicit randomization, the existence of purists hint on equilibrium in beliefs
  - Players cannot guess what opponents are doing
  - Their beliefs about opp are correct on average
  - But, they may not be randomizing themselves
- Field vs. Lab
  - Ostling, Wang, Chou and Camerer (2007), "*Field and Lab Convergence in Poison LUPI Games*," working paper