Dominance-Solvable Games

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(Lecture 4, Micro Theory I)

Dominance

- Dominance
  - Strategy A gives you better payoffs than Strategy B regardless of opponent strategy
- Dominance Solvable
  - A game that can be solved by iteratively deleting dominated strategy

Dominance

- Do people obey dominance?
  - Looking both sides to cross a 1-way street
  - "If you can see this, I can't see you."
  - p-Beauty Contest behavior (guess above 67)
- Will you bet on others obeying dominance?
  - Workers respond to incentives rationally
  - Companies don't use "optimal" contracts
- SOPH: Knowing other's steps of reasoning

Belief of Iterated Dominance

1. Obey Dominance,
2. Believe that others obey dominance,
3. Believe that others believe you'll obey dominance,
4. Believe that others believe that you believe they obey dominance,
5. Believe that others believe that you believe that they believe you obey dominance, etc...

Outline

- A Simple Test: Beard & Beil (MS 94')
- Centipede:
  - McKelvey & Palfrey (Econometrica 92')
- Mechanism Design:
  - Sefton and Yavas (GEB 96')
- Dirty Face:
  - Weber (EE 01')

A Simple Test: Beard and Beil (MS 1994)

Iterated dominance game		
Player 1 Move	Player 2 move	
	l	r
L	9.75, 3	
R	3, 4.75	10, 5

A Simple Test: Beard and Beil (MS 1994)

Treatment	Payoffs from			Frequency		# of Pairs	Threshold P(r|R)
	(L, l)	(R, l)	(R, r)	L	r|R		
1 (baseline)	(9.75,3)	(3, 4.75)	(10, 5)	66%	83%	35	97%
2 (less risk)	(9, 3)	(3, 4.75)	(10, 5)	65%	100%	31	85%
3 (even less risk)	(7, 3)	(3, 4.75)	(10, 5)	20%	100%	25	57%
4 (more assurance)	(9.75,3)	(3, 3)	(10, 5)	47%	100%	32	97%
5 (more resentment)	(9.75,6)	(3, 4.75)	(10, 5)	86%	100%	21	97%
6 (less risk,more reciprocity)	(9.75,5)	(5, 9.75)	(10,10)	31%	100%	26	95%
7 (1/6 payoff)	(58.5,18)	(18,28.5)	(60,30)	67%	100%	30	97%

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Follow-up 1: Goeree and Holt (PNAS 1999)

Condition	# of Pairs	Threshold P(r|R)	Payoffs			Frequency	
			(L)	(R, l)	(R, r)	(L)	(r|R)
Baseline 1	25	33%	(70, 60)	(60, 10)	(90, 50)	12%	100%
Lower Assurance	25	33%	(70, 60)	(60, 48)	(90, 50)	32%	53%
Baseline 2	15	85%	(80, 50)	(20, 10)	(90, 70)	13%	100%
Low Assurance	25	85%	(80, 50)	(20, 68)	(90, 70)	52%	75%
Very Low Assurance	25	85%	(400,250)	(100,348)	(450,350)	80%	80%

Follow-up 2: Schotter-Weigelt-Wilson (GEB 94)

Normal Form	Player 2		Game 1M
Player 1	l	r	Frequency
L	4, 4	4, 4	(57%)
R	0, 1	6, 3	(43%)
Frequency	(20%)	(80%)	

Sequential Form			Game 1S
L	4, 4		(8%)
	l	r	
R	0, 1	6, 3	(92%)
Frequency	(2%)	(98%)	

Normal Form	Player 2			Game 3M
Player 1	T	M	B	Frequency
T	4, 4	4, 4	4, 4	(82%)
M	0, 1	6, 3	0, 0	(16%)
B	0, 1	0, 0	3, 6	(2%)
Frequency	(70%)	(26%)	(4%)	

Sequential Form						Game 3S
T	4, 4	T				(70%)
		0, 1				
				M	B	
			M	6, 3	0, 0	(100%)
			B	0, 0	3, 6	(0%)
Frequency		(13%)		(31%)	(69%)	

Follow-up 2: Schotter-Weigelt-Wilson (GEB 94)

- Schotter et al. (1994)'s conclusion:
- Limited evidence of iteration of dominance (beyond 1-step), or SPE, forward induction
  - Can more experience fix this?
- No for forward induction in 8 periods...
  - Brandts and Holt (1995)
- But, Yes for 3-step iteration in 160 periods
  - Rapoport and Amaldoss (1997): Patent Race

Centipede Game: 4-Move SPNE

- McKelvey and Palfrey (Econometrica 1992)

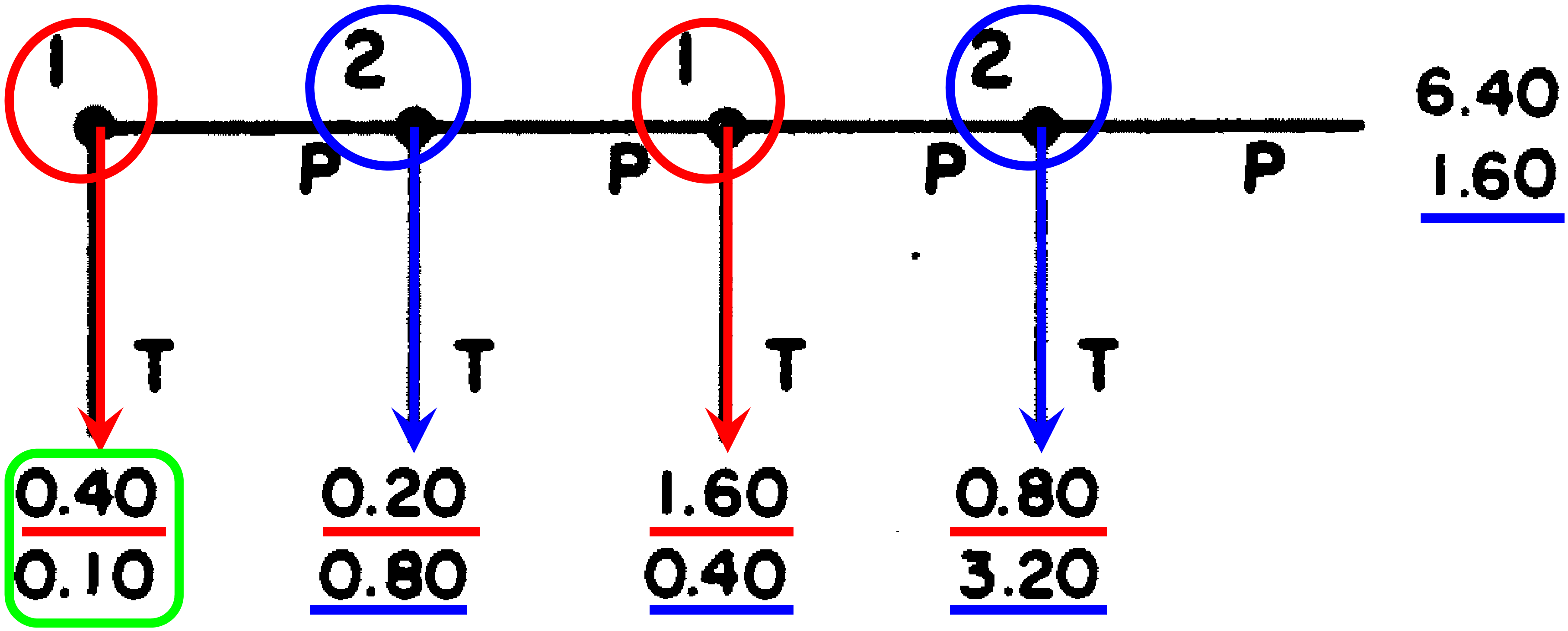


FIGURE 1.—The four move centipede game.

Centipede Game: 6-Move SPNE

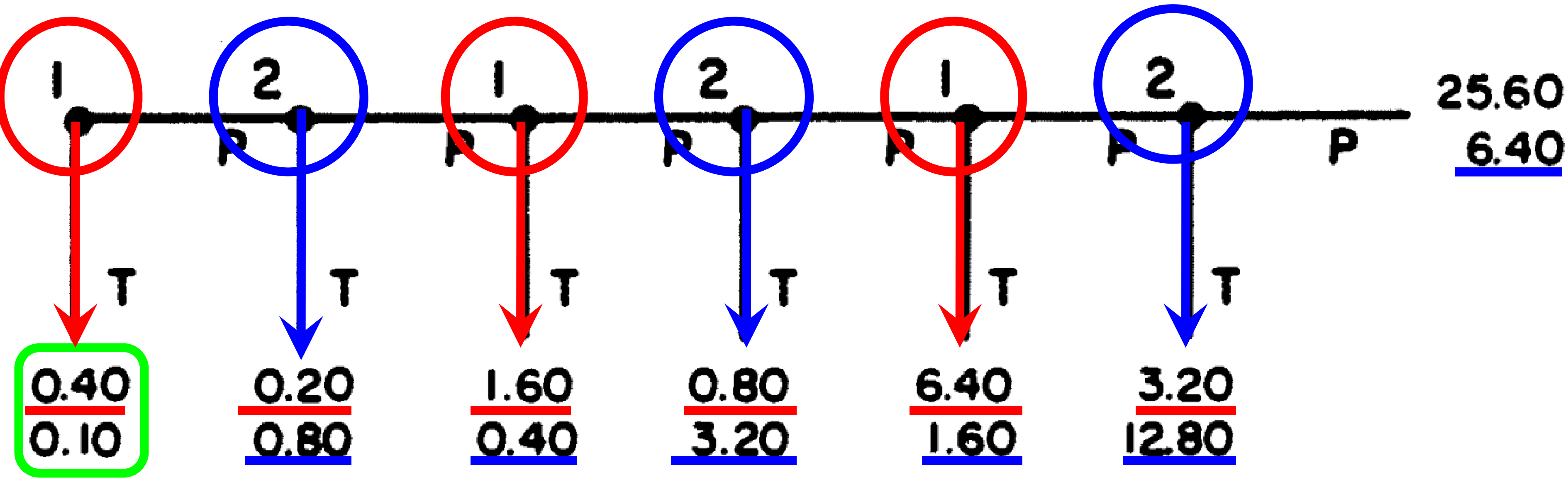


FIGURE 2.—The six move centipede game.

Centipede Game: Outcome

TABLE IIA

PROPORTION OF OBSERVATIONS AT EACH TERMINAL NODE

		Session	N	f_1	f_2	f_3	f_4	f_5	f_6	f_7
Four Move	1	(PCC)	100	.06	.26	.44	.20	.04		
	2	(PCC)	81	.10	.38	.40	.11	.01		
	3	(CIT)	100	.06	.43	.28	.14	.09		
	Total	1–3	281	.071	.356	.370	.153	.049		
High Payoff	4	(High-CIT)	100	.150	.370	.320	.110	.050		
Six Move	5	(CIT)	100	.02	.09	.39	.28	.20	.01	.01
	6	(PCC)	81	.00	.02	.04	.46	.35	.11	.02
	7	(PCC)	100	.00	.07	.14	.43	.23	.12	.01
	Total	5–7	281	.007	.064	.199	.384	.253	.078	.014

Centipede Game: Pr(Take)

TABLE IIB[a]

IMPLIED TAKE PROBABILITIES FOR THE CENTIPEDE GAME

	Session	p_1	p_2	p_3	p_4	p_5	p_6
Four Move	1 (PCC)	.06 (100)	.28 (94)	.65 (68)	.83 (24)		
	2 (PCC)	.10 (81)	.42 (73)	.76 (42)	.90 (10)		
	3 (CIT)	.06 (100)	.46 (94)	.55 (51)	.61 (23)		
	Total 1–3	.07 (281)	.38 (261)	.65 (161)	.75 (57)		
High Payoff	4 (CIT)	.15 (100)	.44 (85)	.67 (48)	.69 (16)		
Six Move	5 (CIT)	.02 (100)	.09 (98)	.44 (89)	.56 (50)	.91 (22)	.50 (2)
	6 (PCC)	.00 (81)	.02 (81)	.04 (79)	.49 (76)	.72 (39)	.82 (11)
	7 (PCC)	.00 (100)	.07 (100)	.15 (93)	.54 (79)	.64 (36)	.92 (13)
	Total 5–7	.01 (281)	.06 (279)	.21 (261)	.53 (205)	.73 (97)	.85 (26)

Centipede Game: Learning Effect (1-5 vs. 6-10)

TABLE IIIB

IMPLIED TAKE PROBABILITIES

COMPARISON OF EARLY VERSUS LATE PLAYS IN THE LOW PAYOFF CENTIPEDE GAMES

Treatment	Game	p_1	p_2	p_3	p_4	p_5	p_6
Four Move	1–5	.06 (145)	.32 (136)	.57 (92)	.75 (40)		
	6–10	.08 (136)	.49 (125)	.75 (69)	.82 (17)		
Four Move	1–5	.00 (145)	.06 (145)	.18 (137)	.43 (112)	.75 (64)	.81 (16)
	6–10	.01 (136)	.07 (134)	.25 (124)	.65 (93)	.70 (33)	.90 (10)

Centipede Game: Mimic Model

- What theory can explain this?
- Altruistic Types (7%): Prefer to Pass
- Selfish Types:
  - Mimic altruistic types up to a point (gain more)
- Unraveling: error rate shrinks over time

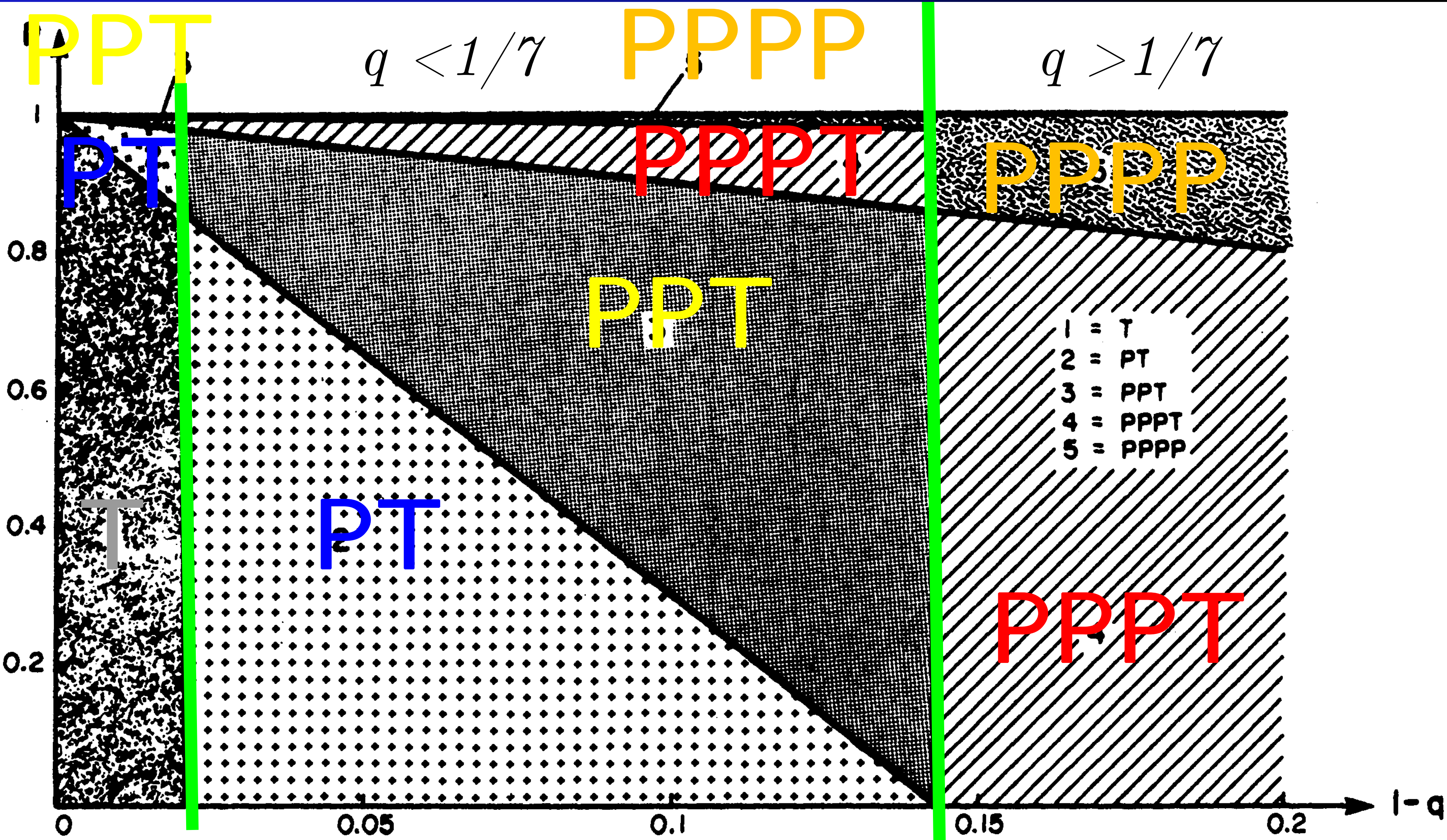
Centipede Game: Mimic Model

- Selfish players sometimes pass (mimic altruist)
- By imitating an altruist one might lure an opponent into passing at the next move
  - Raising one's final payoff in the game
- Equilibrium imitation rate depends directly on the beliefs about the likelihood $(1\text{-}q)$ of a randomly selected player being an altruist.
  - The more likely players believe there are altruists in the population, the more imitation there is.

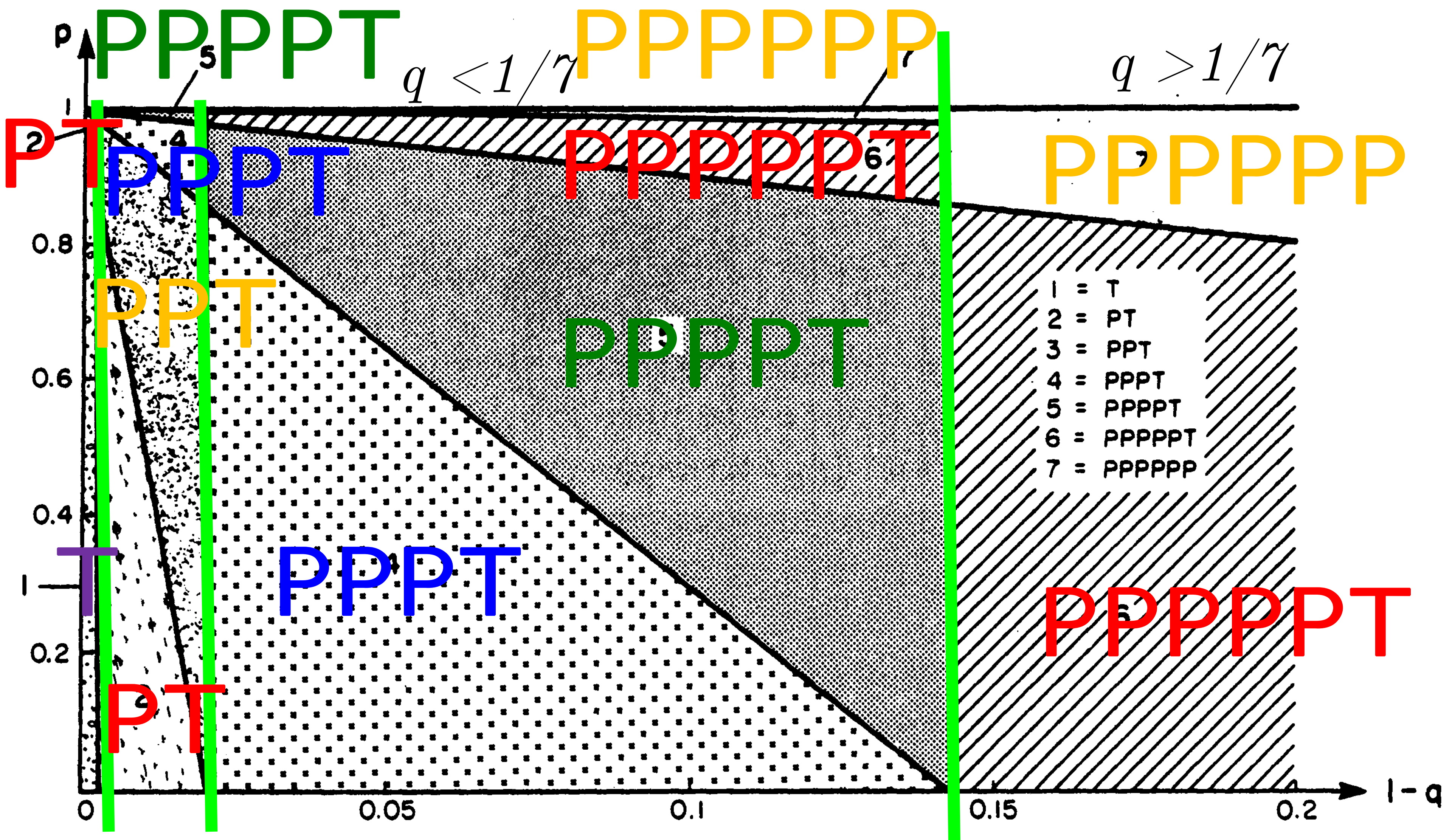
Centipede-Mimic: Predictions for Normal Types

1. On the last move, Player 2 TAKE for any q
2. If $1\text{-}q > 1/7$, both Player 1 and Player 2 PASS
   – (Except on the last move Player 2 always TAKE)
3. If $0 < 1\text{-}q < 1/7$ → Mixed Strategy Equilibrium
4. If $1\text{-}q = 0$ both Player 1 and Player 2 TAKE

Centipede - Mimic Model Equilibrium Outcome



Centipede - Mimic Model Equilibrium Outcome



Centipede Game: Mimic Model Add Noisy Play

- We model noisy play in the following way.
- In game t, at node s, if p^* is the equilibrium probability of TAKE that the player at that node attempts to implement,
- We assume that the player actually chooses TAKE with probability $(1\text{-}\varepsilon_t)p^*$, and makes a random move with probability ε_t
- $\varepsilon_t = \varepsilon e^{-\delta(t-1)}$
- Explains further deviation from mimic model...

Centipede Game: Follow-ups

- Fey, McKelvey and Palfrey (IJGT 1996)
  - Use constant-sum to kill social preferences
  - Take 50% at 1st, 80% at 2nd
- Nagel and Tang (JMathPsych 1998)
  - Don't know other's choice if you took first
  - Take about half way
- Rapoport et al. (GEB 2003)
  - 3-person & high stakes: Many take immediately
  - CH can explain this (but not QRE) – see theory

Mechanism Design

- Pure coordination game with *\$1.20 & \$0.60*
- How can you implement a Pareto-inferior equilibrium in a pure coordination games?
- Abreu & Matsushima (Econometrica 1992)
  - Slice the game into "T periods"
  - F: Fine paid by first subject to deviate
  - Won't deviate if $F > \$1.20/T$
  - Can set $T=1$, $F=\$1.20$; more credible if T large

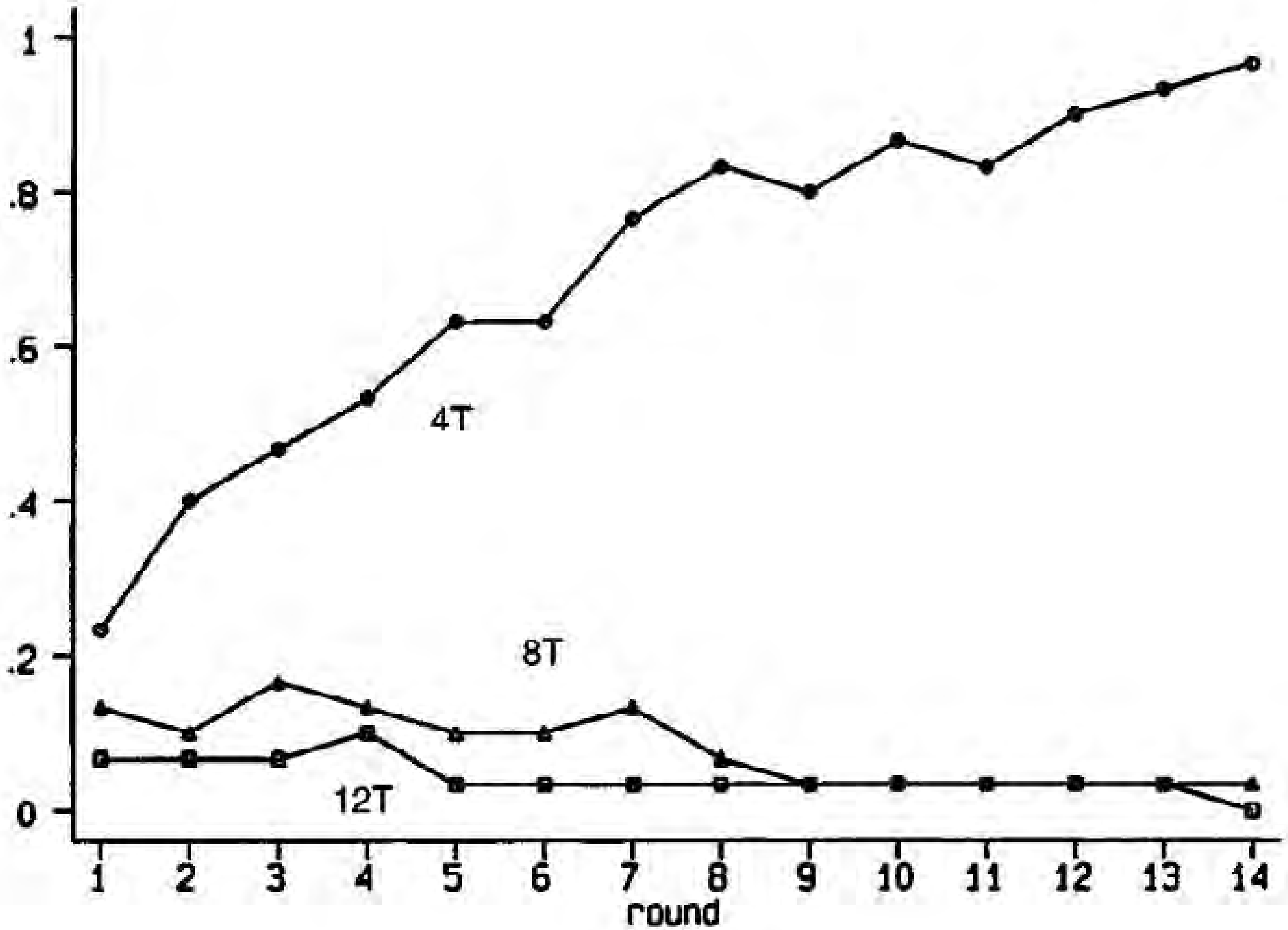
Mechanism Design

- Glazer and Rosenthal (Economtrica 1992)
  - Comment: AM mechanism requires more steps of iterated deletion of dominated strategies
- Abreu & Matsushima (Econometrica 1992)
  - Respond: "[Our] gut instinct is that our mechanism will not fare poorly in terms of the essential feature of its construction, that is, the significant multiplicative effect of 'fines.'"
- This invites an experiment!

Mechanism Design

- Sefton and Yavas (GEB 1996)
- $F=\$0.225$
- $T=4,\ 8,\ or\ 12$
  - Theory: Play inferior NE at $T=8\ or\ 12$, not $T=4$
- Results: Opposite, and diverge...
- Why? Choose only 1 switch-point in middle
  - Goal: switch soon, but 1 period after opponent

Mechanism Design



Mechanism Design

- Glazer and Perry (GEB 1996)
  - Implemental can work in sequential game via backward induction
- Katok, Sefton and Yavas (JET 2002)
  - Doesn't work either
- Can any "approximately rational explanation" get this result?
  - Maybe "Limited steps of IDDS + Learning"

Dirty Face Game

- Three ladies, A, B, C, in a railway carriage all have dirty faces and are all laughing.
- It sudden flashes on A:
- Why doesn't B realize C is laughing at her? Heavens! *I* must be laughable.
  - Littlewood (1953), "*A Mathematician's Miscellany*
- Requires A to think that B is rational enough to draw inference from C

Dirty Face Game: Weber (Exp. Econ. 2001)

- Independent types X (prob=.8) or O (prob=.2)
  - X is like "dirty face"
- Commonly told "At least one player is type X."
  - P(XX) = 0.64→2/3, P(XO) = 0.32→1/3
- Observe other's type
- Choose Up or Down (figure out one is type X)
- If nobody chooses Down, reveal other's choice and play again

Dirty Face Game

		Type	
		X	O
Probability		0.8	0.2
Action	Up	$0	$0
	Down	$1	-$5

Dirty Face Game

- Case XO: Players play (Up, Down)
- Type X player thinks...
  - I know that "at least one person is type X"
  - I see the other person is type O
- So, I must be type X → Chooses Down
- Type O player thinks...
  - I know that "at least one person is type X"
  - I see the other person is type X
- No inference → Chooses Up

Dirty Face Game

- Case XX - First round:
- No inference (since at least one is type X, but the other guy is type X) → Both choose Up
- Case XX - Second round:
- Seeing UU in first
  - the other is not sure about his type
  - He must see me being type X
- I must be Type X → Both choose Down

Dirty Face Game

		Trial 1		Trial 2	
		XO	XX	XO	XX
Round 1	UU	0	7*	1	7*
	DU	3*	3	4*	1
	DD	0	0	0	0
Round 2 (after UU)	UU	-	1	-	2
	DU	-	5	-	2
	DD	-	1*	-	3*
	Other	-	-	1	-

Dirty Face Game

- Results: 87% rational in XO, but only 53% in 2^{nd} round of XX
- Significance:
- Choices reveal limited reasoning, not pure cooperativeness
  - More iteration is better here...
- Upper bound of iterative reasoning
  - Even Caltech students cannot do 2 steps!

Conclusion

- Do you obey dominance?
- Would you count on others obeying dominance?

- Limit of Strategic Thinking: 2-3 steps
- Compare with Theories of Initial Responses
  - Level-k Types: Stahl-Wilson95, CGCB01, CGC06
  - Cognitive Hierarchy: CHC04